ADVANCED
PRACTICAL PHYSICS

(FOR B.Sc. STUDENTS OF INDIAN UNIVERSITIES)

VOL. I & II

GENERAL PROPERTIES OF MATTER, SOUND, LIGHT,
HEAT, MAGNETISM & ELECTRICITY

By

DR. S. S. SHARMA, M.SC., LL.B., P.E.S.,
Professor & Head of the Physics Department,
Govt. College, Bhopal (Madhya Pradesh)

Formerly

Professor & Head of the Physics Department,
Govt. College, Bhopal (Madhya Pradesh)

and

Lecturer in Physics
Agra College, Agra.

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PREFACE TO THE REVISED EDITION

We have great pleasure in bringing out the revised edition of the book. The book has been thoroughly revised and a number of articles have been deleted and new matter has been added. We trust and hope that the book in its present form shall prove to be more useful to the students.

July 1964 The Publisher.

PREFACE TO THE FIRST EDITION

> "Experiment is the interpreter of nature. Experiment never deceives. It is our judgment which sometimes deceives itself, because it expects results which experiment refuses."
>
> —*Leonardo da Vinci*.

We are living in a different age. There was a time when education was considered a luxury meant only for a few. It was something like polish on a gold ornament. A complete and generous education is a prime necessity now, when the competition for existence is growing every moment, the complexities of our social and economic structure are increasing day by day and the struggle for international supremacy is threatening the very civilization.

We are moving fast, in fact; too fast. While the modern inventions have made our actions responsive to touch, the man of tomorrow must know how to keep pace with them. The future man is being manufactured in our laboratories and class-rooms now. To keep pace with his future environments, he must develop in him the technique of maximum output with the minimum of time, energy, and material. The present-day education ought to help him to acquire that. A laboratory course, specially that of Practical Physics as a part of education, has a primary duty in this direction.

As a teacher of Physics at a University stage for nearly twenty years, I have always felt that the laboratory work in Physics is not what it should be, and with the ever-increasing overcrowding in degree classes the quality of performance of students is bound to deteriorate. A teacher of Physics, under the present conditions, is unable to supervise the work of unwieldy groups of students. Hence, there is a strong-felt need of a suitable text-book for degree

classes. The books available at present are either too sketchy in their treatment or are too comprehensive in the treatment of theoretical aspect of the problems discussed. Sometimes, this theoretical treatment is so elaborate that in the mathematical intricacies the student is unable to follow precisely the experimental aspect of the problems. Hence, a happy compromise has to be struck.

In this book, greater emphasis has been laid on the experimental, rather than the theoretical aspect of each experiment. Of course, the principle and the theory of each experiment essential for its successful performance has been fully described and critically discussed. As a matter of fact, before actually taking up an experiment, students should go through this part of the matter for a complete understanding of the principles underlying it. Thereafter, they should pay special attention to the intricacies involved in the skilful manipulation for accurate measurements. These points in the procedure of the experiment have been discussed somewhat in great detail. While describing the steps of procedure for the experiment, attention has been pointedly drawn to the important precautions to be observed at this stage.

The record of observations is an important part of the procedure of the experiment. For this purpose, simple and self-explanatory tables have been drawn and; by way of illustration, actual readings have been inserted in two of three experiments and steps of calculation have been methodically shown. Wherever possible, graphical methods have been introduced.

Figures of instruments and diagrams required for theoretical elucidation have been nearly and accurately drawn. They are mostly sectional and simple. Their important components have been labelled.

I am thankful to the various authors whose work I have freely consulted in the preparation of the book. I am also thankful to several of my colleagues and former students for their suggestions, criticisms, and help in the preparation of the book. I shall feel greatly obliged to those readers who will bring to my notice the shortcomings of the book and forward to me their suggestions for its improvement.

Physics Laboratory, Government College, Gyanpur. (U. P.) **S. S. Sharma.**

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Preface to the Fourth Edition

We have great pleasure in bringing out the Fourth Edition of the book. While preserving the original character of the book, the text has been thoroughly revised and enlarged to incorporate valuable suggestions from our numerous readers in the various universities and colleges. Thus a number of articles have been deleted and new matter has been added. We trust and hope that the book in its present form shall prove to be more useful and shall continue to serve the needs of the students. Suggestions for further improvement of the book shall be gratefully acknowledged.

The Publishers

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S. S. Sharma

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GENERAL PROPERTIES OF MATTER

1

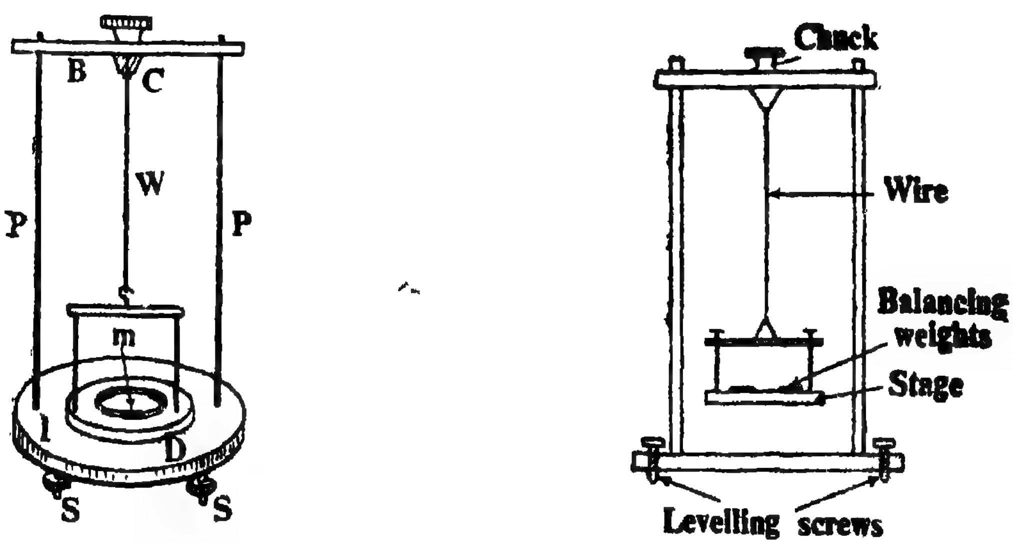
Moment of Inertia

EXPERIMENT—1

Object—To determine the moment of inertia* of a given body (*e. g.*, an annular ring) with the help of an Inertia Table and an auxiliary body (*e. g.*, a disc) whose moment of inertia can be calculated from its dimensions.

Apparatus Required—Inertia table, auxiliary body (disc), vernier callipers, physical balance, weight box, spirit level, stopwatch, and the given body (annular ring).

Description of the Apparatus—The Inertia Table (fig—1 a) consists of a circular aluminium disc D suspended by means



(a) Actual apparatus **(b) Sectional diagram**

Fig—1. Inertia Table.

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* For a detailed study of Moment of Inertia read author's book "A Critical Study of Practical Physics and Viva-Voce".

of a fairly stout wire W fixed at the top to a chuck C which is fixed to a cross-bar B between two pillars P, P standing vertically on a heavy iron base I which is provided with three levelling screws S. On the upper face of the aluminium disc are drawn several concentric circles, with the help of which a body can be easily placed symmetrically on the disc. Near the circumference of the disc there is also one concentric groove, in which three balancing weights are placed. By changing the relative positions of these weights in the groove the disc can be made horizontal. To protect the instrument from draughts the whole apparatus is generally enclosed in a glass case. The sectional diagram of the apparatus is shown in fig—1 (b).

Formula Employed—The moment of Inertia I_1 of the given body is determined with the help of the following formula :—

$$I_1 = I_2 \times \frac{T_1^2 - T_0^2}{T_2^2 - T_0^2} = \frac{Mr^2}{2} \times \frac{T_1^2 - T_0^2}{T_2^2 - T_0^2}$$

where I_2 = moment of inertia of the auxiliary body (disc) [M = mass of the disc ; r = its radius].
T_0 = time-period for the inertia table stage alone
T_1 = time-period with the given body on the stage
T_2 = time-period with the auxiliary body on the stage.

PRINCIPLE AND THEORY OF THE EXPERIMENT

If a rigid body suspended by means of fairly thin wire be given a slight rotation in a horizontal plane and then released, it begins to execute to and fro motion about the wire as axis. These oscillations of the body are known as *torsional oscillations* whose time-period T is given by the formula—

$$T = 2\pi\sqrt{I/c} \qquad \ldots \qquad \ldots \qquad (1)$$

where I is the moment of inertia of the body about the wire as axis and c is a costant* for the wire and is known as the restoring couple per unit twist produced in the wire.

Now if I_0 is the moment of inertia of the stage with its two small pillars and the cross-bar attached between them about the wire as the axis of rotation and if T_0 is the time-period for the torsional oscillations of the stage, then

$$T_0 = 2\pi\sqrt{I_0/c} \qquad \ldots \qquad \ldots \qquad (2)$$

* The value of this constant is given by the formula, $c = n\pi r^4/2l$ (see expt—7), where r is the radius and l is the length of the wire. n is a constant known as the modulus of rigidity for the material of the wire. From formula-(1) it is clear that in order to increase the value of T, the value of c should be small, *i. e.*, the value of r (radius) of the wire should be small and its l (length) should be great. *Under these circumstances accuracy in the measurement of the time-period is increased.*

Now if the given body of unknown moment of inertia I_1 is placed on the stage with its centre of gravity on, and the axis of rotation coincident with, the axis of the wire, and if T_1 be the time-period, then

$$T_1=2\pi\sqrt{\frac{I_0+I_1}{c}} \qquad \ldots \qquad \ldots \qquad (3)$$

Now this body is removed and another body of known moment of inertia I_2 *is similarly placed* on the stage, the time-period T_2 under this circumstance is given by :—

$$T_2=2\pi\sqrt{\frac{I_0+I_2}{c}} \qquad \ldots \qquad \ldots \qquad (4)$$

Squaring (2) and (3) and then dividing (3) by (2) we have

$$\frac{T_1^2}{T_0^2} = \frac{I_0+I_1}{I_0} = 1 + \frac{I_1}{I_0}$$

Hence
$$\frac{I_1}{I_0} = \frac{T_1^2}{T_0^2} - 1 = \frac{T_1^2 - T_0^2}{T_0^2} \qquad \ldots \qquad (5)$$

Similarly from equations (2) and (4), we have

$$\frac{I_2}{I_0} = \frac{T_2^2 - T_0^3}{T_0^2} \qquad \ldots \qquad \ldots \qquad (6)$$

Dividing (5) by (6) we have

$$\frac{I_1}{I_2} = \frac{T_1^2 - T_0^2}{T_2^2 - T_0^2}$$

$$\therefore \qquad I_1 = I_2 \times \frac{T_1^2 - T_0^2}{T_2^2 - T_0^2} \qquad \ldots \qquad (7)$$

Now as the given auxiliary body is a disc whose moment of inertia about an axis passing through its centre of gravity and perpendicular to its plane is equal to $Mr^2/2$, we have

$$I_1 = \frac{Mr^2}{2} \times \frac{T_1^2 - T_0^2}{T_2^2 - T_0^2} \qquad \ldots \qquad (8)$$

Thus by observing the values of the quantities occurring on the right-hand side of (8), the value of I_1 can be calculated.

[**Note**—Students should carefully note the formulae for the values of moment of inertia of the following solids :—

Circular Disc (of radius r)

(i) about an axis passing through its centre and perp. to its plane ... $\frac{1}{2}$ Mr^2

(ii) about a diameter as axis ... $\frac{1}{4}$ Mr^2

Annular Ring (of radii r_1, r_2)

about on axis passing through its centre and perp. to its plane ... $\frac{1}{2} M (r_1^2 + r_2^2)$

Right Cylinder (of length l and radius r)

(i) about its own axis ... $\frac{1}{2} Mr^2$

(ii) about an axis passing through its centre and perp. to its axis ... $M (l^2/_{12} + r^2/_4)$

Sphere (of radius r)

about a diameter ... $\frac{2}{5} Mr^2$

In the above formulae M represents the mass of the solid.]

Method—

(i) Before taking observations the adjustment of this apparatus should be carefully done. With the help of the levelling screws and a spirit level set the heavy iron base horizontal. Now with the help of a plumb-line adjust the positions of the balancing weights in the groove in such a way that *the aluminium disc becomes horizontal.* Now slightly rotate the disc in its own plane and release it, thereby inducing it to execute torsional oscillations'* With the help of an accurate stop-watch note the time for a known number of oscillations,† and from it find out the value of T_0. In order that the oscillations may be counted easily and accurately, a *reference mark* may be made on the vertical rim of the aluminium disc and a *small vertical pointer* may be placed in front of it on the experimental table.

(ii) Next place on the inertia table stage the body (the annular ring) whose moment of inertia is required, and with the help of concentric circles drawn on the disc adjust it in such a way that the horizontality of the stage is secured.† In this position the axis of the wire (which is also the axis of rotation) passes through the centre of gravity of the body.

Now set the combination oscillating and by noting the time as before calculate the time-period** T_1.

* In this process no other type of vibrations, (*e. g.*, pendular vibrations) should be produced in the stage. Remember also that in setting the stage to oscillate, the suspension wire should not be twisted beyond the elastic limit.

† The number of oscillations counted should be large—preferably twenty-five, since by noting the time four times, the mean time-period can be very easily calculated.

† In effecting the horizontality of the stage with the body on it *the balancing weights in the groove should, under no citcumstances be disturbed.* This is important.

** It is obvious that T_1 will be greater than T_0.

(iii) Remove the body and now in its place put the auxiliary body (in this case, the disc) in such a way that the axis of the wire passes through its centre of gravity. Determine as before the time-period†† T_2.

(iv) Weigh the disc and find out its diameter by means of a vernier callipers and thus calculate its moment of inertia about the axis of rotation.

Using the value of I_2 and substituting the mean values of T_0, T_1 and T_2 in the formula calculate the value of I_1.

Observations—

[A] *Measurement of the diameter of the disc*

S. No.	Observed diameter of the disc	Corrected diameter of the disc	Remarks
1.	......cm	......cm	(1) Vernier constant =...cm
2.			(2) Zero error =...cm
⋮			Mass of the disc =...cm
	Mean	......cm	

[B] *Readings for the determination of* T_0, T_1, T_2.

Least count of the stop-watch = ...sec

S. No.	No. of oscillations	Time taken			Mean		
		Inertia table alone	Inertia table + given body	Inertia table + auxiliary body	T_0	T_1	T_2
1.	25	...sec	...sec	...sec			
2.	,,				...sec	...sec	...sec
3.	,,						
4.	,,						

†† T_4 will also be greater than T_0, but nothing can be said about the relative magnitudes of T_1 and T_2.

Calculations—

Mean corrected radius of the disc = ...cm
Moment of inertia of the disc about a
vertical axis passing through its C. G. = $\frac{1}{2}Mr^2$ = ...gm-cm²

$$\text{Now} \qquad I_1 = \frac{T_1^2 - T_0^2}{T_2^2 - T_0^2} \times \frac{Mr^2}{2}$$

$$= \text{......gm-cm}^2$$

Result*—The moment of inertia of the given annular ring about a vertical axis passing through its centre of gravity as determined with the help of inertia table = ...gm-cm²

Precautions and Sources of Error

(1) After securing the horizontality of the stage of the inertia table in the first adjustment, the balancing weights in the groove should, under no circumstances, be disturbed in the subsequent determinations, otherwise the moment of inertia of the stage about the axis of rotation shall get changed.

(ii) The motion of the stage should be confined in the horizontal plane and should be torsional in character. All other types of motion should be completely checked.

(iii) The bodies should be so placed on the stage that their axis always coincides with the axis of rotation and their centre of gravity also lies on this axis.

(iv) Although in the derivation of the formula, $T = 2\pi\sqrt{I/c}$, no assumption regarding the magnitude of the angle of twist is made, yet the wire should not be twisted beyond elastic limit, otherwise the torsional couple will not be proportional to the angle of twist.

(v) Periodic times should be measured very accurately since they occur squared in the formula.

(vi) Now the auxiliary solid may not have a uniform density throughout, in that case its moment of inertia obtained from the formula $Mr^2/2$ will not be quite correct and hence will effect the final result. This will then constitute a source of error.

* The result so obtained experimentally may be verified by calculating the moment of inertia of the annular ring with the help of the formula—

Moment of inertia of the annular ring about a vertical axis passing through its C. G. = $M\,(r_1^2+r_2^2)/2$, where M is its mass and r_1, r_2 are its radii.

(vii) Secondly, the manner of counting the number of oscillations and the recording of time with a stop-watch are not susceptible of great accuracy and hence the result will also be accordingly effected.†

[**Note**—The accuracy of the result in this experiment depends chiefly upon two factors : (1) the largeness of the difference between the time-periods of the combination and the stage of the inertia table alone, and (2) the accuracy in the determination of these time-periods. The first condition is fulfilled by making the stage of the inertia table of a light metal (aluminium), so that its moment of inertia may be small as compared to that of the combination.

The second condition is realised by noting the time for a large number of oscillations with a stop-watch of small least count. Further, the values of the time-periods should be increased. This can be done by increasing the length of the wire and by reducing its radius. In practice, a wire nearly 50 cm long and 0·1 cm in diameter is found quite satisfactory for this purpose.]

EXPERIMENT—2

Object—To determine the moment of inertia of a flywheel about its own axis of rotation.

Apparatus Required—The given flywheel, thread, weight box, vernier callipers, metre scale and stop-watch.

Description of the Apparatus—A flywheel is simply a massive disc of large diameter with a long axle of comparatively small diameter. The wheel is so adjusted on its bearings that its centre of gravity lies on the axis of rotation.

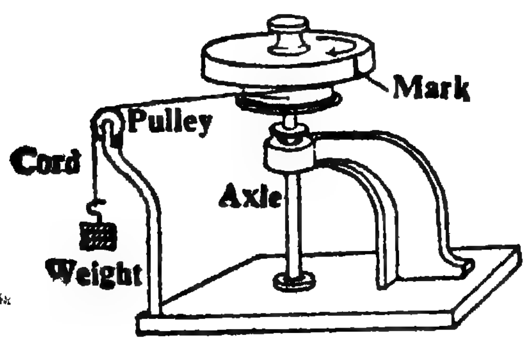


Fig. 2
Flywheel with axle vertical.

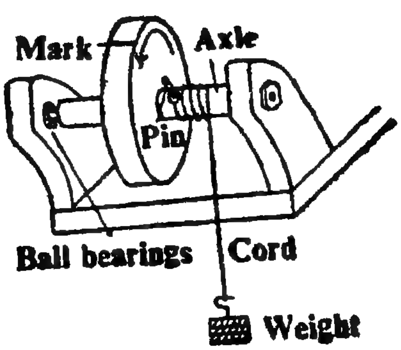


Fig. 3
Flywheel with axle horizontal.

† To obtain a very accurate value of the moment of inertia, the periodic times are noted with the help of a chronometer and the oscillations are counted with the help of telescope and scale method by fixing a tiny mirror on to the suspeosion wire.

In the above diagrams vertical and horizontal patterns of the flywheel adjusted on its bearings have been shown. At some point on the axle, or in a cylindrical rim on the wheel itself, either a small hole or a small peg is provided. A brass pin is made to fit into the hole and is tied firmly to a good length of the cord. If, instead of a hole a peg be found, a simple loop is made in the end of the cord and then slipped over the peg. The cord having been attached in one of these ways, the wheel is turned so as to wind the cord round the rim a few times. The cord is passed over a pulley if the axle of the wheel is vertical (fig.–2), or allowed to hang straight down if the axle is horizontal (fig.–3). To the free end of the cord is attached a mass of suitable magnitude. To facilitate the counting of the revolutions of the wheel, a pointer is attached to the bracket and an easily visible mark is made on the rim of the wheel.

Formula Employed*—The moment of inertia I of the flywheel is calculated by making use of the following formula :—

$$I = \frac{m\,(gh\,t^2 - 8\pi^2 n_2^2 r^2)}{8\pi^2 n_2\,(n_1 + n_2)}$$

where m = mass suspended from the cord.

h = height through which m falls before being detached from the axle.

n_1 = number of revolutions made by the wheel when the mass m falls through the height h.

n_2 = number of revolutions made by the flywheel to come to rest after the mass is detached.

t = time taken by the wheel for making n_2 revolutions.

r = radius of the axle.

PRINCIPLE AND THEORY OF THE EXPERIMENT

The idea of moment of inertia is obtained from a consideration of the kinetie energy of a rotating body. Thus it is by measurement of the kinetic energy of a rotating body that we usually measure its moment of inertia. The experimental determination of the moment of inertia I of a body is usually carried out by giving

* Since the flywheel has a large moment of inertia, the kinetic energy of the descending mass may be neglected in comparison with the energy of the wheel. Under this condition the formula given here takes the simplified form :—

$$I = \frac{mgh\,t^2}{8\pi^2 n_2\,(n_1+n_2)}$$

This formula may be employed in actual practice. **With this formula there is no need of measuring the radius (r) of the axle.**

to the body a definite or measurable quantity of energy E, and by measuring the resulting angular velocity ω from the relation :—

$$E = \tfrac{1}{2} I\omega^2 \quad \ldots \quad \ldots \quad (1)$$

The energy imparted to the wheel is supplied by the fall of a known weight through a measured height, and the resulting angular velocity is determined by finding the time taken by the wheel to perform a number of revolutions which can be easily counted.

If now the mass be allowed to fall, it will lose its potential energy, and the potential energy so lost will be converted partly into kinetic energy of translation due to the motion acquired by the falling mass itself and partly into kinetic energy of rotation of the flywheel. If we neglect the frictional losses for the time being, we may state from the principle of the conservation of energy that

Potential Energy lost by the falling mass = Kinetic Energy gained by the mass + Kinetic Energy gained by the wheel

Now if the mass suspended be m gms and if it fall through a vertical distance h cm before the string is released from the wheel, the potential energy is mgh ergs. Just as the end of the string is pulled off from the rim, the mass has acquired, say, a velocity equal to v cm per second, and the wheel an angular velocity ω radians per second. The kinetic energy of the falling mass at this instant is thus $\frac{1}{2} mv^2$ and the kinetic energy of rotation of the flywheel is $\frac{1}{2} I\omega^2$. Thus, *neglecting friction*, we have

$$mgh = \tfrac{1}{2} mv^2 + \tfrac{1}{2} I\omega^2 \quad \ldots \quad \ldots \quad (2)$$

Determination of h—The most convenient way of getting an accurate value of h is to arrange the length of the cord so that the end separates from the wheel just as the bottom of the falling mass touches the ground. If the mass be started with its base in level with the table, the height through which it falls while attached to the wheel is equal to the height of the table above the floor.

Determination of v **and** ω—After the string has become detached from the wheel, the wheel continues to revolve for a considerable time. Its angular velocity, however, decreases on account of friction and eventually the wheel comes to rest again. If the friction be assumed constant, the wheel will be retarded uniformly, and the *average angular velocity* taken over the whole time required to come to rest will be equal to *one-half* the initial angular velocity ω. If the wheel makes n_2 revolutions after the string has become detached, and takes t seconds to come to rest, the average angular velocity, while coming to rest, is given by

Average angular velocity, $\omega' = 2\pi n_2/t$ radians per sec.

Therefore ω, the angular velocity at the moment when the string gets detached is given by

$$\omega = 2\omega' = \frac{4\pi n_2}{t} \text{ radians/sec.}$$

Having found ω, v is calculated by means of the relation :—

$$v = r\omega$$

where r is the radius of the cylindrical rim (or of the axle) round which the cord is wound.

Correction for Friction—If the friction of the supporting bearings is considerable, it must be accounted for. Suppose a certain amount of work W is done against friction every time the wheel revolves *once*, and therefore an amount of work n_1W is done against friction, where n_1 is the number of revolutions performed by the wheel when the mass m falls through the height h. Hence equation (2) is modified to

$$mgh = \tfrac{1}{2} mv^2 + \tfrac{1}{2} I\omega^2 + n_1W \quad \ldots \quad \ldots \quad (3)$$

since the work n_1W is done while the mass m is losing its potential energy.

Now, after the string is detached from the flywheel, the wheel possesses a certain amount of kinetic energy ($= \frac{1}{2} I\omega^2$). This energy is gradually lost in overcoming friction, the whole amount being absorbed in a certain number of revolution n_2. Hence

$$\tfrac{1}{2} I\omega^2 = n_2W$$

or

$$W = \frac{1}{2} \times \frac{I\omega^2}{n_2}$$

and therefore

$$n_1 W = \frac{n_1}{n_2} \times \tfrac{1}{2} I\omega^2 \quad \ldots \quad (4)$$

Substituting the value of n_1 W from (4) in relation (3), we have

$$mgh = \tfrac{1}{2} mv^2 + \tfrac{1}{2} I\omega^2 + \frac{n_1}{n_2} \times \tfrac{1}{2} I\omega^2$$

$$= \tfrac{1}{2} mv^2 + \tfrac{1}{2} I\omega^2 \left(1 + \frac{n_1}{n_2}\right)$$

the friction correction being represented by the term n_1/n_2 inside the bracket. Thus, by solving the last relation further, we have

$$I = \frac{2\,mgh - mv^2}{\omega^2\left(1 + \frac{n_1}{n_2}\right)}$$

But

$$v = r\omega = \frac{4\pi n_2 r}{t}$$

$$\therefore \qquad I=\frac{m(ght^2-8\pi^2n_2^2r^2)}{8\pi^2n_2(n_1+n_2)} \qquad \ldots \qquad \ldots \qquad (5)$$

This formula is employed for calculating the moment of inertia of the given flywheel.

Method—

(i) Take a string whose length may just be equal to the height of the axle from the floor. Make an ordinary loop at its one end and slip it on to the small peg* on the axle of the flywheel and wrap the string completely and *uniformly* round the axle counting the number of turns wound. To the order end of the thread tie a *suitable* mass†.

(ii) Let the mass be now released. Count the number of revolutions‡ (n_1) which the wheel makes before the loop comes off the peg and the mass is detached from the axle.

As soon as the mass is detached from the axle, start the stop-watch and count the number of revolutions (n_2) which the wheel makes before finally coming to rest. Stop the stop-watch immediately and thus determine the time (t) for which the wheel continues to rotate after the detachment of the mass from its axle.

(iii) Now with the help of a vernier callipers measure the diameter** of the axle at a number of points along two mutually perpendicular directions and thus calculate the mean radius of the axle.

Measure the length (correct to a mm. only)†† of the cord with

* Sometimes instead of the peg a hole is provided on the axle. Into this hole can be inserted a brass pin which will serve the purpose of a peg to which the thread can be tied.

† The mass should not be so great that the wheel moves very fast and the counting of rotations becomes difficult. At the same time the mass should not be so small that the wheel requires a push to set it in motion.

‡ The number of revolutions so counted must be equal to the number of turns of the cord wound on the axle.

** These readings should be neatly entered in a tabular form, in which the constants of the vernier callipers should also be inserted.

†† The radius of the axle should be measured more accurately than the length of the string, since the former is a much smaller quantity than the latter. Moreover, r^2 occurs in the formula, hence any mistake committed in the determination of r will double itself in the result. It is due to this reason that t should also be observed very carefully and n_2 should be counted more accurately than n_1.

the help of a metre scale and thus determine h, the distance through which the mass descends.

(iv) For the same value of m and h take at least three sets of observations for n_1. n_2 and t and employ their mean values for the evaluation of the moment of inertia of the flywheel. Repeat the experiment in this way with different masses and strings of different lengths and find out* the mean value of the moment of inertia of the flywheel about its axis of rotation.

Observations—

[**A**] *Measurement of the diameter of the axle.*

S. No.	Reading along any diameter	Reading along a perp. diameter	Mean diameter	Remarks
1	.........cm	.........cm	......cm	(i) Vernier const. = . cm (ii) Zero error = ...cm
⋮				
		Mean	......cm	

[**B**] *Measurement of* h, n_1, n_2 *and* t

S. No.	m	h	n_1	n_2	t	Mean n_1	Mean n_2	Mean t
1	...gm	...cm	......	......	...sec	......	... ..	...sec
2								
3								

Calculations—

Mean observed diameter of the axle = ...cm
Mean corrected diameter of the axle = ...cm
∴ Mean radius of the axle = ...cm

* Make use of logarithmic tables for calculation work (see example given on the next page) and express the result in gm-cm².

Now $$I = \frac{m\,(ght^2 - 8\pi^2 n_2^2\, r^2)}{8\pi^2 n_2\,(n_1 + n_2)}$$

$= \ldots\ldots\ldots$ gm-cm^2

Result—The moment of inertia of the given flywheel about its axis of rotation $= \ldots\ldots$ gm-cm^2.

[**Example**—In a certain experiment for the determination of the moment of inertia of a flywheel, the following readings were obtained :—

(i) $m = 500$ gm., (ii) $h = 92{\cdot}5$ cm, (iii) $n_1 = 15$,
(iv) $n_2 = 18$, (v) $t = 16{\cdot}3$ sec, (iv) $r = 0{\cdot}95$ cm.

$$\text{Now } I = \frac{500\,(981 \times 92{\cdot}5 \times 16{\cdot}3^2 - 8 \times 3{\cdot}14^2 \times 18^2 \times 0{\cdot}95^2)}{8 \times 3{\cdot}14 \times 18\,(15 + 18)}$$

(a) Solving the Numerator :—

log 500 = 2·6990		log 500 = 2·6990	
log 981 = 2·9917		log 8 = 0·9031	log 3·14
log 92·5 = 1·9661	log 16·3	2log 3·14 = 0·9938	= 0·4969
2log 16·3 = 2·4244	= 1·2122	2 log 18 = 2·5106	log 18
Sum = 10·0812			= 1·2553
		2log 0·95 = $\bar{1}$·9554	log. ·95
		Sum = 7·0619	= $\bar{1}$·9777

[**Note**—Now it will be clear to the students that the number corresponding to the logarithm 7·0619 will be much smaller than the number corresponding to 10·0812. **Hence in an actual experiment the second term in the numerator may be neglected.**

However, if this term is also retained, *be cautious in further simplifying the numerator*.]

(b) Solving the Denominator

log 8 = 0·9031	
2 log 3·14 = 0·9938	log 3·14 = 0·4969
log 18 = 1·2553	
log 33 = 1·5185	
Sum = 4·6707	

Now log I = 10·0812 − 4·6707 = 5·4105

∴ I = Antilog 5·4105 = $2{\cdot}573 \times 10^5$ gm-cm^2]

Precautions and Sources of Error—

(i) The loop slipped over the peg should be sufficiently loose otherwise when the string has unwound itself there may be a tendency for the string to rewind itself in the opposite direction.

(ii) The diameter of the string should be negligible in comparison to that of the axle. If the string is of appreciable thickness, its radius should be added to the radius of the axle to get the effective value of r.

(iii) The diameter of the axle should be measured at a number of points along its length and at each point readings of diameters in two mutually perpendicular directions should be observed.

(iv) The stop-watch should be started immediately the string leaves the axle.

(v) Care should be taken that the flywheel starts of its own accord and no push is imparted to it. The mass tied to the end of the cord should be such as is capable to overcome friction at the bearings and thus it automatically starts falling.

(vi) In this method the exact moment when the string detaches itself from the axle cannot be accurately ascertained and hence the values of n_1, n_2 and t cannot be determined with sufficient accuracy. This constitutes a source of error.

(vii) In the derivation of the above formula it has been assumed that the frictional force remains constant while the angular velocity of the flywheel changes from ω to zero. But as the force of friction depends on the magnitude of the velocity (being less at higher velocity), this assumption is unwarranted.*

For acccurate result the angular velocity of the flywheel is calculated by a method, (*e. g.*, by using a tuning fork) in which no such assumption is made.

2

Acceleration Due to Gravity

EXPERIMENT—3

Object—To study the variation of T with h for a bar-pendulum and to determine,

(i) the radius of gyration of the bar about an axis passing through its centre of gravity and perpendicular to its plane ;
and (ii) the value of acceleration due to gravity at......*

Apparatus Required—The bar-pendulum, knife-edge for suspending the pendulum, metre scale. spirit level and a stop-watch.

Description of the Apparatus—A simple form of compound pendulum designed by D. Owen in 1939 is shown in the accompanying figure. It consists of a metallic bar nearly a metre long, in which a series of circular holes of nearly 5 mm. diameter are bored at equal distances (nearly 2 cm.) along its length. With the help of these holes the bar can be suspended from a knife-edge and made to oscillate. The knife-edge† is fixed in a platform supported on three screws ; the hinder one of which is adjustable, thereby the platform can be made horizontal.

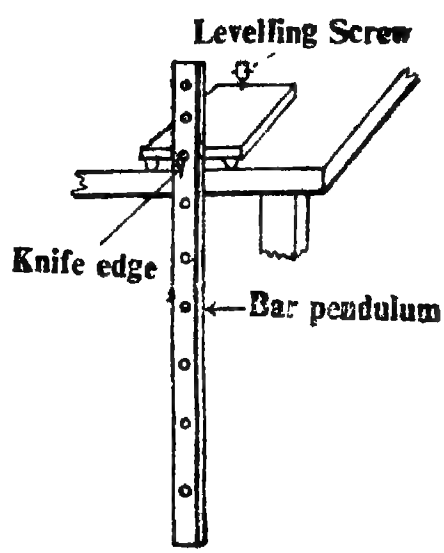


Fig. 4
Bar Pendulum

Formula employed—The value of g can be calculated from any of the following formulae :—

$$g = \frac{4\pi^2}{T_0^2} \times (2k) \qquad \dots \qquad (1)$$

---

* Name the place where the experiment is being conducted.

† There can be slight variations in the manner of suspending the pendulum with the knife-edge.

The minimum time-period T_0, and 2k (where k is the radius of gyration of the bar about an axis passing through its centre of gravity and parallel to the axis of rotation) can be read from the graph.

Also, $$g = 4\pi^2 \times \frac{h_1 + h_2}{T^2} \qquad (2)$$

where T = time-period

$h_1 + h_2$ = length of the equivalent simple pendulum.

Both of these quantities can be read off from the graph.

The radius of gyration can also be calculated from the formula :—

$$k = \sqrt{h_1 \times h_2} \qquad \ldots \qquad \ldots \qquad (3)$$

PRINCIPLE AND THEORY OF THE EXPERIMENT

If the compound pendulum be allowed to oscillate about a horizontal knife-edge passing successively through each hole, and a graph be plotted taking the periods of oscillation as ordinates and corresponding distances* of the axis of suspension from the centre of gravity of the bar as abscissae, a graph of the type shown below will be obtained (see Fig—5).

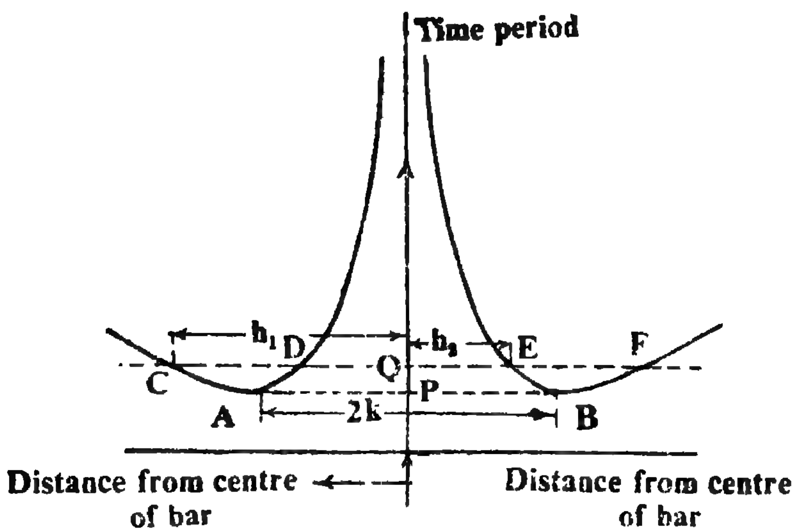


Fig. 5
T—h graph for a compound pendulum.

Instead of measuring these distances from the centre of gravity of the bar, *they can be measured from one end of the bar* and the graph plotted. A similar graph will be obtained. In this graph, the abscissa of O (the intersection of the time-axis with the distance-axis) will give the position of the centre of gravity of the bar pendulum.

Now we know that the time-period T of a compound pendulum is given by :—

$$T = 2\pi\sqrt{\frac{k^2 + h^2}{hg}}$$

where, h is the distance of its centre of gavity from the point of suspension, and k is the radius of gyration about a parallel axis passing through the centre of gravity.

When the axis of suspension passes through the centre of gravity (*i. e.*, when $h = 0$), the periodic time becomes infinitely great. If the axis is at an infinite distance the periodic time is again infinite. Consequently, there must be some intermediate position for which the periodic time is a minimum. Now T will be a minimum when $(k^2 + h^2)/h$ is minimum. But,

$$\frac{k^2 + h^2}{h} = \frac{(k - h)^2 + 2kh}{h} = \frac{(k - h)^2)}{h} + 2k$$

This is clearly a minimum when $k = h$.

Thus the minimum time-period

$$T_0 = 2\pi\sqrt{\frac{k^2 + k^2}{kg}} = 2\pi\sqrt{\frac{2k}{g}} \qquad \ldots (1)$$

From the graph, $\qquad T_0 = OP$

and $\qquad 2k = AB \qquad \ldots (2)$

Now any line drawn parallel to the distance axis will cut, in general, the curve in four points such as C, D, E and F which are situated symmetrically about the time-axis. If T be the periodic time corresponding to these points, then

$$T = 2\pi\sqrt{\frac{h_1 + h_2}{g}} \qquad \ldots (3)$$

where $\qquad T = OQ$

$h_1 = QC = QF$

$h_2 = QD = QE$

or $\qquad h_1 + h_2 = CE = DF$

Again, $k = \sqrt{h_1 h_2} = \sqrt{QC \times QD} = \sqrt{QF \times QE} \qquad \ldots (4)$

From (1) and (3) we can calculate* the value of g, while from (2) and (4) we can obtain the value of k.

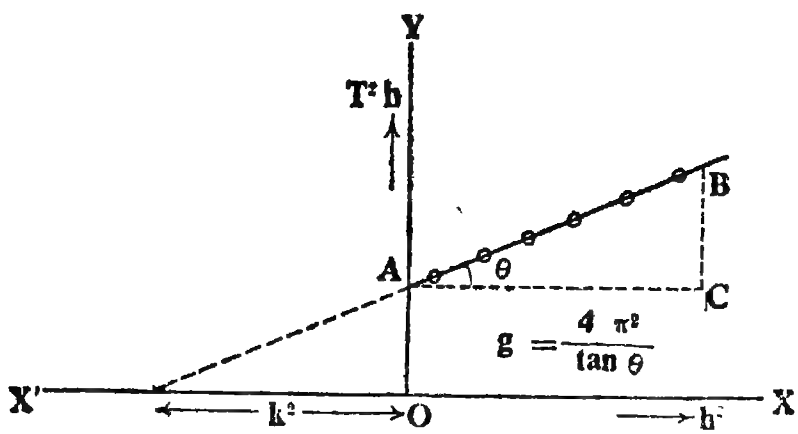
* It is often very difficult to locate the exact positions of the minima points on the graph, hence it is advisable to use equation (3) for the calculation of g and equation (4) for the calculation of k.

[**Note**—An alternative method to find the mean value of g and k is as follows :—

The equation $T = 2\pi\sqrt{\frac{k^2 + h^2}{hg}}$ can be written as

$$T^2h = \frac{4\pi^2}{g}h^2 + \frac{4\pi^2}{g}k^2$$

By plotting T^2h against h^2, a curve (straight line) as shown in



Fig—6
Straight line graph for a bar pendulum.

fig—6 is obtained. The slope (= BC/AC) of this curve is given by

$$\tan\theta = \frac{4\pi^2}{\;}$$

Hence, $$g \quad \frac{4\pi^2}{\tan\theta}$$

The intercept on the x-axis gives directly the value of k^2 whence the radius of gyration (k) can be evaluated.]

Method—

(i) First of all make the knife-edge horizontal with the help of the levelling screws provided with the platform and test the horizontality with a spirit level. Suspend the pendulum about the knife-edge from the hole nearest to one end. Tie a loop of cotton thread on the bar at its lower end and tie a fairly long piece of thread to this loop. Now with the help of this thread *displace the bar slightly* to one side and keep it in this position by attaching the thread to a stand kept nearly. Burn the thread when the bar will be set free and begin to oscillate about the knife-edge *in a vertical*

plane. Note the time with a stop-watch for a known number of oscillations* and from this calculate the periodic time.

(ii) In this way determine the periodic time of the bar when it is successively suspended† from the holes.

(iii) Now with the help of a metre scale measure the distance of the positions of the knife-edge from one end of the bar. Then plot a graph between the periodic times (T) and the corresponding distances of the points of suspension from the end of the bar, for drawing the graph take the time-periods as ordinates and the distance of the points of suspension from the end of the bar as abscissae. The graph will be similar to one shown in Fig—5 and will consist of two symmetrical branches.

(iv) Join A and B (see Fig—5). The line AB cuts the T-axis at P. The abscissa of P gives the position of the centre of gravity of the bar. Draw any line CDEF perpendicular to the T-axis and cutting the curve in four points C, D, E and F, Measure QC and QF (the mean of which gives h_1), QD and QE (the mean of which gives h_2), and also OQ (which gives the corresponding periodic time T). Then from the formula (3) given above calculate the value of g.

Again, with the help of formulae (2) and (4) given above calculate the value of k, the radius of gyration of the bar.

Observations—

Readings for the Measurement of h *and* T.

Least count of the stop-watch = ...sec.

No. of the hole	Distance of point of suspension from one end	No. of oscillations	Time taken		Periodic time (T)
			min.	sec.	
1	.........cm	25	...	...	
		25	...	...	......sec
		25	...	...	
		25	...	...	
2					
⋮					
⋮					

* To facilitate the counting of oscillations correctly, place a pointer (or make a mark on the wall behind the bar) coincident with the mean position of the pendulum.

† As the pendulum is made to oscillate from the hole which are

(*See next page*)

Calculations—

From the graph

(i) $\begin{cases} QC = \ldots cm. \\ QF = \ldots cm. \end{cases}$ $\therefore$ Mean $h_1 = \ldots cm,$

$\begin{cases} QD = \ldots cm. \\ QE = \ldots cm. \end{cases}$ $\therefore$ Mean $h_2 = \ldots cm.$

$\therefore h_1 + h_2 = \ldots\ldots\ldots cm.$

Also $T = OQ = \ldots\ldots\ldots sec.$

$$\therefore \quad g = 4\pi^2 \times \frac{h_1 + h_2}{T^2}$$

$$= \ldots\ldots\ldots cms/sec.^2$$

(ii) $2\,k = AB = \ldots\ldots\ldots cm.$

$T_0 = OP = \ldots\ldots\ldots sec.$

$$\therefore \quad g = 4\pi^2 \times \frac{2k}{T^2}$$

$$= \ldots\ldots\ldots cms/sec^2$$

(iii) $k = \sqrt{h_1\, h_2}$

$= \ldots\ldots\ldots cm.$

Also $k = \frac{1}{2}\,AB = \ldots\ldots\ldots cm.$

Result—The graph depicting the relation between the time-period of a compound pendulum and the distance of the point of suspension from one end of the bar is attached herewith.

(i) The radius of gyration of the bar about an axis through its centre of gravity and perpendicular to its plane $= \ldots\ldots\ldots$ cms.

(ii) The value of g at ...... $= \ldots\ldots$ cm/sec.^{2}

[Standard value of g $= \ldots\ldots$ cm/sec.^{2}

$\therefore$ Error $= \ldots\%$]

[**Note**—Calculate the value of 'g' and k by drawing the straight line graph as shown in fig—6. This method is very instructive.]

Precautions and Sources of Error—

1. Before starting the experiment make the knief-edge horizontal. This adjustment will keep the pendulum oscillating in a vertical plane, and secondly the bar shall not tend to slip off the knife-edge during oscillations.

(*Cont. prev. page*)

near to the centre, the time-period increases, hence very few oscillations can be timed in these positions. When the bar is suspended from the central hole it may not even be set to vibration, since its weight does not have any moment about the fulcrum which now passes through the centre of gravity of the bar.

2. In the theoretical deduction of the formula it has been assumed that sin $\theta=\theta$, where θ is the angular deflection of the bar. Hence to satisfy this condition the bar should not be displaced more than 5° from its mean position.

3. To get very exact points on the curve in the vicinity of the minimum period, the time should be observed very carefully. Try to note the time for 100 oscillations except for points very close to the centre of gravity of the bar, where, due to large time-periods, few oscillations can be timed.

4. Before taking observations see that the pendulum is oscillating in the vertical plane only, and that all other irregular motions, if any, have subsided.

5. The curves on the graph should be smoothly drawn.

6. The manner of observing the oscillations is far from satisfactory ; secondly the time-period has not been corrected for (i) finite arc of swing, (ii) air effects, (iii) curvature of knife-edge, and (iv) yielding of support, hence the result is not free from errors due to these causes.

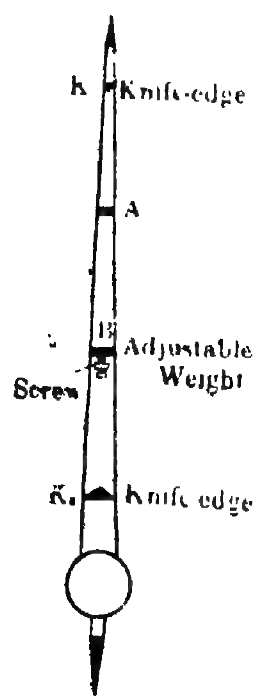
EXPERIMENT—4

Object—To determine the acceleration due to gravity at ....* with the help of a Kater's pendulum.

Apparatus Required—Kater's pendulum, a simple pendulum, a stop-watch, a telescope, and a reading microscope.

Description of the Apparatus—Kater's reversible pendulum consists of a long metallic rod which is sufficiently heavy at one end so that the centre of gravity of the system is much nearer one end than the other. It is provided with two fixed or movable knife-edges K_1 and K_2 about which the pendulum can be supported and made to vibrate like an ordinary pendulum. On the rod are provided two adjustable weights A and B to make the periods of the pendulum about the two knife-edges the same, the smaller weight B is used for finer adjustments only. This adjustments is done with the help of a screw provided with B.

This pendulum was constructed by Captain Kater of England more than a century ago and is employed for very accurate determinations of g.



Fig—7
Kater's Pendulum

* Name the place where the experiment is being conducted.

pendulums oscillate together, as seen in the field of view of the telescope.

Now by inverting the compound pendulum, oscillate it about the other knife-edge, and check that both the pendulums still swing together.

(iii) Having thus assured that the time-periods T_1 and T_2 are very nearly equal, proceed to measure them accurately. For this purposê remove the simple pendulum and set the compound pendulum oscillating by slightly displacing it from its mean position. As soon as the vertical line on the piece of paper crosses the vertical cross wire, begin counting the oscillations and note the time with an accurate stop-watch for a large number of oscillations (say, 100). Repeat the process a number of times and calculate the mean periodic time T_1. Similarly, determine the mean time-period T_2 about the other knife-edge. These two time-periods shall be very nearly equal.

(iv) Now balance the pendulum on a *sharp* wedge and mark the position of its centre of gravity. Then place the pendulum alongside a standard scale in such a manner that the knife-edges and the marks on the scale can be simultaneously focussed by a travelling microscope. Determine the distance of the knife-edge K_1 from the nearest scale mark as well as the distance of the knife-edge K_2 from the nearest scale mark to it. Thus calculate the accurate value of $(h_1 - h_2)$ by noting the distance between the two reference marks on the scale and then adding (or subtracting) to it the distances of K_1 and K_2 from the respective marks as found with the help of the microscope. Then determine approximately the distances h_1 and h_2 of K_1 and K_2 respectively from the centre of gravity, and then substituting their values in the formula (5) given above, calculate the value of g.

Observations—

[A] *Readings for the determination of Periodic Times.*

Least count of the stop-watch = ...sec.

S. No.	No. of oscillations	Time about one kinife-edge	Time about the other knife-edge	T_1	T_2
1.	100	...min...sec.	...min...sec.	...sec.	...sec.
⋮					
			Mean	...sec.	...sec.

[B] ***Measurement of h_1 and h_2.***

(i) Least count of the microscope = ...cm.

(ii) Position of the knife-edge K_1 = ...cm.

(iii) Position of the knife-edge K_2 = ...cm.

(iv) Position of the C.G. of the bar = ...cm.

Calculations—

$$h_1 = \ldots\text{cm.}$$
$$h_2 = \ldots\text{cm.}$$
$$\therefore\ h_1 - h_2 = \ldots\text{cm.}$$

Also, $$h_1 + h_2 = \ldots\text{cm.}$$

Now, $$g = \frac{8\pi^2}{\left[\dfrac{T_1^2 + T_2^2}{h_1 + h_2} + \dfrac{T_1^2 - T_2^2}{h_1 - h_2}\right]}$$
$$= \ldots\ldots\text{cm/sec}^2$$

Result—The value of the acceleration due to gravity at*...as determined with the help of a Kater's pendulum = ...cm per sec per sec.

[Standard value = ...cm/sec^2
∴ Error = ...%]

Precautions and Sources of Error—

(1) If the knife-edges are movable they should be fixed and adjusted parallel to each other.

(2) In the derivation of the formula for the periodic time of the pendulum it has been assumed that $\sin\theta = \theta$, where θ is its angular deflection. Hence the amplitude of vibration should not exceed 5°.

(3) The motion of the two pendulums should be confined to a vertical plane only. All other types of motion should be eliminated.

(4) While focussing the vernier microscope, the graduated scale surface should be coplanar with the edges of the knife-edges and take care of the back-lash error.

(5) Special attention should be paid to the accurate determination of the periodic times of the pendulum. For this purpose, a stop-watch reading upto one-tenth of a second should be employed and time for a large number of oscillations should be recorded.

* Mention here the name of the place where the experiment has been performed.

(6) As in the derivation of the formula no account has been taken for the effects produced by (a) air drag, (b) finite arc of the swing, (c) curvature of the knife-edges and (d) the yielding of the support, hence the value of g obtained will not be absolutely accurate.

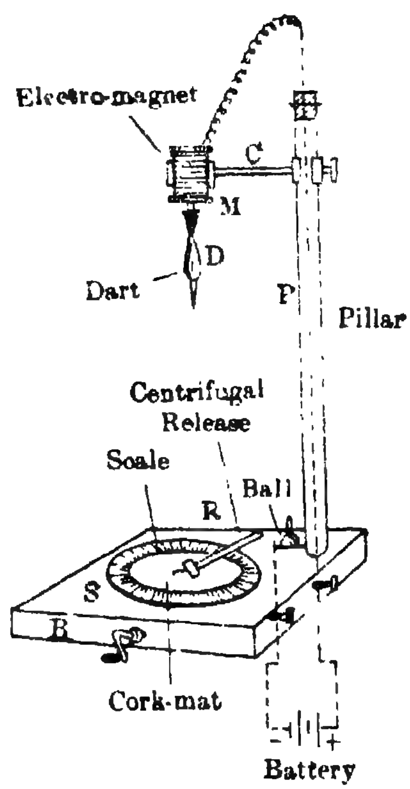
(7) If the scale used for the measurement of $(h_1 + h_2)$ is not a standard one, the graduations will be unreliable and thus there can be an error in its measurement. Similary, if there is any error in the graduations of the stop-watch, the result will be adversely effected.

EXPERIMENT—5

Object—To determine the acceleration due to gravity at...with the help of Microid's apparatus.

Apparatus Required—Microid's apparatus, a four-volt battery a metre scale, and a stop-watch.

Description of the Apparatus—Microid's apparatus is shown in the accompanying figure. It consists of a gramophone motor, whose turn-table is fitted with a cork-mat carrying a circular scale (S) divided into a fairly large number of divisions. At one corner of the box containing the motor is fixed a vertical iron pillar (P) which is four to five feet long. A sliding contact (C) on the pillar carries at its free end an electro-magnet (M) whose height about the turn-table can be varied. When the electro-magnet is excited with a battery (Ba), it supports at its lower end a specially constructed dart (D). The electrical circuit of the electro-magnet contains two metallic plates which are separated from each other by an air gap, which can be closed by a ball resting on a hole. The circuit is automatically broken with the help of a centrifugal release (R) which is fitted on the scale. When the motor acquires a constant speed the centrifugal release flies over and displaces the ball, thereby breaking the electrical circuit. The magnet consequently loses its magnetism, thereby releasing the dart which falls freely under gravity and its pointed end pierces the cork-mat attached below. When the ball is released from its seat it is automatically trapped by a device incorporated in the apparatus.



Fig—8
Microid's Apparatus.

Formula Employed—The value of g is calculated with the help of the formula :—

$$g = \frac{2h}{t^2}$$

where h is the height of the dart above the cork-mat, and t is the time taken by the dart in falling through this distance.

PRINCIPLE AND THEORY OF THE EXPERIMENT

This method is based on the direct use of the *second equation of motion.* We know that if u be the initial velocity of a moving particle, which is subjected to a uniform acceleration f, then the distance, s travelled by the particle in time t is given by

$$s = u\,t + \tfrac{1}{2} f\,t^2$$

Now, if a body falls freely from rest ($u=0$) through a vertical distance h in time t, then the above equation takes the following form :—

$$h = \tfrac{1}{2}\,g\,t^2$$

or $$g = \frac{2\,h}{t^2}$$

Thus, if we can measure the time t of the vertical fall h we can calculate the value of g with the help of this equation. In the

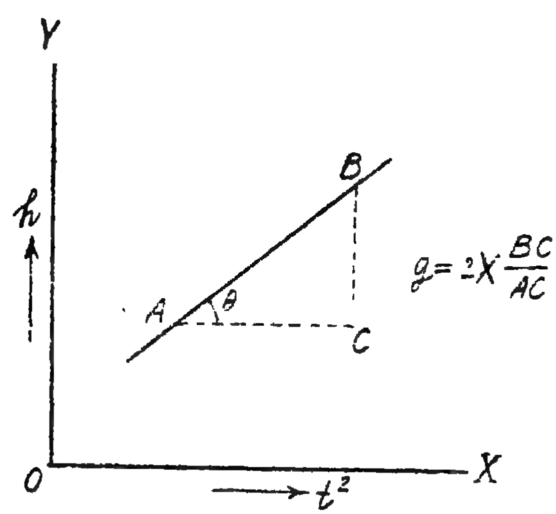


Fig. 9
t^2—h graph for a body falling under gravity.

Microid's apparatus the time of fall of the dart as measured with the help of the scale attached on the turn-table of the motor. Each division of the scale corresponds to 0·01 second. Thus the time t is known fairly accurately.

Further, the value of g can also be obtained graphically. If we draw a graph between h and t^2, we get a straight line as shown in fig—9. Twice the slope (=tan θ) of this straight line gives the value of g, that is,

$$g = \frac{2\,h}{t^2} = 2 \times \frac{BC}{AC}$$

Method—

(i) First of all *calibrate the circular scale* attached on the cork-mat *i. e.*, find out the value of time to which each small division of the scale corresponds. For this purpose, connect a 4-volt battery of accumulators to the two terminals provided on the case of the instrument and start the motor. When the motor has acquired a constant speed (which takes nearly two revolutions), find out the time for a number of revolutions (say, 25) with an accurate stop-watch and from this value calculate the time required by the disc to complete one revolution. Repeat this process a number of times and calculate the mean value of the time required for one revolution. Dividing this time by the number of divisions on the circular scale, calculate the time corresponding to each division of this scale. In this way the circular scale is calibrated.

(ii) Now test the *alignment of the electro-magnet with the zero of the scale*. For this purpose, complete the circuit of the electro-magnet by placing the ball in the hole and support the dart from the electro-magnet. Adjust the centrifugal release in such a way that it projects over the edge of the cork-mat and rotate the disc showly with your hand till the centrifugal projection comes in contact with the ball. Hold the disc practically stationary and slightly displace the ball. The dart shall fall down. If the alignment is correct, it should fall on the zero of the scale. If not, adjust the magnet or note down the zero error.

(iii) Now replace the ball and the dart in their positions. Measure the height (h) of the pin point of the dart above the cork-mat with a metre scale. Throw over the centrifugal release to the in-position and start the motor. After a few revolutions of the disc the motor shall acquire a constant speed. The centrifugal release shall fly over and displace the ball, thereby breaking the electric circuit. The electro-magnet loses its magnetism, consequently the dart falls down, and gets stuck up in the cork-mat. Now stop the motor and note down the reading on the circular scale where the dart penetrates the mat. Correct this reading for the zero-error noted earlier, and thus find the time of fall t of the dart. Repeat this operation at least thrice and calculate the mean value of t corresponding to this height.

(iv) Repeat the process for different heights of the dart. Calculate the value of g for each set separately and then find the mean value of g.

(v) Draw a graph between t^2 (represented along the x-axis) and h (represented along the y-axis). Measure AC and BC (Fig—9), and then calculate the value of g with the help of the formula given above.

Observations—

[A] *Readings for the calibration of the circular scale.*

Least count of the stop-watch = ... sec.

No. of divisions on the circular scale = ...

S. No.	No. of revolutions of the disc	Time taken		Time taken for one revolution (in sec)	Mean
		min.	sec.		
1.					
2.					...sec
:					
10.					

[B] *Alignment of the magnet with the zero of the scale.*

S. No.	Zero error (in div)
1.	
2,	
3.	
4·	
5.	

Mean = ... div.

[C] *Readings for the determination of h and t.*

S. No.	Height of the dart above the scale (h)	Readings on the scale where the dart penetrates the mat	Mean uncorrected time	Time corrected for zero error	t^2
1.		... div	... sec	... sec	...sec^2
:		... div			
:	...cm	... div			
:					

Calculations—

Time corresponding to one small division on the circular scale = ... sec.

Set I $g = \frac{2h}{t^2} = ... = ...$ cm/sec^2

[**Note**—Make similar calculations for other sets]

$\therefore$ Mean value of g = ... cm/sec^2

Again, from the graph,

BC = ...cm and AC = ... sec^2

$\therefore$ $g = 2 \times \frac{BC}{AC} = ...$ cm/sec^2

Result—The value of acceleration due to gravity at......as obtained with Microid's apparatus is,

(i) by calculation = ... cm/sec^2

(ii) by graphical method = ... cm/sec^2

[Standard value of g = ... cm/sec^2

$\therefore$ Error = ...%]

Precautions and Sources of Error—

(1) When the ball is placed in the hole, it should be gently rotated in the hole. This process will displace the dust particle and will ensure a good electrical contact.

(2) Before performing the main experiment, the alignment of the electro-magnet with the zero of the circular scale should be tested, and if there is a zero-error it should be noted down.

(3) The counter-weight on the centrifugal release should be so adjusted that the ball is displaced by it only when the motor has acquired a constant speed.

(4) As the time of fall (t) occurs raised to the second power and is a small quantity, it should be determined very accurately. For this purpose, the scale should be calibrated with a stop-watch reading upto one-tenth of a second.

(5) The graph between t^2 and h should be ploted on a large graph paper and the scales should be so chosen that they represent fully the accuracies in the quantities which are to be ploted on the graph paper.

(6) The straight-line graph should be smoothly drawn. The points should be evenly distributed about it and they should be as close to it as possible.

[Note—The accuracy of the result obtained by this method depends primarily on the accuracy with which the time of fall of the dart is measured. As each division of the circular scale corresponds to 0·01 sec and as the division is fairly wide, it is possible to estimate 1/10 of a division, and they can read upto 0·001 sec. Hence the time of fall can be very accurately determined. It is for this reason that this method gives reasonably accurate results.]

3

Elasticity

EXPERIMENT—6

Object—To determine the value of Young's modulus from the flexure of a beam supported on two knife-edges and loaded at its middle point.

Apparatus Required—The given beam, micrometer screw, a Leclanche cell, a shunted galvanometer (or a voltmeter), a hanger, slotted half kgm. weights, vernier callipers, and a screw gauge.

Description of the Apparatus—The apparatus (fig—10) required for this experiment consists of two knife-edges across which the beam can be supported, and a hanger supported from the middle of the beam, where suitable loads can be applied to the beam. The depressions at the centre are measured with a sensitive micrometer screw* which is supported just above the middle point of the beam.

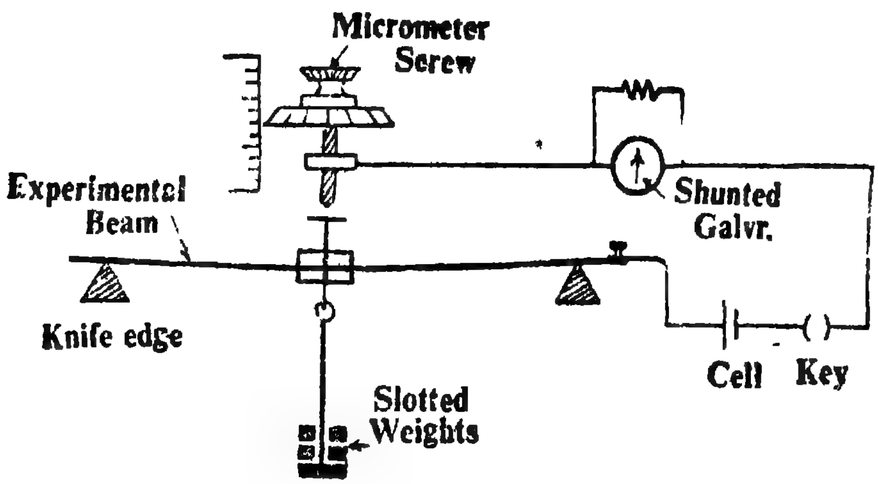


Fig. 10

Apparatus for the determination of Y.

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* The *Optical Lever Method* for measuring the depression of the beam is fully described towards the end of this experiment.

The exact contact of the screw with the beam is detected with the help of an electric circuit containing a cell and a shunted galvanometer.*

Formula Employed—The Young's moduls (Y) for a bar of rectangular cross-section is given by the following relation :—

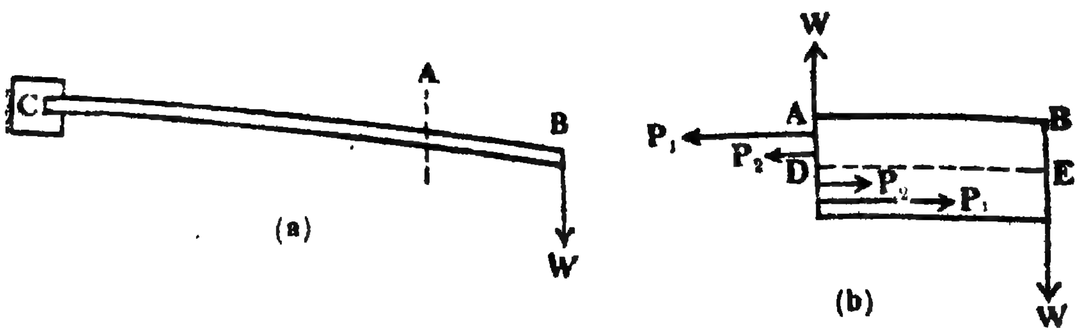
$$Y = \frac{Mg.\ L^3}{4bd^3\delta}$$

where, M = load suspended from the beam
L = length of the beam between the knife-edges
b = breadth (the horizontal side) of the beam
d = depth (*i. e*, the vertical side) of the beam
δ = depression of the beam corresponding to the load M.

PRINCIPLE AND THEORY OF THE EXPERIMENT

When a beam is bent from its natural shape by the action of applied forces, it will recover its original form on removal of those forces, provided that no part of it has been strained beyond the elastic limit.

In fig—11 (a) a beam CB is shown clamped at one end (C) and supporting an applied load at the free extremity (B). Such a

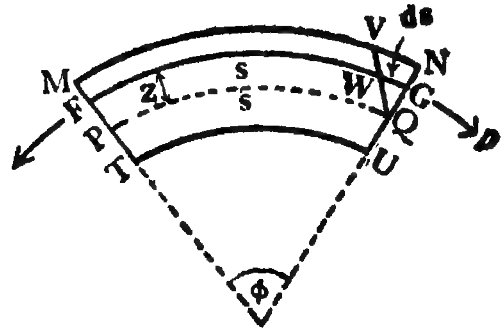


Fig—11
Forces acting in a bent beam.

system is called a *cantilever.* If we imagine a section of the beam to be drawn at the point A, the internal forces over the section A, applied by the remainder AC of the beam, must, together with the external load W, keep the part AB in equilibrium. The force W acting vertically downwards at B is balanced by an equal vertical force W acting upwards at A (fig—b). These two forces constitute a couple of moment W×AB, called the *bending moment* At A, and thus there must be, in addition, an internal couple of equal moment

* A voltmeter or an electric bell may also be employed for this purpose, but the use of the galvanometer or the voltmeter is preferable.

and opposite sense. Certain lines along the length of the beam are extended, others are compressed, while some are unaltered in length. The latter lie in a surface, called the *neutral surface*, which is parallel to the axis about which bending occurs. Thus DE (Fig. 11 b) which is the intersection of the neutral surface by the plane of the diagram, retains its original length. Fibres within the beam above DE will be extended, and the extension will increase with greater distance from the neutral surface, while fibres below DE will undergo longitudinal contractions. The resultant elastic reaction will produce forces such as p_1, p_2 in both parts of the beam, and these will constitute a system of couples, whose resultant may be called the *moment of resistance*, and balances the bending moment $W \times AB$. We therefore conclude that the combined moments, due to forces p about the point D, are equal and opposite to the external bending moment.



Fig—12
Section of a bent beam

Now, to obtain an expression for the depression consider an element MNUT (fig—12) of the beam. The neutral surface PQ subtends angle ϕ at its centre of curvature, and if the radius of curvature is R, then

$$PQ = R\phi$$

Draw QV parallel to PM, and, since FG is the length of a stretched fibre, situated at a distance z above the neutral surface, FW=PQ=s (say), is the normal length, while WG = ds is the extension which it has undergone. Thus its

$$\text{tensile strain} = \frac{ds}{\quad}$$

and hence, $$Y = \frac{p/\alpha}{ds/s}$$

where α is the cross-sectional area of the fibre, Y is the Young's modulus for the material of the beam, and p is the magnitude of the internal force which produces this extension ds. But

$$PQ = s = R\phi, \quad \text{and} \quad ds = z\phi$$

Hence $$\frac{ds}{s} = \frac{z}{R}$$

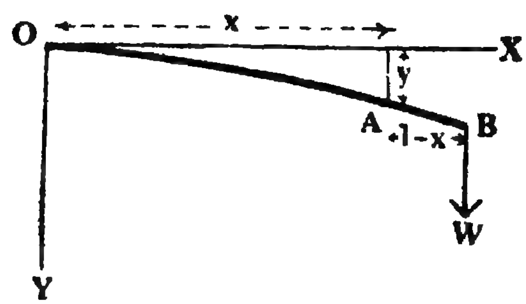
and $$P = \frac{1}{R} \times z\alpha$$

The moment of p about Q is $pz = \frac{Y}{R} \times \alpha z^2$, and thus the internal bending moment—or the moment of resistance—which is the sum of all such terms. is

$$\Sigma pz = \frac{Y}{R} \times \Sigma \alpha z^2$$

the quantity $\Sigma a z^2$ is analogous to the moment of inertia about the neutral axis, and is called the *geometrical moment of inertia* of the cross-section about the axis. It is equal to Ak^2, where A is the cross sectional area and k is the radius of gyration. We thus have :

$$\text{Internal Bending Moment} = \frac{YAk^2}{R} \quad \ldots \quad \ldots \quad (1)$$



Fig—13
Depression of a loaded cantilever.

To apply this fundamental equation, let us choose the co-ordinate axes OX, OY (fig—13) along, and perpendicular to, the unstrained position of the beam. Let the co-ordinates of A be x, y and let us suppose the curvature of the beam is small. The co-ordinates of B are then l, δ where δ is the depression of B, due to bending, and l is the length of the beam. The external bending moment at A is W $(l—x)$ and if we consider a short length ds of the beam its curvature at a point is $\frac{d\psi}{ds}$ where ψ is the angle which the tangent make with the x—axis. Hence the curvature

$$\frac{1}{R} = \frac{d\phi}{ds} = \frac{d}{ds}(\tan\psi) = \frac{d^2y}{dx^2}$$

since ψ is small, and thus $\psi =$, $\tan\psi$ Hence

$$W(l-x) = \frac{Y}{R}Ak^2 = YAk^2\frac{d^2y}{dx^2} \quad \ldots (2)$$

By integrating (2) we have

$$YAk^2 \frac{dy}{dx} = W\left(lx - \frac{x^2}{2}\right) + C_1$$

When $x = 0$, $\frac{dy}{dx} = 0$, so that $C_1 = 0$, thus

$$YAk^2 \frac{dy}{dx} = W\left(lx - \frac{x^2}{2}\right) \qquad (3)$$

By further integrating (3) we have

$$YAk^2y = W\left(\frac{lx^2}{2} - \frac{x^3}{6}\right) + C_2$$

Now, when $y=0$, $x=0$, hence $C_2 = 0$, thus

$$YAk^2y = W\left(\frac{lx^2}{2} - \frac{x^3}{6}\right) \qquad \ldots (4)$$

At B, where $x = l$, the maximum displacement, δ from the horizontal position occurs, hence

$$\delta = \frac{Wl^3}{3YAk^2} \qquad \ldots (5)$$

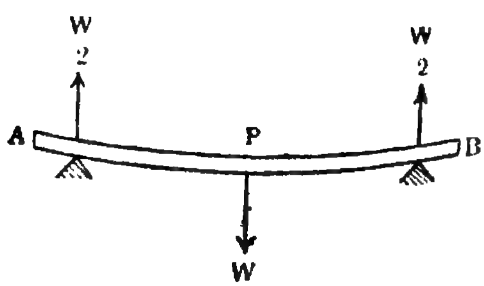


Fig. 14
Depression of a beam loaded in the middle.

In our experiment the beam is supported at, or near, its ends (fig–14), and carries a load W at the centre-point P. The external forces acting are W/2 at each end, due to the support thrusts, and W acting downwards at P. The tangent at P will be horizontal, and thus each half of the beam is equivalent to a cantilever, clamped at the point P and displaced by an end force W/2. The relative elevation δ of A (or B) above p is consequently obtained from (5) by putting W/2 for W and $l/2$ for l. Thus

$$\delta = \frac{Wl^3}{48\,Y.Ak^2} \qquad \ldots \qquad \ldots (6)$$

For a bar of rectangular cross-section (breadth=b, thickness=d), $A = bd$ and $k^2 = d^2/12$. Substituting these values we have

$$\delta = \frac{Wl^3}{4\,Y\,bd^3}$$

or

$$Y = \frac{Wl^3}{4\,bd^3.\,\delta} \qquad \ldots (7)$$

Relation* (7) is employed for the determination of Y for the material of the beam. In this formula l represents that length of the beam *which is situated between the two knife-edges.*

Method—

(i) Place horizontally on the knife-edges the given experimental beam such that its smallest edge is vertical and its *equal lengths project beyond the knife-edge.*† Then suspend the hanger for placing weights from the stirrup resting on the bar midway between the knife-edges and also parallel to them.

(ii) Then connect the electric circuit. If a galvanometer is used, *it should be shunted* and the shunt should not be removed during the course of the experiment. If a voltmeter is employed, care should be taken to connect its *positively marked terminal* to the higher potential point of the circuit.

(iii) Bring the micrometer screw in contact with the beam when it is unloaded and not the readings. Place gently a kgm-wt on the hanger. The contact of the screw point will be broken. Turn the screw slowly in one direction till the contact is just made. Note the readings of the micrometer screw. In this way go on loading the beam in equal steps of a kgm-wt and go on recording the corresponding micrometer readings till the maximum permissible load is applied.‡

(iv) When the maximum load has been added and its depression noted, turn the screw a little more in the same direction thereby *overtouching* the beam. Now slowly turn the screw in the reverse direction till its contact with the boam is just broken and note the reading. Than move up the screw through a distance which is less than the depression corresponding to a kgm-wt and then remove one kgm-wt from the hanger. This beam would go up and make the contact with the screw. Now move up the screw slowly till the contact is just broken and note the

For a further discussion of this equation see the Note given at the end of this experiment.

The reactions at the knife-edges will be equal only when this condition is fulfilled—a condition which is presumed to exist in the derivation of the formula.

It has been found experimentally that in the case of metals Hooke's law fails if the elongation exceeds $\frac{1}{1000}$ cm. per cm. Thus the highest strained filament of the beam, (*i. e.*, the outermost filament), should not be strained beyond this limit. For instance, for a bar whose $l = 80$ cm., d = 5 mm., the maximum permissible value of depression = 1·05 cm. nearly.

reading. Repeat this process* till one by one all the weights are removed.

(v) Next take the mean of the two readings of the micrometer screw for various loads. Now calculate the mean depression (δ) of the beam as shown in the observation table. For this purpose the readings should be so coupled that no reading is used twice otherwise that particular reading shall become useless when the mean is struck. Due to the same reason the mean depression should not be calculated by taking the difference† between successive readings.

(vi) Measure with a screw gauge the thickness‡ (d of the formula) of the bar at several places along its entire length in between the knife-edges and then find out the mean thickness corrected for zero error of the instrument. Similarly take a number of readings for the breadth (b of the formula) of the beam with the help of a verneir callipers. Then calculate the value of Y with the help of the formula given above (see equation—7).

Observations—

[A] *Readings for the measurement of depression of the beam.*

L. C. of the micrometer screw

(i)	Pitch of the screw	= ·05 cm.
(ii)	No. of disc divisions	= 100
(iii)	∴ Least count	= ·0005 cm.

* In moving the screw up or down care should be taken to avoid the back-lash error. For this purpose *continue the motion of the screw in one direction only.*

† For example, if a set of six readings be denoted by a, b, c, d, e and f, their successive differences are b—a, c—b,.........etc, and their mean is

$$\frac{(b-a)+(c-b)+.........+(f-e)}{5}=\frac{f-a}{5}.$$

Thus, by this procedure any additional accuracy which might have accured through taking six readings is lost completely, the result depending entirely upon the first and the last readings. Each of the intermediate readings is taken into account twice, once positively and second time negatively, and is thus without effect on the result.

In the procedure adopted in the table each reading is taken into account once only, the result depends, therefore on *all* the readings taken and is correspondingly more accurate.

‡ In the expression for Y given above depth of the beam occurs as d^3 and is also a small quantity, hence it has to be measured very accurately with a screw gauge.

S. No.	Load in kgm-wt (M)	Micrometer screw reading* Load increasing	Load decreasing	Mean	Depression for 2 kgm (δ)
1	0·0	2·4240cm	2·4350cm	2·4295cm	
2	0·5	2·5405 „	2·5525 „	2·5465 „	
3	1·0	2·7765 „	2·7725 „	2·7745 „	(in cms)
4	1·5	2·9930 „	2·9530 „	2·9730 „	(5)—(1)=0·7560
5	2·0	3·1815 „	3·1895 „	3·1855 „	(6)—(2)=0·7690
6	2·5	3·3050 „	3·3260 „	3·3155 „	(7)—(3)=0·7535
7	3·0	3·5455 „	3·5105 „	3·5280 „	(8)—(4)=0·7725
8	3·5	3·7465 „	3·7445 „	3·7455 „	
				Mean	0·7628 cm.

[B] Length of the beam between the knife-edges = 103·5 cm.

[C] *Readings for thickness and breadth of the beam.*

S. No.	Observed thickness	Observed breadth	Remarks
1.	.........cm.	.........cm.	L. C. of the screw gauge =......cm. Zero error of the screw gauge =......cm. Vernier constant =.........cm. Zero error of the vernier callipers =.........cm.
⋮			
⋮			
⋮			
Mean	............cm	............cm	

* If optical lever arrangement has been used in the experiment, record here "Optical Lever Reading". Then in the last column we shall get the values of x of formula—(9) given ahead in the body of the text.

Calculations—

(i) Corrected thickness of the beam, d = 0·5375 cm.

(ii) Corrected breadth of the beam, b = 2·37 cm.

Now
$$Y = \frac{MgL^3}{4bd^3\delta}$$

$$= \frac{2000 \times 981 \times (103{\cdot}5)^3}{4 \times 2{\cdot}37 \times ({\cdot}5375)^3 \times {\cdot}7628}$$

$$= 19{\cdot}36 \times 10^{11} \text{ dynes/cm}^2$$

[**Calculation work—**

Numerator		*Denominator*
log 2000 = 3·3010		log 4 = 0·6021
log 981 = 2·9917		log 2·37 = 0·3747
3 log 103·5 = 6·0447	log 103·5 = 2·0149	3 log ·5375 = $\bar{1}$·1912 (log ·5375 = $\bar{1}$·7303)
Sum = 12·3374		log ·7628 = $\bar{1}$·8824
0·0504		Sum = 0·0504
Diff. = 12·2870		

Antilog = $19{\cdot}36 \times 10^{11}$]

Result—The value of the Young's modulus for the material (...) of the given beam as found by the method of bending = ... dynes/cm²

[Standard value = ...dynes/cm²

Error = ...or......%]

Precautions and Sources of Error—

1. The beam should be placed on the knife-edges symmetrically so that equal portions project on both sides of the knife-edges.

2. The attachment for the hanger should be placed centrally on the beam and it should be parallel to the knife-edges. This will ensure symmetrical loading.

3. The loads should be placed or removed from the hanger as gently as possible and the readings should be recorded only after waiting for sometime, so that the thermal effects produced in the specimen get subsided.

4. The mean depression of the beam should be calculated according to the procedure adopted in the table. It should never be calculated by taking difference between two consecutive readings.

5. To avoid back-lash error in the micrometer screw, it should always be turned in the same direction before taking a reading.

6. As the thickness d of the beam occurs as d^3 in the formula employed, and is also a small quantity, it should be accurately measured with a screw gauge at several points along the length of the beam situated between the knife-edges.

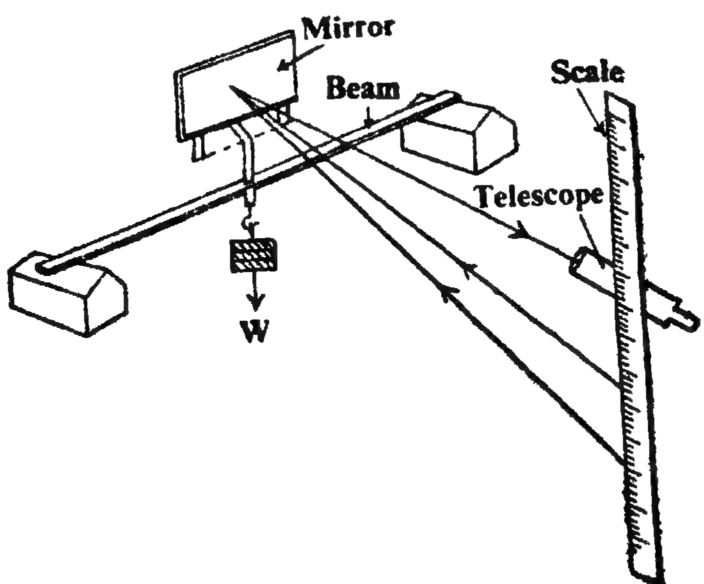
7. Proper care should be taken in using the galvanometer or the voltmeter. The former should remain shunted throughout the experiment; the latter, if used, should have its positive terminal connected to the higher potential point of the circuit.

8. When the load on the hanger is changed the bar slides slightly over the knife-edges and its motion is opposed by a horizontal force of friction and consequently the result is effected by error due to the presence of this frictional force.

OPTICAL LEVER METHOD FOR MEASURING DEPRESSION

Great accuracy is attained by employing a sensitive micrometer screw. For instance, if the pitch of the screw be ½ mm. and the disc be divided into 100 divisions, depressions can be measured with an accuracy of 0·0005 cm. Another device, though not so accurate, consists of the optical lever arrangement whereby the depressions can be measured sufficiently accurately. If the distance between the mirror and the scale (see below) be 2 meters, depressions can be measured with an accuracy of 0·002 cm.

Description of the Apparatus—The optical lever (fig—15) consists of a plane mirror rigidly mounted in a frame such that two of its legs lie in the plane of the mirror while the third leg projects forward. The optical lever is placed near the middle of the beam with its two hind legs on a fixed support and with its front leg resting on the middle point of the beam, the three legs being nearly in the same horizontal plane and the line joining the two hind legs parallel to the length of the beam. When the beam is loaded, its central portion is depressed and consequently the projecting leg moves down and thereby tilts the



Fig—16
Optical lever

mirror through a small angle. This tilt is measured by means of a telescope and scale arrangement.

Principle of use—Let the initial position (fig—16) of the mirror be M_1. In this position a certain mark (say, E) on the graduated scale is seen through the telescope T. After loading the beam, a depression equal to δ is produced and the mirror tilts through an angle θ. This position of the mirror is represented in the diagram by M^2. Now some other mark (say, F) on the scale will appear on the cross-wire of the telescope. Let the distance EF be x. Now it is obvious that the direction of the incident ray must have changed through 2θ in order to send the reflected ray in its original direction. Now, as these angles are usually small,

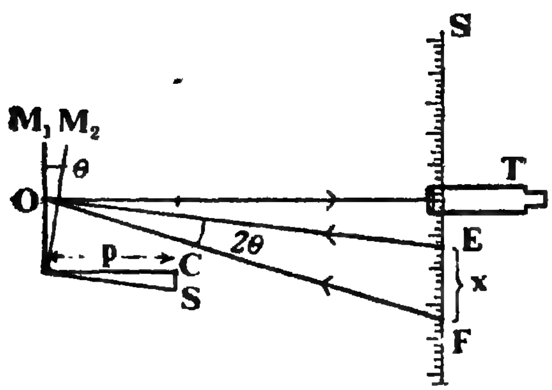


Fig. 16
Theory of an optical lever

$$2\theta = \frac{EF}{D} = \frac{x}{D} \tag{8}$$

where D is the distance between the mirror and the telescope.

If the distance of the front leg from the line joining the two hind legs be p, then (since depression CS $=\delta$) we have,

$$\theta = \frac{\delta}{p}$$

Substituting this value of θ in equation—(8) we have the depression of the beam given by the relation,

$$\delta = \frac{}{2D} \tag{9}$$

Thus by noting the value of x the value of the depression δ can be calculated.

Method—

For measuring δ with the optical arragement place a scale vertically in front of the mirror at a distance of nearly one metre, and close to the scale at the same height as the mirror place a telescope firmly clamped on a rigid stand. Focus the eye-piece on the crosse-wire till it is distinctly seen. Now, for focussing the telescope on the image of the scale in the mirror, first focus it on the mirror itself and then push in the tube of the telescope until the image of the scale appears in the field of view. Note down the position of

the horizontal cross-wire of the telescope on the image of the scale. After loading the beam note down again the position of the cross-wire on the image. The difference between the two readings gives the value of x of the formula given above. Increase the load on the hanger by equal steps of half-a-kilogram, noting down the position of the horizontal cross-wire of the telescope on the image of the scale after the addition of each load on the hanger. Now decrease the load on the hanger by the same stages upto zero value of the load and note down the position of the horizontal cross-wire on the image of the scale in the mirror for various loads. Find out the mean of the two readings taken with increasing and decreasing loads. From these mean values culculate the mean shift x for a particular load *by coupling the readings* as shown in the table given above for the miscrometer readings.

Now measure the distance D between the mirror and the scale. To measure the lever arm p, place the optical lever on the left-hand side of the sheet of your note-book, and press it lightly so that the impressions of its feet are produced on the sheet. From these impressions measure the perpendicular distance of the projecting foot from the line joining the two hind legs which gives p. Now, by knowing the multiplying factor (p/2 D) of the optical lever, calculate the value of the depression δ from the formula, $\delta = xp/2D$, given above.

Observation

[A] *Readings for the measurement of the depression of the beam.*

(i) Lever arm, *i. e.*, the distance p of the front foot from the line joining the other two. = ... cm

(ii) Distance D of the scale from the mirror. = ... cm

[Note—Now make a table for the record of the readings as given above. Other readings, *e. g.*, length, breadth, thickness of the beam should also be properly recorded.]

Calculations

Multiplying factor of the optical lever, p/2D = ...

$$\therefore \quad \delta = \frac{x\,p}{2\,D} = \quad \ldots \quad \text{cm}$$

Thus $\gamma = \ldots$ etc.

[Note—For *precaution and sources of error*, see the main experiment given above.]

A NOTE ON THE DERIVATION OF THE FORMULA

In deriving relation (5) we have considered the internal forces giving rise to a moment of resistance whose magnitude equals $W \times AB$. As a matter of fact these internal forces also give rise to a shearing stress whose value is W/bd. This causes a shear of the beam, producing a lowering of the end B relative to C. This effect, however, is small compared with the depression of B due to the bending, as proved below.

The depression (δ_1) due to the shearing stress W/bd is given by the relation—

$$\delta_1 = \frac{Wl}{bdn}$$

where n is the modulus of rigidity.† In this expression δ_1/l is the shear produced. Again, the depression δ produced by bending is given by (since $Ak^2 = bd^3/12$)—

$$\delta = \frac{Wl^3}{3Y.Ak^2} = \frac{4\,Wl^3}{Y.\,bd^3}$$

$$\text{Hence,} \quad \frac{\delta_1}{\delta} = \frac{Wl}{bdn} \times \frac{Y\,bd^3}{4\,Wl^3} = \frac{Y}{4n}\left(\frac{b}{l}\right)^2$$

For a beam generally employed for experimental purposes the quantity $(b/l)^2$ is exceedingly small.† Thus δ_1 is small compared with δ and the whole depression of B is therefore sensibly equal to δ, and thus the effect due to shearing stresses can be neglected.

EXPERIMENT—7

Object—To determine the modulus of rigidity for the material (.........)* of a rod by using the horizontal pattern of the twisting apparatus.

† n is given by the formula—

$$\text{Rigidity} = \frac{\text{Shearing stress}}{\text{Shearing strain}} = \frac{W/bd}{\delta_1/l}$$

† For instance, if $b = 2{\cdot}5$ cm. and $l = 100$ cm. we have $(b/l)^2 = 0{\cdot}000625$, which is a negligibly small quantity.

* Mention here the name of the material of the rod provided in the experiment.

Apparatus Required—Twisting apparatus, slotted kgm—wts. (preferably half kgm-wts), metre scale, screw gauge and vernier callipers.

Description of the Apparatus—The horizontal pattern of the twisting apparatus is depicted in the following figure :—

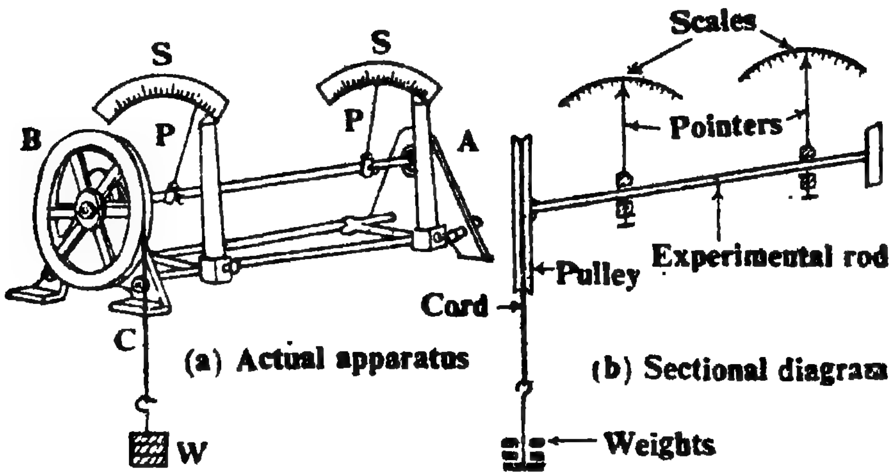


Fig. 17. Twisting apparatus.

The experimental rod is fixed at one end to a block A (Fig. 17 a) and the other end is attached to a steel axle to the pulley B. Two (or three) pointers (P, P) can be clamped to the rod at any desired points and these move over circular scales (S, S) graduated in degrees. The twist is produced by winding a cord C round the pulley and suspending weignts W from the hanger attached to its lower end. A sectional diagram of the actual apparatus is shown in Fig. 17 (b).

Formula employed—The rigidity n for the material of the rod is given by the formula—

$$n = \frac{180.\ Mg.\ D.\ l}{\pi^2\ r^4.\ \phi}$$

where
M = load suspended
D = diameter of the pulley
l = length of the rod suffering twist
r = radius of the rod
ϕ = angular twist produced in the rod.

PRINCIPLE AND THEORY OF THE EXPERIMENT

Let us consider a cylinder fixed at one end and twisted at the other by means of a couple of moment C, whose axis coincides with the axis of the cylinder. The angular displacement ϕ, at a distance

l from the fixed end, is proportional both to *l* and C. This is an example of pure shear, since there can be no change either in the length, or the radius of the cylinder. Each circular cross-section is rotated about the axis of the cylinder by an amount which is determined by its distance from the fixed point.

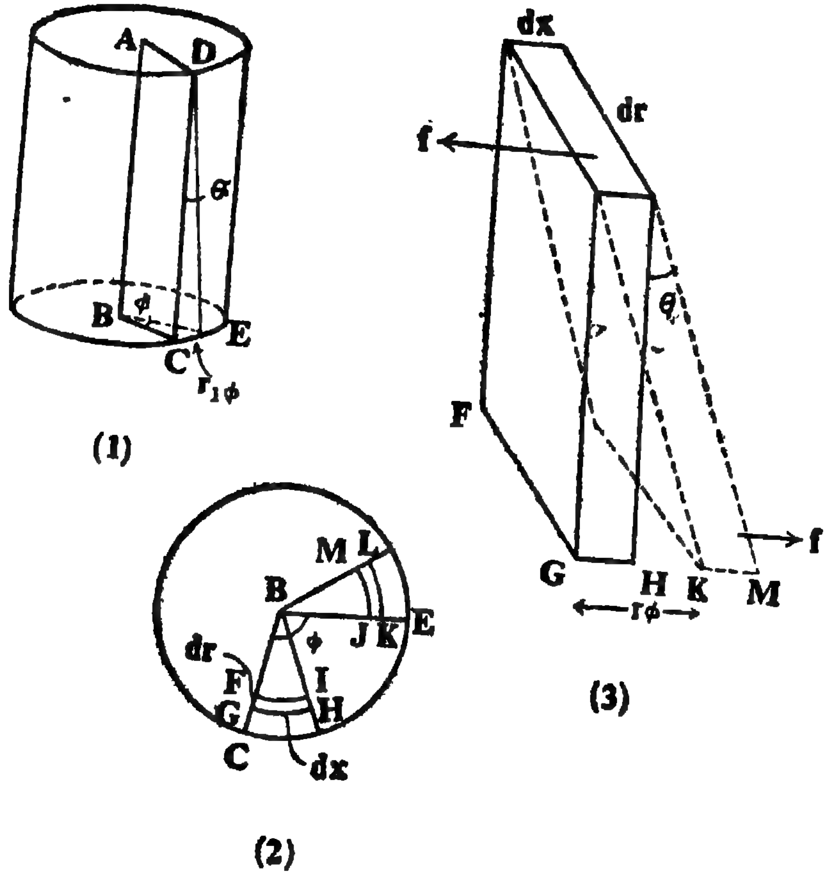


Fig. 18
Twisting of a cylinder.

Thus in Fig. 18 (1) above, a wedge of the cylinder is strained from the position ABCD into the position ABED, so that CE = $r_1\varphi$ where r_1 is the radius of the cylinder. In Fig. 18 (2) an element FGHI of the lower end is moved JKLM. Now, if BF = r and FG = dr, while GH = dx, then the parallelepiped with FGHI as base is sheared as shown in Fig. 18 (3), where GK = $r\phi$. The force f acting tangentically on the face FGHI and producing the shear θ, constitutes a shearing stress of magnitude f/dr.dx. Thus the modulus of rigidity according to its definition is given by the relation—

$$\overline{\text{dr. dx. }\theta}$$

Now, since $l\theta = r\phi$, or $\theta = r\phi/l$, the above relation reduces to—

$$n = \frac{f.l}{dr.dx.r\phi}$$

or $$f = n.\,dr.\,dx.\,\frac{r\phi}{l}$$

This force f has a moment fr about the axis of the cylinder and thus the total moment C is given by

$$C = \frac{n\phi}{l}\int\int r.^2drdx$$

The integral of dx must be taken round the circle of radius r, and hence

$$C = \frac{n\phi}{l}\int_0^{r_1} r^2dr(2\pi r)$$

$$= \frac{2\pi n\phi}{l}\int_0^{r_1} r^3.dr$$

$$= \frac{n\pi r_1^4\phi}{2i}$$

In this experiment, the couple C is applied by hanging a weight Mg over the pulley whose moment is equal to Mg. D/2 where D is the diameter of the pulley. Thus

$$\frac{n\pi r_1^4\phi}{2l} = \frac{MgD}{2}$$

or $$n = \frac{Mgdl}{\pi r_1^4\phi}$$

If ϕ is measured in degrees it should be converted in radians and its value is then given by $\pi\phi/180$. Thus

$$n = \frac{180MgDl}{\pi^2 r_1^4\phi}$$

which is the required relation.*

* In the above derivation r_1 has been assumed, due to obvious reasons, to be the radius of the cylinder. If r_1 is put equal to r the formula given under the heading "Formula Employed" follows.

Method—

(i) Wind a cord on the pulley and to the free and attach a hanger and see that the vertical portion of the cord is tangential to the pulley. Clamp the pointers and suitable distances from the fixed end of the rod and adjust the pointers near the zero of the graduated scales. Take the initial readings of the pointers with no load on the hanger (which should be placed on the table for recording the zero readings of the pointers).

(ii) Now put gently a load on the hanger. The pulley will rotate till the couple due to torsional reaction of the rod is balanced by the applied couple. Note the positions of the pointers now. In this way go on gradually increasing the load on the hanger in equal steps of the load till the maximum permissible load* is applied and go on noting down the corresponding readings of the pointers. Repeat, in the same fashion, with decreasing loads. The reading of the same pointer on the scale for a certain load taken either during increasing steps or during decreasing steps should agree very closely.

(iii) Now suspend the hanger on the other side of the pulley and repeat the above procedure. *This will eliminate any error due to the excentricity of the pulley about the rod.* By coupling the readings in a suitable manner (as explained in the procedure of Exp.–7) calculate the mean twist in this case as also in the previous case. Then obtain the mean twist ϕ corresponding to the load M.

(iv) With the help of a screw gauge measure the diameter of the rod *at several places and at each place in two mutually perpendicular directions.* Similarly, determine the mean radius of the pulley with the help of a vernier callipers.

* Before applying loads to the rod students should try to have some idea of the maximum permissible load. It has been found experimentally that for all metals Hooke's law holds only when the shear does not exceed $(1/3)^\circ$. In experiments for finding the rigidity the shear should never exceed $(1/9)^\circ$. Thus the angle of twist ϕ (which is equal to $l\theta/r$) should be less than $(l/9r)^\circ$ and hence the maximum load should be well below that value which is sufficient to produce this shear. For instance, if in an experiment a rod 5 mm. thick and 25 cm. long is subjected to torsional couple, then the maximum value of $\phi = 11^\circ$.

Observations—

[A] *Readings for the determination of the twist.*

[**Cord to the Left of the Pulley**]

Length of the rod under twist = ...cm.

Serial No.	Load on the hanger	Reading of the pointers with load increasing			Reading of the pointers with load decreasing			Mean Twist
		I Pointer	II Pointer	Twist	I Pointer	II Pointer	Twist	
1	..kgm	....	....	......	....	....	......	......
⋮								
⋮								
⋮								

[**Cord to the Right of the Pulley**]

[**Note**—Make a similar table and record the readings]

[B] *Reading for the determination of the diameter of the rod.*

S. No.	Reading along any direction	Reading along a perp. direction	Mean observed diameter	Remarks
1	.........cm.	.........cm.	...... cm.	L. C. of the screw gauge = ... cm. Zero error = ... cm.
⋮				
⋮				
⋮				
			...... cm.	

(C) *Readings for the determination of the diameter of the pulley*

S. No.	Observed diameter	Corrected diameter	Remarks
1	.........cm.	.........cm.	
⋮			L. C. of the vernier callipers = ...cm.
			Zero error = ...cm.
⋮			
	Mean	......cm.	

Calculations—

(i) *Twist ϕ corresponding to load M* :—

S. No.	Load	Twists			Mean twist (ϕ) corresponding to load (M)
		Load increasing	Load decreasing	Mean	
1	......kgm.	...	...	A	
⋮				B	(D) − (A) =...
				C	(E) − (B) =...
				D	(F) − (C) =...
				E	
6				F	
					Mean twist=...

(ii) Mean corrected diameter of the rod = ...cm.

∴ Mean corrected radius of the rod = ...cm.

(iii) Mean corrected radius of the pulley = ...cm.

Now, $n ... \dfrac{180 \text{ Mg D}l}{\pi^2 r^4 . \phi}$

.........dynes/cm²

Result—The modulus of rigidity for the material (...........) of the rod a is determined by statical method using horizontal pattern of the twisting apparatus = ...dynes/cm².

[Standard value = ...dynes/cm².

$\therefore$ Error = ...dynes/cm² = ...%]

Precuations and Sources of Error—

1. The cord should be so hung that its vertical portion is *tangential* to the pulley. Under this condition only the force applied shall act tangentially to the pulley.

2. The load should be placed or removed gently and the reading should be taken only after waiting for some time. The weights should be increased or decreased in equal and suitable steps.

3. The loads should never exceed the maximum permissible value otherwise Hooke's law will not be obeyed.

4. Angles of twist should be recorded by twisting the rod both clockwise and anti-clockwise. This will eliminate any error due to wrong centring of the pulley.

5. Mean twist should be calculated by suitably coupling the reading so that no reading occurs twice. Under no circumstances should the mean twist be calculated by taking successive differences of consecutive reading.

6. If the cord employed be of sufficient thickness then the effective radius of the pulley should be obtained by adding half the thickness of the cord to the observed radius of the pulley.

7. The diameter of the rod* should be recorded at a number of places along the experimental length and at each place along two mutually perpendicular directions.

8. As the instrument is not provided with a double pointer extending both ways along the diameter of a circular scale, the readings for the angular twists are not free from error due to eccentricity of the axis of the rod with respect to the circular scale.

9. The scale and the pointer method† of measuring angle is not an accurate one.

---

* The radius of the rod occurs raised to the fourth power and at the same time it is comparatively a small quantity, hence it is the most important quantity for experimented determination.

† In more refined pattern of the instrument the angular deflections are measured with the help of a lamp and scale arrangement.

10. The pattern of the apparatus carries only one pulley. Thus a single force can be applied to the end of the rod. This force produces a side-pull also, which introduces friction between the rod and its bearing. Thus, free twisting of the rod is hindered.*

ADDITIONAL EXPERIMENTS

No. 8 (a) & 8 (b)—To verify that the angle of twist for a given length of a rod is proportional to the couple applied to it and also to verify that the single of twist is proportional to the length of the rod twisted.

The reading obtained in the main experiment can be employed for the verification of these exercises. For this purpose, plot a graph between M and ϕ for each value of the length of the rod twisted by taking M along the x-axis and ϕ along the y-axis. Each graph between M and ϕ will be a straight line.

From these graphs find out the best value of ϕ/M corresponding to each l. Depict the relation between ϕ/M and l by drawing another graph which will also be found to be a straight line.

Here it may be added that the modulus of rigidity can be calculated by making use of this graph. By taking two suitable points lying on this graph, the difference between their abscissae will give $(l_1 - l_2)$, while the difference between their ordinates will give $(\phi_1 - \phi_2)/M$. Thus we have

$$n = \frac{180 \text{ Mg D} (l_1 - l_2)}{\pi^2 r^4 (\phi_1 - \phi_2)}$$

EXPERIMENT—8

Object—To determine the modulus of rigidity for the material of a wire by statical method using Barton's apparatus.

Apparatus Required—Barton's apparatus, slotted kgm-wts. (preferably half-kgm.-wts), metre scale, screw gauge and vernier callipers.

Discription of the Apparatus—In this pattern of the twisting apparatus (Fig. 19) the upper end of the wire (whose modulus of

This error hes been eliminated in the vertical pattern of the apparatus, known as Barton's apparatus (see expt.—8), which also eliminates the error due to eccentricity of the rod with respect to the circular scale.

rigidity is to be determined) is firmly clamped at A and at the bottom to a heavy cylinder S of brass, thus the wire is kept taut and straight. The parallel flexible cords leave opposite sides of the cylinder tangentially, and passing over two frictionless pulleys (P_1, P_2) carry at the other ends two identical pans. Equal loads placed in the two pans impart twist to the rod, which is read along its length by two or three pointers* moving over graduated circles (B, C, D in the figure). The robust base of the apparatus is provided with levelling screws (L in the figure).

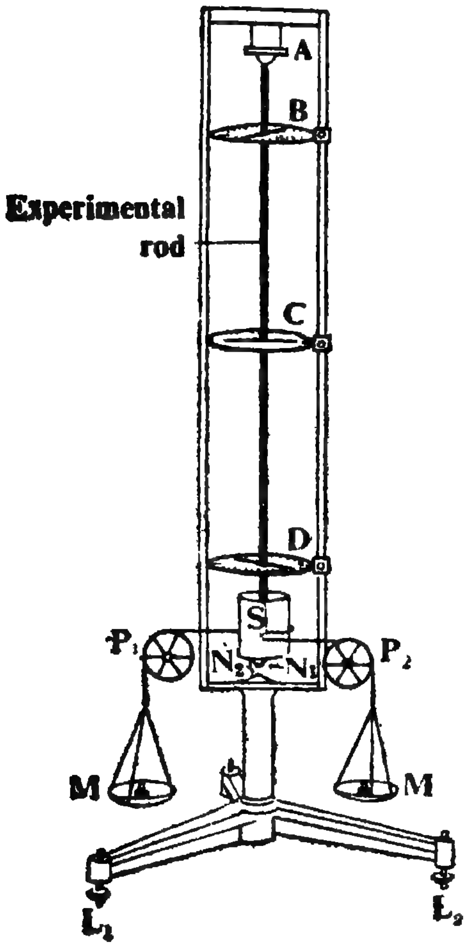


Fig. 19

Barton's Apparatus

Formula Employed—The modulus of rigidity n is calculated by the formula—

$$n = \frac{360.\ Mg.\ Dl}{\pi^2\ r^4\ \varphi}$$

where M = load suspended from each cord

D = diametar of the heavy cylinder

l = portion of the rod suffering twist

r = radius of the rod

ϕ = twist† produced in the rod.

PRINCIPLE AND THEORY OF THE EXPERIMENT

As proved in the previous experiment, the couple c due to torsional reaction is given by—

$$c = \frac{n\pi r^4}{2l}.\ \phi$$

In more elaborate form of the apparatus two mirrors can be attached at two desirable points on the wire and the twists measured with a lamp and scale arrangement.

In Barton's apparatus comparatively thinner wires can be employed for experimental purposes than is the case with the horizontal pattern described in the previous experiment. Hence it is obvious that the value of angular twist produced by a given couple is much greater in the vertical pattern and consequently be more accurately measured.

where the twist ϕ is expressed in radians. However, If ϕ be measured in degrees, then

$$c = \frac{n\pi r^4 \phi}{2l} \times \frac{\pi}{180}$$

If this twist in the wire is produced by placing a load M in each of the pans, then the couple due to equal loads in the two pans is MgD where D is the diameter of the heavy cylinder attached to the lower end of the wire. Equating these two couples we have

$$\frac{n\pi^2 r^4 \phi}{360.l} = MgD$$

Hence $$n = \frac{360.MgDl}{\pi^2 r^4 \phi}$$

Method—

(i) Before starting the actual experiment make the following adjustments :—

(a) With the help of the levelling screws provided at the base of the instrument level* the apparatus such that the experimental wire passes through the centres of the circular graduated scales.

(b) Slip a flexible, sufficiently stout, cord over the small peg provided on the curved surface of the cylinder. Pass the cord over the pulleys and attach two identical pans to the free ends. Adjust the heights and positions of the pulleys in such a manner that the thread leaves the cylinder *tangentially* at two diametrically opposite points and the portions of the cord lying in between the cylinder and the pullys are horizontal, parallel, and tangential to the cylinder and in the place of the respective pulleys.

[**Note**—For securing the above adjustment, the heights of the pulleys should be so adjusted that their tops are at the same level as pin provided on the surface of the cylinder. Further, the pulleys should also be adjusted sideways till their planes are tangential to the cylinder.

By the above adjustment, correct and symmetrical torsional couple shall be applied. When loads will be placed in the pans, the cylinder shall turn about its axis, it shall not be drawn sideways. To test this during the course of the experiment, see that the alignment of the pointed ends (N_1 and N_2) is not disturbed.]

* When this adjustment is properly made the pointer N_1 provided at the bottom of the heavy cylinder is just above N_2 provided in the frame (see Fig. 19).

(c) Adjust the pointers on the graduated circular scales in such a way that they are free to move and they do not *just* touch the scales. Fix them preferably on the zero marks of the scales.

(ii) Take the zero readings of the pointers after they have been fixed at two suitable points* on the vertical wire.

Gently place equal loads (say, half a kgm.) in each of the two pans. This will cause the cylinder to rotate till the couple due to torsional reaction of the wire balances the applied couple. The pointers will consequently be displaced. Note down the new positions of the pointers.† The difference between the two readings of the same pointer will clearly give the angle of twist for the corresponding load.

(iii) Now go on increasing the load in the pans in equal steps till the maximum permissible load‡ is reached, and go on noting down the positions of the pointers.

(iv) Next decrease the loads in equal steps till the original condition of no load in the pans is reached and note the reading of the pointers after each decrease of the load.

(v) Now calculate the mean angular twist in the manner as shown in the observation table. The readings should be so coupled that no reading is included twice in the operation. Then obtain the mean twist ϕ corresponding to a load M.

(vi) With the help of a screw gauge measure the diameter of the wire *at several points along its length* (between the graduated circles which have been employed for the measurement of ϕ) and at each place *along two mutually perpendicular directions.* In the same way, measure the diameter of the heavy cylinder with the help of a vernier callipers. Now calculate the value of rigidity with the help of the formula given above.

* These two points can be sufficiently spaced to give a good value of the angular deflection, since l is directly proportional to ϕ.

† To avoid the error due to eccentricity of the pointer with respect to the scale, read both ends of the pointer and take the mean of the two readings.

‡ To get an idea of the magnitude of this load see the previous experiment.

Observations—

[A] *Readings for the determinations of Twist*

Length of the wire under twist = ...cm.

Load Increasing

S. No.	Load on the pans	Readings of the Pointers						Twist
		I Pointer			II Pointer			
		I End	II End	Mean	I End	II End	Mean	
1	... kgm	...	...	...	...	...	...	...
......								

Load Decreasing

[**Note**—Make a similar table and record the readings.]

[B[*Readings for the determination of the diameter of the wire.*

S. No.	Reading along any direction	Reading along a perp. direction	Mean observed diameter	Remarks
1 : 10	.........cm.	.........cm.	......cm.	L.C. of screw gauge=...cm. Zero error =...cm.
		Mean	......cm.	

[C] *Readings for the determination of the diameter of the cylinder.*

S. No.	Reading along any direction	Reading along a perp. direction	Mean observed diameter	Remarks
1	............cm.	............cm.	......cm.	L.C. of vernier callipers =...cm.
⋮				Zero error =...cm.
5				
		Mean	......cm.	

Calculations—

(i) *Twist ϕ corresponding to load M.*

S. No.	Load	Twists			Mean twist (ϕ) corresponding load (M)
		Load increasing	Load decreasing	Mean	
1	...kgm.	...	...	A	
⋮				B	(D) — (A) =...
⋮				C	(E) — (B) =...
⋮				D	(F) — (C) =...
⋮				E	
6				F	
					Mean twist =...

(ii) Mean corrected diameter of the wire = ...cm.

∴ Mean corrected radius of the wire = ...cm.

(iii) Mean corrected diameter of the cylinder=......cm.

Now
$$n = \frac{360\ Mg.\ Dl}{\pi^2\ r^4\ \varphi}$$
$$= ......\text{dynes/cm}^2$$

Result—The modulus of rigidity for the meterial (......) of the given wire as determined by statical method using Barton's apparatus = ...dynes/cm.[1]

[Standard value = ... dynes/cm.^{2}
∴ Error =...dynes/cm.^{2}=...%]

Precautions and Sources of Error—

(1) Before starting the experiment the base should be levelled (with the help of the levelling screws) and the strings and the pulleys should be so adjusted, that the two portions of the strings between the cylinder and the pulleys are *horizontal, parallel to each other, tangential to the cylinder and are situated in the plane of the pulleys.* Under this circumstance a pure couple of moment mgd shall be applied. This will simply rotate the cylinder and will not pull it sideways.

(2) After putting loads in the two pans, and before taking the readings, check each time that the pointed ends (N_1 and N_2) are just one above the other, otherwise the wire shall be eccentric with respect to the circular scales and in that case the difference in the readings at the two ends of the same pointer shall be too much.

(3) The loads should never exceed the maximum permissible value otherwise Hooke's law shall cease to be valid.

(4) Mean twist should be calculated by suitable coupling the readings so that no reading is included twice in the operation. Under no circumstances should the mean twist be calculated by taking successive differences of consecutive readings.

(5) If the cord employed be of sufficient thickness, the effective diameter of the cylinder should be obtained by adding the thickness of the cord to the observed diameter of the cylinder.

(6) The diameter of the wire* should be recorded at a number of points along the length of the wire and at each point along two mutually perpendicular directions.

(7) The diameter of the cylinder should also be determined at a number of places and along two mutually perpendicular directions.

(8) In this apparatus pointers moving over graduated circles are employed for reading angular deflections. This is obviously not a very sensitive method, hence the accuracy of the result is limited.

(9) If the pulleys are not frictionless, error will be introduced

*The radius of the wire occurs raised to the fourth power and at the same time it is comparatively a small quantity, *hence it is the most important quantity for experimental determination.*

in the value of the force applied by means of the loads and consequently the result shall be adversely affected.

[**Note**—For additional exercises with this apparatus, see the previous experiment].

EXPERIMENT—9

Object—Te determine the value of the modulus of rigidity by dynamical method using a sphere.

Apparatus Required—The given wire, a heavy sphere, a robust support with a chuck for fixing the wire, stop-watch, metre scale, vernier callipers, physical balance of high capacity, and weight box.

Description of the Apparatus—This is essentially a torsion pendulum. It consists of a heavy metal sphere (Fig. 20) fixed by means of a chuck to the lower end of the given experimental wire, the upper end of which is fixed to a chuck in a rigid support. When the sphere is slightly rotated and then released, it executes torsional oscillations. To facilitate the counting of the oscillations, the sphere is provided at its bottom with a pointer.

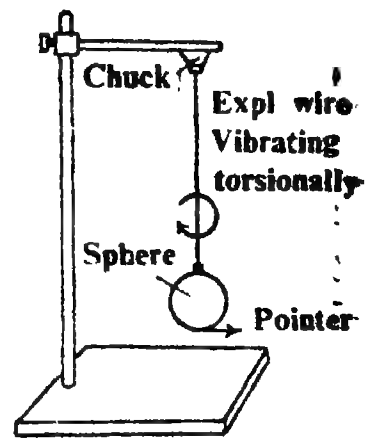


Fig. 20
Torsion pendulum

Formula Employed—The modulus of rigidity n of the material of the wire is given by the formula :—

$$n = \frac{8\pi I l}{T^2 r^4}$$

where I = moment of intertia of the sphere.

$$= \frac{2}{5} MR^2 \quad [M = \text{mass of the sphere},\ R = \text{radius of the sphere}]$$

l = length of the wire between the chucks
T = time period of the torsion pendulum
r = radius of the wire.

PRINCIPLE AND THEORY OF THE EXPERIMENT

If I be the moment of inertia of the sphere and θ be the angular displacement at any instant the equation of motion of the sphere is given by

$$I.\ \frac{d^2\theta}{dt^2} + c\theta = O \qquad (1)$$

where c is the torsional couple per unit radian twist due to the torsional reaction of the wire.

Equation (1) evidently represents a simple harmonic motion whose period is given by

$$T = 2\pi\sqrt{I/c} \quad \dots \quad \dots \quad (2)$$

But* $$c = \frac{n\pi r^4}{2l} \quad \ldots \quad \ldots \quad (3)$$

where n is the rigidity, r the radius and l the length of the wire. Substituting the value of c in (2) we have

$$T = 2\pi\sqrt{I \cdot 2l/n\pi r^4}$$

or $$T^2 = \frac{8\pi Il}{nr^4}$$

whence $$n = \frac{8\pi Il}{T^2r^4} \quad \ldots \quad \ldots \quad (4)$$

Now the moment of inertia of a sphere about and diameter is given by the formula—

$$I = \frac{2}{5} MR^2$$

where M = mass of the sphere, and R = its radius. Hence with the help of equation (4) the value of n for the material of the wire can be calculated.

[**Note**—(1) If in place of the sphere any other solid is given, then the appropriate value of its moment of inertia should be substituted in (4). For instance, if a disc of mass m and radius R be provided then the moment of inertia of the disc about a vertical axis passing through its centre of gravity is given by $I = mR^2/2$.

If a disc is employed for the torsional pendulum, its clamp-axis must be situated exactly at the centre, otherwise the disc will not remain horizontal. But the disc has two advantages over the sphere. Firstly, for equal masses the disc shall have a larger moment of inertia, hence thicker wires can be employed. Secondly, in a disc uniform distribution of mass is easier to attain than in a sphere. In large spheres air-bubbles are often left enclosed, hence use of the formula for the calculation of its moment of inertia is not always justified.

(2) The Inertia Table can also be employed for the determination of the rigidity of the wire. In that case if I_0 be the moment of inertia of the stage along with its pillars and cross-bar, then

$$T_0 = 2\pi\sqrt{I_0/c}$$

when T_0 is the time period of the stage.

Now an auxiliary body of regular shape (say, a metallic ring) whose moment of inertia I can be calculated from its dimensions is

*For this see the Theory of Experiment–7.

placed tentrally on the stage, then the time period T of the combination is given by

$$T = 2\pi\sqrt{(1 + I_0)/c}$$

From these two formulae, we easily obtain

$$n = \frac{8\pi I l}{r^4 (T^2 - T_0^2)}$$

Now it may be added here that this method suffers from two serious objections. Firstly, it is assumed tacitly in the formula that c remains constant, but actually *it changes with a change in the load on the wire.* Secondly, the moment of inertia of the auxiliary body is calculated on the assumption of uniform density throughout. But the condition of *perfect homogeneity of materials is not fully complied with in practice.* Both these errors are, however, eliminated by using a Maxwell's needle. (For this see Expt.-10).

Method—

(i) Take a fairly long and thin wire* whose modulus of rigidity is to be determined. From the lower end of the wire suspend the heavy metallic sphere and fix rigidly the upper end of the wire in the chuck provided for this purpose in the support.

(ii) *Turn the sphere slightly* and release it, thereby producing the torsional oscillations of the sphere. Count a number of oscillations and note the time for the same with the help of an accurate stop-watch. Repeat this process a number of times and calculate the mean time period.

[Note—In this experiment, the sphere should execute pure torsional oscillations. Hence while turning the sphere to twist the wire, the centre of gravity of the sphere should not get displaced, otherwise the sphere shall also execute pendular oscillaiions.

Further, in order to count the oscillations, make a reference mark on the experimental table just below the pointer. Begin counting the oscillations only when the pointer just crosses the reference mark. By this procedure, least error shall be introduced in the measurement of T, since in a simple harmonic motion the velocity is largest at its mean position.]

(iii) With the help of a metre scale measure the length of the experimental wire (between the chucks only). Measure the diameter of the wire at several points *along its entire length and at each points along two mutually perpendicular directions.*

* A wire half a metre long and one mm. in diameter is quite satisfactory for this purpose.

(iv) Measure the diameter of the sphere along several directions with the help of a verneir callipers. Weigh the sphere in a physical balance of suitable capacity. From these measurements calculate the moment of inertia of the sphere. Then calculate† the rigidity of the wire with the help of the formula [Equation—(4)] given above.

Observations—

[C] *Readings for the determination of the time period.*

S. No.	No. of oscillations	Time taken	Time period	Remarks
1.	....	...min...sec.	.........sec.	L. C. of the stop-watch = ...sec. Length of the wire = ...cm. Mass of the sphere = ... gm.
⋮				
⋮				
		Mean	.... sec.	

[R] *Readings for the determination of the diameter of the wire.*

S. No.	Diameter along one direction	diameter along a perp. direction	Mean observed diameter	Remarks
	...............cm.	...............cm.	......cm.	
1				L. C. of the screw gauge = ...cm.
⋮				
⋮				Zero error = ...cm.
⋮				
⋮				
10				
		Mean	......cm.	

† The above procedure can be varied a little. By taking different lengths of the wire, the corresponding time periods are determined. Then either graphically or by calculation the mean value of l/T^2 is obtained. Then with the help of this value of l/T^2 the rigidity of the wire is calculated out.

[C] *Readings for the determination of the diameter of the sphere.*

S. No.	Observed diameter	Corrected diameter	Remarks
1	......cm.	......cm.	L. C. of the vernier callipers= ......cm.
⋮			Zero error = ...cm.
5			
	Mean	.........cm.	

Calculations--

Radius (R) of the sphere = ...cm.
Mass (M) of the sphere = ...gm.
∴ Moment of inertia
of the sphere $= \frac{2}{5} MR^2$
= ......gm-cm^2.

Again

Corrected diameter of the wire = ...cm.
∴ Radius of the wire = ...cm.

Now

$$n = \frac{8\pi Il}{T^2 r^4}$$

= ......dynes/cm^2.

Result--The modulus of rigidity for the material (...........) of the given wire as found by dynamical method = ...dynes/cm^2.

[Standard value = ...dynes/cm^2 : Error =...%]

Precautions and Sources of Error--

(1) The wire chosen should be free from kinks. It should be fairly thin and long. This will result in increasing the time-period which will consequently be measured with greater accuracy.

(2) The sphere should be permitted to execute no other type of ocilllations except the torsional ones.

(3) In the derivation of the formula no restriction is laid on the magnitude of the angular twist, still the wire should never be twisted beyond the elastic limit, otherwise the restoring couple per unit twist due to torsional reaction will not be proportional to the twist itself.

(4) Since the radius of the wire is a very small quantity and in the formula it occurs raised to the fourth power, hence the diameter should be recorded very accurately. It should be measured at a number of points along the whole length of the wire and at each point the diameter should be recorded along two mutually perpendicular directions.

(5) While using the micrometer screw care should be taken to avoid back-lash error.

(6) In this determination, the moment of inertia of the sphere has been calculated from its dimensions on the assumption that the density of its material is uniform throughout its entire mass. But this condition of perfect homogeneity of materials is not fully complied with in practice. This imperfection obviously entails an error in the value of the rigidity.

A NOTE ON RIGIDITY DETERMINATIONS

During the manufacturing process the material of the wires is made to flow when they are made by drawing them through a circular aperture. Consequently, the outer layers of the wires are close-grained and tough as compared to those layers which are situated near the central core. It is for this reason that *the rigidity determined from experiments employing thinner wires is found to be more than that from experiments on thicker wires of the same material.*

Secondly, students will find that the value of *rigidity determined by a dynamical method is somewhat more than the value obtained by a statistical method.* This is due to the fact that under the action of a torque the wire does not acquire its final twist at once but the twist increases with time, thereby in the statical experiment an increased value of twist is obtained. In the dynamical method the torque is applied for a short time only and the time of oscillation is so short that this yield is not produced.

EXPERIMENT—10

Object—To determine the value of the modulus of rigidity of the material of a given wire by dynamical method using a Maxwell's needle.

Apparatus Required—Maxwell's needle, the given wire, metre scale, stop-watch, physical balance, weight box, and screw gauge.

Description of the Apparatus—Maxwell's needle (Fig.—21) consists of a hollow brass tube of length, say L, suspended by a vertical wire attached to its middle point with the help of a chuck. The other end of the wire is attached to another chuck fixed in a wall clamp. The suspension wire can carry a mirror M with the help of which the torsional oscillations of the needle can be counted with facility and accuracy. The tube can accommodate four brass cylinders each of which is of length L/4. The four cylinders are identical in dimensions, but two of them are solid (S, S) and equal in mass to each other and the remaining two are hollow (H, H) also equal in mass to each other.

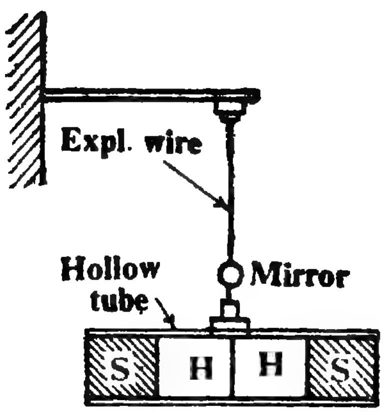


Fig —21
Maxwell's needle

Formula Employed—The rigidity n of the material of the wire is given by the following formula :—

$$n = \frac{2\pi\, l\, (M_2 - M_1)\, L^2}{r^4\, (T_1^2 - T_2^2)}$$

where
- l = length of the wire
- r = radius of the wire
- M_1 = mass of the hollow cylinder
- M_2 = mass of the solid cylinder
- T_1 = time-period when the solid cylinders are placed in the middle
- T_2 = time-period when the solid cylinders are placed in the middle
- L = length of the brass tube.

PRINCIPLES AND THEORY OF THE EXPERIMENT

Let the two hollow cylinders be placed in the middle of the tube and solid ones at the ends (as shown in the above diagram) and let the combination be slightly rotated in a horizontal plane and then released. Under these circumstances the needle will execute a simple harmonic motion and the time-period T_1 will be given by

$$T_1 = 2\pi\sqrt{I_1/c} \quad \ldots \quad (1)$$

where I_1 is the moment of inertia of the combination about the wire as axis, and c is the restoring couple per unit twist due to torsional reaction.

Now let the hollow and solid cylinders be interchanged so that the two solid cylinders occupy the central position. If the new time-period be T_2, then

$$T_2 = 2\pi\sqrt{I_2/c} \quad \ldots \quad (2)$$

where I_2 is the moment of inertia* of the new combination about the wire as axis of rotation.

Squaring (1) and (2) and then subtracting (2) from (1), we have

$$T_1^2 - T_2^2 = \frac{4\pi^2}{c}(I_1 - I_2)$$

$$\therefore \quad c = \frac{4\pi^2 (I_1 - I_2)}{T_1^2 - T_2^2} \quad \ldots \quad (3)$$

$$\text{But}\dagger \quad c = \frac{n\pi r^4}{2l} \quad \ldots \quad (4)$$

where n is the rigidity, l is the length and r is the radius of the wire hence from equations (3) and (4) we have

$$\frac{n\pi r^4}{2l} = \frac{4\pi^2 (I_1 - I_2)}{T_1^2 - T_2^2}$$

$$\text{or} \quad n = \frac{8\pi l\,(I_1 - I_2)}{r^4 (T_1^2 - T_2^2)} \quad \ldots \quad (3)$$

Now we have to evaluate $(I_1 - I_2)$. For this purpose, let M_1 and M_2 be the masses of each of the hollow and solid cylinders respectively. Also let I_0 I' and I'' be the moments of inertia of the hollow tube, the hollow cylinder, and the solid cylinder respectively about a vertical axis passing through their centres of gravity. Then applying the theorem of parallel axes we have—

$$I_1 = I_0 + 2I' + 2M_1\left(\frac{L}{8}\right)^2 + 2I'' + 2M_2\left(\frac{3L}{8}\right)^2$$

$$\text{and} \quad I_2 = I_0 + 2I'' + 2M_2\left(\frac{L}{8}\right)^2 + 2I' + 2M_1\left(\frac{3L}{8}\right)^2$$

* The moment of inertia of the combination in the second case will be different from the first one since the distribution of mass about the axis of rotation has undergone a change.

† For this, see the Theory of Experiment-7.

Hence
$$I_1 - I_2 = \frac{L^2}{4}(M_2 - M_1) \qquad \ldots \qquad (4)$$

where L is the length of the tube.

Substituting this value of $(I_1 - I_2)$ from (4) in (3) we have—

$$n = \frac{2\pi l\,(M_2 - M_1)\,L^2}{r_4\,(T_1^2 - T_2^2)} \qquad \ldots \qquad (5)$$

This expression can be employed for determining the value of the rigidity* of the material of the suspension wire.

Method—

(i) Take a fairly thin and long wire of the material whose modulus of rigidity is to be determined. For this purpose, a wire nearly 50 cm. long and 1 mm. thick is quite preferable. Suspend from its lower end the Maxwell's needle and place a pointer in front of the mirror for counting the oscillations.† Place all the cylinders inside the tube in such a way that the two hollow cylinders occupy the central position and the solid ones are towards the two ends and *that no part of the cylinders projects outside the tube.*

(ii) Now make the Maxwell's needle perform torsional oscillations by slightly rotating it in a horizontal plane and then releasing it. Then with the help of an accurate stop-watch note the time of a number of oscillations which are counted by observing the transits of the mark on the mirror past the pointer.‡ Repeat the observation a number of times and thus determine the mean value of the time-period T_1

(iii) Next interchange the positions of the hollow and solid cylinders taking care that *they are properly fitted inside the tube.* Determine the time-period T_2.

(iv) Now with the help of a metre scale measure the length L of the hollow tube and that of the wire (l) between the two chucks. Next determine the mean value of $(M_2 - M_1)$ with the help of a physical balance. For this purpose, place a solid cylinder on the left pan of the balance and the hollow one on the right and add extra weights on this pan till equilibrium is attained. These extra weights give the value of $(M_2 - M_1)$. Similarly, determine its value for the other pair and thus calculate the mean value.

* The value of rigidity so determined is free from errors arising from causes discussed in Experiment—9.

† A telescope can also be employed for this purpose.

‡ If a telescope is used, count the oscillations with reference to the point of intersection of the cross-wires.

(v) With the help of a screw gauge *determine the diameter of the wire at several places and at each place along two mutually perpendicular directions.* Tabulate all these readings and then calculate the value of the rigidity of the material of the wire with the help of the formula given above.

Observations—

(A) *Readings for the determination of the periodic times.*

Least count of the stop-watch = ...sec.

S. No.	No. of oscillations	Determination of T_1		Determination of T_2	
		Time taken	T_1	Time taken	T_2
1	....	...min...sec.	......sec.	...min...sec.	......sec.
......					
......					
		Mean	......sec.	Mean	......sec.

[B] *Readings for the determination of the diameter of the wire.*

S. No.	Reading along one direction	Reading along a perpendicular direction	Observed diameter (Mean)	Remarks
1	......cm.	.........cm.	.........cm.	Least count = ...cm. Zero error = ...cm.
......				
......				Length of the wire. = ...cm
10				
		Mean	.........cm.	

[C] (i) Length of the hollow tube = ...cm.

(ii) Difference between masses of one pair of solid and hollow cylinders = ...cm.

(iii) Difference between masses of the other pair of solid and hollow cylinders = ...gm.

$\therefore$ Mean $(M_2 - M_1)$ = ...gm.

Calculations—

Mean radius of the wire = ...cm.

Now
$$n = \frac{2\pi l (M_2 - M_1) L^2}{r^4 (T_1^2 - T_2^2)}$$
= ....* dynes/cm.^{2}

Result—The value of the modulus of rigidity for the material (.........) of the wire = ...dynes/cm.^{2}

[Standard value ....* = ...dynes/cm.^{2}
Error = ...%]

Precautions and Sources of Error

1. The wire chosen for this experiment should be free from kinks. It should be fairly thin and long. This will result in increasing the value of the time-period, which will then be measured with greater accuracy.

2. So that the moment of inertia of the tube remains constant it should remain horizontal throughout the whole experiment.

3. The needle should be permitted to execute no other type of motion except the torsional oscillations.

4. Although theoretically no restriction is laid on the magnitude of the angular twist, yet the wire should never be twisted beyond the elastic limit.

5. Care should be taken to measure the diameter of the wire accurately. It should be measured at a number of points along the whole length of the wire and at each point the diameter should be recorded along two mutually perpendicular directions.

6. While measuring the diameter of the wire, avoid backlash error of the micrometer screw.

7. The chief sources of error in this experiment are : (i) The cylinders, specially the hollow ones, may not be symmetrical. In that case the formula does not truly apply. (ii) The clamp may not be exactly in the middle of the hollow tube. In that case the

* Use logarithmic tables for calculation work. $(T_1^2 - T_2^2)$ can be factorised for this purpose.

tube will not remain horizontal. (iii) If the wire has corroded, there is an uncertainty in the measurement of its radius. Actually, the measurement of r involves the greatest source of error.

EXPERIMENT—11

Object To determine the Young's modulus, modulus of rigidity, and Poisson's ratio for a material in the form of a wire by Searle's method.

Apparatus Required—Two identical inertia bars, a wire of the given material, stop-watch, thread, physical balance, weight box, metre scale, vernier callipers and screw gauge.

Formula Employed—The Young's modulus (Y), the rigidity (n) and Poisson's ratio (σ) are calculated by making use of the following formulae :—

$$Y = \frac{8\pi\, Il}{T_1^2\, r^4} \tag{1}$$

$$n = \frac{8\pi\, Il}{T_2^2\, r^4} \tag{2}$$

$$\frac{T_2^2}{2T_1^2} - 1 \tag{3}$$

where I = moment of inertia of the bar about a vertical axis through its centre of gravity.
l = length of the wire between the two clamping screws.
r = radius of the wire.
T_1 = time-period when the two bars are executing S.H.M. together.
T_2 = time-period for the torsional oscillations of a bar.

PRINCIPLE AND THEORY OF THE EXPERIMENT

Two exactly identical bars AB and CD (Fig.—22) are joined at their centres by a wire GG of the material whose elastic constants are to be determined. The system is suspended by two parallel torsionless threads so that *in the equilibrium position the bars are parallel to each other and lie in the same horizontal plane*. If the two bars be slightly rotated through equal angles in opposite directions and then be released, the bars will begin vibrating in a horizontal plane with the same period about their supporting threads. During this process, the wire will be bent from an approximately circular

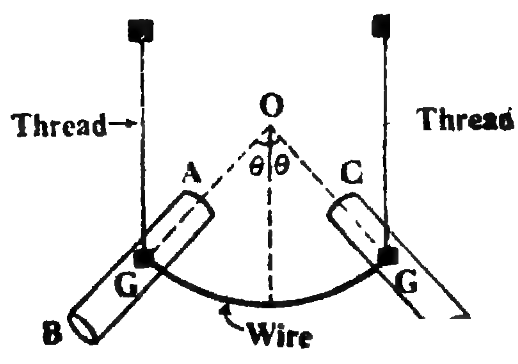


Fig.—22
Searle's apparatus for Y, n and σ

curve on one side to the straight line joining its ends to a similar curve on the other side.

Neglecting the torsion of the threads, the only couple acting on the bars is that due to the bending of the wire GG. Now the couple (*i. e.*, the bending moment) produced in this case is given by*—

$$\text{Couple} = \frac{Y.\,Ak^2}{R} \qquad \ldots \qquad \ldots \qquad (1)$$

where Y is the Young's modulus for the material of the wire, (Ak^2) is the geometrical moment of inertia $\left(= \pi r^2 \times \frac{r^2}{4} = \frac{\pi r^4}{4}\right.$, where r is the radius of the wire) and R is the radius of curvature of the arc GG. Now, from the figure it is easy to see that

$$R = \frac{l}{2\,\theta} \qquad \ldots \qquad \ldots \qquad (2)$$

where l is the length of the wire. Hence

$$\text{the couple} = \frac{\pi r^4.Y\theta}{2l} \qquad \ldots \qquad \ldots \qquad (3)$$

If I be the moment of inertia of the bar about the vertical axis passing through its centre of gravity, and $\frac{d^2\theta}{dt^2}$ its angular acceleration, then we have

$$I\frac{d^2\theta}{dt^2} + \frac{\pi r^4 Y}{2l}\,.\,\theta = 0$$

or

$$\frac{d^2\theta}{dt^2} + \frac{\pi r^4 Y}{2Il}\,.\,\theta = 0$$

The motion of the bar is, therefore, simple harmonic whose time-period is given by—

$$T_1 =$$

whence

$$Y = \frac{8\,\pi\,I\,l}{r^4\,T_1^2} \qquad \ldots \qquad \ldots \qquad (4)$$

* For the derivation of this relation, refer to Expt.–6.

If the length of the bar be L and R be its radius, then its moment of inertia*

$$I = M\left(\frac{L^2}{12} + \frac{R^2}{4}\right)$$

where M is the mass of the bar.

Now the threads of the bars are removed and one of the bars is fixed horizontally on a support while the other is suspended below from the wire (Fig.–23). Now if the lower rod is slightly turned in its own plane and then released it will execute torsional oscillations and the time-period T_2 will be given by

$$T_2 = 2\pi\sqrt{I/c}$$

or $$c = \frac{4\pi^2 I}{T_2^2} \qquad \ldots \quad (5)$$

where c is the restoring couple per unit twist. But†

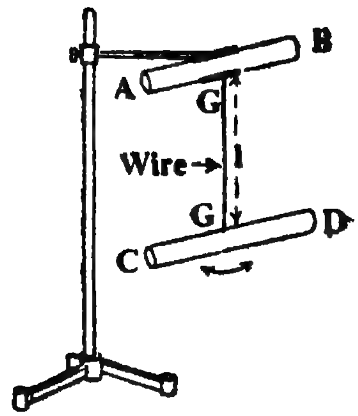


Fig.–23
Torsional oscillations of a rod.

$$c = \frac{n\pi r^4}{2l} \qquad \ldots \quad (6)$$

where n is the rigidity for the material of wire. Thus

$$\frac{n\pi r^4}{2l} = \frac{4\pi^2 I}{T_2^2}$$

whence $$n = \frac{8\pi\, Il}{r^4 . T_2^2} \qquad \ldots \quad (7)$$

Now from equations (4) and (7), we have

$$\frac{Y}{n} = \frac{T_2^2}{T_1^2} \qquad \ldots \quad (8)$$

But‡ $$Y = 2n\,(1 + \sigma) \qquad \ldots \quad (9)$$

---

* If the bar of a rectangular cross-section and its length and breadth (the horizontal side) be a and b respectively, then

$$I = M\left(\frac{a^2 + b^2}{12}\right)$$

† For the derivation of this formula, see Expt.–7.

‡ For this relation, see any standard text-book on General Properties of Matter.

Hence
$$\sigma = \frac{Y}{2n} - 1$$
$$= \frac{T_2^2}{2T_1^2} - 1 \quad \ldots \quad (10)$$

Relations (4), (7) and (10) are the required results, whence the elastic modulii, Y, n and σ can be evaluated.

Method—

(i) Take two *torsionless cotton threads of exactly the same length* (which may be, say, 50 cms.) and with their help suspend the two inertia bars from a rigid support so that the axes of the bars and the wire are coplanar. See that the bars hang *parallel* to each other and that they are *horizontal.* Then by slightly turning the rods in opposite directions in their own plane and then releasing them, set the bars to oscillation.* With the help of an accurate stop-watch, note the time for at least 25 oscillations. Repeat the process four times and thus calculate the mean time-period T_1.

(ii) Now after removing the cotton threads clamp one of the bars to a support so that the other bar hangs from the vertical wire. Rotate slightly the bar in its own plane and then release it so that it excutes torsional oscillations. Find the mean time-period T_2 in this case also.

(iii) Now measure the length of the wire with a metre scale and measure its diameter as usual. Again, measure the lengths of the bar with a metre scale and the diameter with a vernier callipers. Then by weighing them separately in a physical balance calculate their separate moments of inertia and thus find the mean value‡ of I. Then with the help of appropriate formula given above calculate the required elastic modulii.

* This process can be greatly facilitated by drawing the two ends A and C (Fig.–22) slightly towards each other with the help of a loop of a thin thread slipped over them and (when the system is at rest) by burning the thread.

† *For counting the oscillations*, a pointer can be set up close to the end (facing the observer) of one of the bars and a mark on the bar made in line with the pointer when the bar is at rest.

‡ Rigorously speaking, this mean should be utilised in the evaluation of Y. For the evaluation of n, the value of the moment of inertia of the bar actually undergoing torsional oscillations should be used.

Observations—

[A] *Readings for the determniation of time-periods.*

Least count of the stop watch = ...sec.

S. No.	No. of oscillations	Determination of T_1		Determination of T_2	
		Time taken	T_1	Time taken	T_2
1	25	...min...sec.	......sec.	...min...sec.	......sec.
2	25				
3	25				
4	25				
		Mean	......sec.	Mean	......sec.

[B] *Readings for the diameter of the wire.*

S. No.	Reading along any direction	Reading along a perpendicular direction	Mean observed diameter	Remarks
1	.........cm.	............cm.	......cm.	L. C. of the screw gauge = ...cm. Zero error = ...cm.
⋮				
10				Length of the wire = ...cm.
		Mean	.........cm.	

[C] *Readings for the diameter of the bar.*

S. No.	Reading along any direction	Reading along a perp. direction	Mean observed diameter	Remarks
1	........cm.	......=cm.	........cm.	L. C. of the vernier callipers = ... cm. Zero error = ... cm. Length of the bar = ...cm. * Mass of the bar = ...cm. *
:				
:				
:				
		Mean	......cm.	

Calculations—

Mean correct radius of the bar =......cm.

$$I = M\left(\frac{L^2}{12} + \frac{R}{4}\right)$$

$$= \text{........gm.-cm.}^2\dagger$$

Again, mean corrected radius of the wire=......cm.

Hence $$Y = \frac{8\pi Il}{T_1^2 r^4}$$

$$= \text{........dynes/cm.}^2$$

$$n = \frac{8\pi Il}{T_2^2 r^4}$$

$$= \text{........dynes/cm.}^2$$

and $$\sigma = \frac{T_2^2}{2T_1^2} - 1$$

$$= \text{......}$$

---

* If the bars are not exactly identical then the mass and other dimensions of the second bar should also be recorded in the same table.

† If the bars are not found to be exactly identical, calculate the moment of inertia of the second bar also, and then calculate the mean I.

Result—The values of the elastic constants for the material (......) of the wire are

$Y=$......dynes/cm.^{2}. (Standard value=.........dynes/cm.^{2} Error=......%)

$n=$......dynes/cm.^{2}. (Standard value =......dynes/cm.^{2} Error=......=......%)

$\sigma=$......

Standard value=.........

(Error=......=......%)

Precautions and Sources of Error—

(1) While taking observations for T_1, the bars should be made to execute oscillations in the horizontal plane only. All other types of undesirable oscillations should be checked as far as possible.

For this purpose, the two points where the upper ends of the threads are fixed should have a distance equal to that between the hooks provided on the two bars, and the lengths of the two threads should be so adjusted that the bars and the wire stay in a horizontal plane.

(2) In this determination, the amplitude should be small, so that the threads remain vertical and the wire is not strained beyond the elastic limit.

(3) In order that the bending of the wire may be symmetrical, the cotton thread loop, used to start the oscillations should touch the rods at equal distance from the respective ends. Further, it is preferable to burn the upper half of the loop so that the thread falls off immediately.

(4) While determining the periodic time for the torsional oscillations, the angular twist may be kept large (since in the derivation of the formula involving this quantity no assumption is made regarding its magnitude) but care should be taken that the wire is not twisted beyond the elastic limit.

(5) Both these time-periods should be measured very accurately, since any error in their determination will be doubled in the result (the periods occur raised to the second power in all the three formulae employed in this experiment).

(6) The radius of the wire is a small quantity and moreover it occurs in the fourth power in the formulæ for Y and n, hence the diameter of the wire should be measured very carefully. For this purpose, the diameter should be recorded at several places along the whole length of the wire and at each place it should be measured along two mutually perpendicular directions.

(7) While taking readings for the diameter of the wire, error due to back-lash error of the screw gauge should be avoided.

(8) The diameters of the rods should also be observed at several points along their lengths and at each point along two mutually perpendicular directions.

(9) The chief source of error in the experiment is involved in the determination of T_1. In this case, the undesirable motions which accompany the required oscillatory motion of the bars cannot be completely eliminated.

[**Note**—This method has two distinct advantages in its favour. One is the determination of the Poisson's ratio. The determination of this important elastic constant reduces simply to the determination of T_1 and T_2 which can be measured sufficiently accurately. There is no need of knowing Y and n in terms of which the Poisson's ratio is expressed.

Secondly, the bulk modulus of elasticity (K), which is also expressible* in terms of Y and n, can be evaluated from a knowledge of these elastic constants. Now if we calculate the value of K from the values of Y and n as determined by this method it will be more accurate than the value obtained from the value of Y (obtained by extension method using Searle's micrometer screw method) and value of n (obtained from Maxwell's needle method). In the former a very long wire is employed while in the latter a comparatively shorter wire is used.]

EXPERIMENT—12

Object—To determine the Poisson's ratio for rubber.

Apparatus Required—Rubber tube, rubber stopper with a bore, metal sleeve with a hook, metre scale, a small pointer, burette, and slotted weights (preferable in sets of 200 or 250 gm.)

Formula Employed—The Poisson's ratio (σ) for rubber is calculated from the following formula :—

$$\sigma = \frac{1}{2}\left[1 - \frac{1}{A} \times \frac{dV}{dL}\right]$$

where A = area of cross-section of the rubber tube $= \pi D^2/4$ (D = its diameter)

dV = small increase in the volume of the tube when stretched by a small weight

* **Bulk modulus of elasticity** $K = \dfrac{nY}{3(3n - Y)}$

dL =the corresponding increase in the length of the tube.

PRINCIPLE AND THEORY OF THE EXPERIMENT

Suppose, a rubber tube is hung vertically and a small weight is suspended from its lower end. The rubber tube will consequently be stretched a little, thereby increasing the internal volume and the length of the tube a little. The area of cross-section will obviously be diminished. Let the increase in volume be dV and the corresponding increase in the length and decrease in the area of cross-section of the tube be respectively dL and dA, and let the original value for the volume, length and area of cross-section be V, L and A respectively. Now

$$V+dV=(L+dL)\,(A-dA)$$
$$=AL-LdA+AdL-dL.dA$$
$$=V-L.dA+A.dL$$

since $AL=V$ and the last term, being the product of two small quantities, is negligible. Thus

$$dV=A.dL-L.dA \qquad \ldots \qquad \ldots \qquad (1)$$

Again $$A=\pi R^2=\pi D^2/4 \qquad \ldots \qquad \ldots \qquad (2)$$

where R and D are respectively the inner radius and the diameter of the tube.

Differentiating (2), we get

$$dA=\frac{d}{2}.\,D.dD = 2A.\,\frac{dD}{D} \qquad \ldots \qquad \ldots \qquad (3)$$

Substituting this value of dA (3) in (1), we have

$$dV=A.dL-\frac{2AL}{D}.\,dD$$

$$\therefore \quad \frac{dV}{dL}=A-\frac{2AL}{D}\times\frac{dD}{dL}$$

Hence $$\frac{L}{D}.\,\frac{dD}{dL}=\frac{1}{2}\left[\,1-\frac{1}{A}\times\frac{dV}{dL}\,\right] \qquad \ldots \quad (4)$$

But, by definition,*

$$\sigma=\frac{L}{dL}\times\frac{dD}{D} \qquad \ldots \qquad \ldots \quad (5)$$

* **The ratio between the lateral strain (dD/D) and the longitudinal strain (dL/L) is called Poisson's ratio.**

Hence $$\sigma = \frac{1}{2}\left[1 - \frac{1}{A}\cdot\frac{dV}{dL}\right] \qquad (6)$$

Thus, by measuring small changes in volume and length consequent upon stretching the rubber with a small weight, σ can be found out.

Method—

(i) Procure a rubber tube as is used in bicycle tyres. Close one end firmly with a rubber stopper having a hole, and the other end with a metal sleeve provided with a hook below. Suspend the tube vertically with the sleeve end below (Fig.–24). Fix up vertically a burette through the bore in the stopper. Set up a metre scale vertically on one side of the tube and attach a small horizontal pointer on the sleeve and adjust the positions of the scale and the pointer in such a way that the latter does not *just* touch the former.

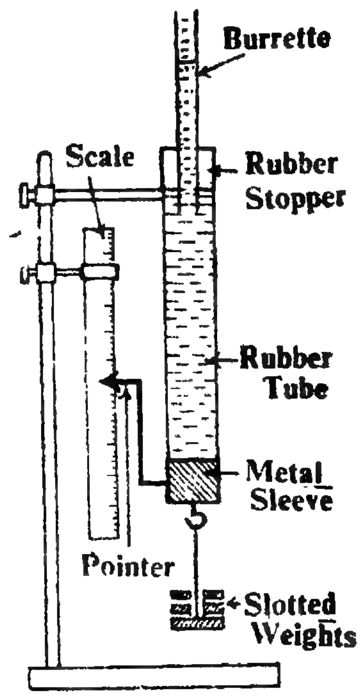


Fig.–24
σ for rubber

(ii) Through the upper end of the burette pour water so that the tube and major part of the burette are filled with it. Measure* the inner diameter of the tube at several places with the help of a vernier callipers and from the mean value of the diameter calculate the area of inner cross-section of the tube.

(iii) Now note the positions of the pointer on the scale and of the water meniscus in the burette. Place very gently a weight on the hook. After waiting for some time take the readings of the meniscus and the pointer. Thus, the values of dV and dL can be calculated from the two readings of the meniscus and the pointer.

(iv) Now go on putting loads† in equal steps and go on taking down the readings of the pointer and the water meniscus.

* It is better to measure the inner diameter of the tube with a travelling microscope in several mutually perpendicular directions.

† Only the maximum permissible load should be applied.

Similarly, take the readings when the loads are progressively decreased till there is no load on the hook.

(v) Now calculate the mean of the two readings of the pointer for the same load on the hook obtained with increasing and decreasing loads. Subtracting the mean reading for no load on the hook from the mean reading for any other load, calculate dL (change in length of the rubber tube) for various loads applied. Similarly, calculate dV (change in volume) from the readings of the burette. Then calculate separately for each set of observation the value of Poisson's ratio and find its means value.

[**Note**—The value of dV/dL can also be obtained graphically. For this pupose, draw (i) a graph between load (M) and water meniscus readings, (ii) another graph between load and pointer readings. The two graphs shall be straight lines. From the first graph calculate dV/dM and from the second calculate the value of dV/dM. By combining the two, get the value of dV/dL, which may be used to calculate the value of Poisson's ratio.]

Observations

[**A**] *Readings for the inner diameter of the tube.*

S. No.	Reading along any direction	Reading along a perp. direction	Mean observed diameter	Remarks
1				
⋮				Vernier constant = ...cm.
⋮				Zero error = ...cm.
10				
		Mean		

[**B**] *Readings for the determination of dV and dL.*

Serial No.	Lead	Readings of the pointer: Load increasing	Readings of the pointer: Load decreasing	Readings of the pointer: Mean	Change in length (dL)	Readings of the burette: Load increasing	Readings of the burette: Load decreasing	Readings of the burette: Mean	Change in volume (dV)	Poisson's ratio (σ)
1	0 gm	...cm.	...cm.	A		...c.c.	...c.c,	A′		
2	200 gm			B	B—A			B′	B′—A′	...
3	400 gm			C	C—A			C′	C′—A′	...
4	600 gm			D	......			D′	...	...
5	800 gm			E	......			E′	...	
6	1000 gm			F	......			F′	...	
									Mean	...

Calculations—

Mean corrected diameter of the tube = ...cm.

Hence area of cross-section of the tube

$$A = \frac{\pi D^2}{4} = \text{.........sq. cm.}$$

First Set

$$\sigma = \frac{1}{2}\left[1 - \frac{1}{A} \times \frac{dV}{dL}\right]$$

$$= \text{.........}$$

[**Note**—Calculate in this way the value of σ for all the sets of observations.]

$\therefore$ Mean σ = ...

Result—The value of Poisson's ratio for rubber = ...*

* Remember there is no unit for Poisson's ratio, since it is a ratio between two strains.

Precautions and Sources of Error—

(1) The theory of the experiment assumes that the extension dL is much smaller than L, henee extension should not be very large.

(2) For the validity of Hooke's law it is essential that the extension of the substance does not exceed the elastic limit. Hence, the load suspended from the end of the tube should be well within the maximum permissible load.

(3) There should be no air bubbles anywhere in the apparatus.

(4) Inner diameter of the rubber tube should be measured at several places and at each place along two mutually perpendicular directions.*

(5) Loads should be of the order of 200 or 250 gms. They should be applied or removed gently. After each addition or removal of load, wait for some time so that the apparatus acquires equilibrium conditions.

EXPERIMENT—13

Object. To determine the restoring force per unit extension of a spiral spring by statical and dynamical methods and also to determine the mass of the spring.

Apparatus Required. A spiral spring, a scale pan, slotted half kgm. weights, stop-watch, metre scale and a pointer.

Formula Employed. By performing the statical experiment the force of restitution (f) for unit extension of the spring can be calculated from the following formula :—

$$f = \frac{Mg}{l} \qquad \ldots \qquad (1)$$

where M=the load suspended from the lower end of the vertically suspended spring.

l=the corresponding extension produced in the spring.

Again, by performing the dynamical experiment, the force of restitution can be calculated from the formula :—

$$f = \frac{4\pi_2 (M_1 - M_2)}{T_1^2 - T_2^2} \qquad \ldots \qquad (2)$$

* The area of cross-section A can be obtained directly by finding the volume of water that fills length L of the tube.

where T_1=time-period when mass M_1 suspended at the lower end is allowed to oscillate.

T_2=time-period when mass M_2 at the lower end oscillates.

The mass of the spring (m_0) is given by the following formula :—

$$m_0=3\left[\frac{M_1T_2^2-M_2T_1^2}{T_1^2-T_2^2}\right] \quad \ldots \quad (3)$$

where the symbols have already been explained.

THEORY AND PRINCIPLE OF THE EXPERIMENT

Let a mass M be suspended from a spring and let it consequently be extended through l. If f be the restoring force per unit extension, then its value for an extension l will clearly be fl. In the equilibrium state

$$fl = Mg$$

$$\text{or} \qquad f=\frac{Mg}{l} \quad \ldots \quad (4)$$

If the extension produced by any load M be determined, then f can be easly calculated from equation—(4).

If the mass at the end of the spring be now displaced vertically downward and subsequently released, then for small amplitudes the restoring force will be proportional to the displacement x (or the restoring force will be equal to f x) and the equation of motion of the spring will be :—

$$M\frac{d^2x}{dt^2}+f\,x=o$$

which is obviously the equation for a Simple Harmonic Motion, whose time-period is given by

$$T=2\pi\sqrt{\frac{M}{f}} \quad \ldots \quad (5)$$

In this deduction no account has been taken of the mass of the spring. Now it can be shown* that the effect of the mass of the spring is the same as though a load equal to one-third the mass of the spring has been suspended at the end of a weightless spring. Hence, if the mass of the spring be m_0 then the formula (5) is modified to the form :—

$$T=2\pi\sqrt{\frac{M+m_0/3}{f}} \quad \ldots \quad (6)$$

To evaluate m and f from relation (6) we should form two equations. For this purpose, if the experiment be performed with

For the derivation of this result, see "Advanced Practical Physics" by Worsnop and Flint, page 93.

two masses successively suspended from the lower end of the spring and the corresponding time-periods be T_1 and T_2, then

$$T_1^2=4\pi^2\ \frac{M_1+m_0/3}{f} \qquad \ldots \qquad (7)$$

$$T_2^2=4\pi^2\ \frac{M_2+m_0/3}{f} \qquad \ldots \qquad (8)$$

Substracting (8) from (7), we have

$$T_1^2-T_2^2=\frac{4\pi^2}{f}(M_1-M_2)$$

whence
$$f=\frac{4\pi^2\,(M_1-M_2}{T_1^2-T_2^2)} \qquad \ldots \qquad (9)$$

Again, by dividing (7) by (8), we have

$$\frac{T_1^2}{T_2^2}=\frac{M_1+m_0/3}{M_2+m_0/3}$$

which on simplification yields

$$m_0=3\left[\frac{M_1T_2^2-M_2T_1^2}{T_1^2-T_2^2}\right] \qquad \ldots \qquad (10)$$

Equations (4), (9) and (10) are the required ones.

[**Note**—A graphical method can also be employed for the evaluation of f and m_0 for a spiral spring. The formula for the time-period can be put down as follows :—

$$T^2=\frac{4\pi^2}{f}.(M+m_0)=\frac{4\pi^2}{f}.M+\frac{4\pi^2}{f}.m_0$$

Hence, if we draw a graph between T^2 and M, we shall get a straight line (Fig.–25). The slope (tan θ) of this line is given by

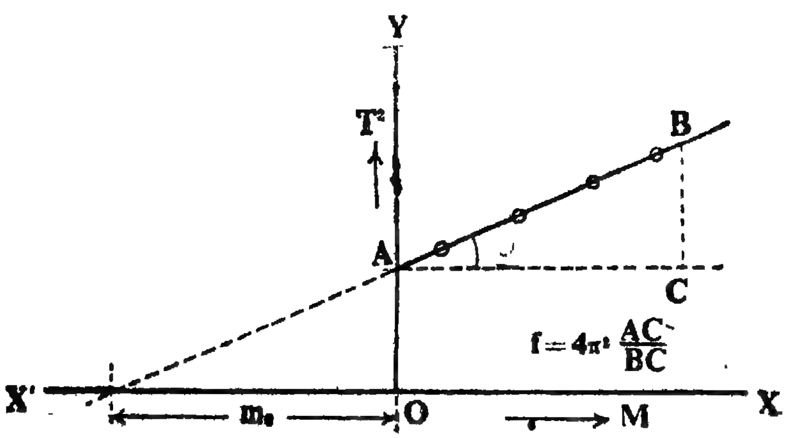


Fig.–25
T^2—M graph for a spiral spring

$$\tan\theta = \frac{4\pi^2}{f}, \quad \text{hence } f = \frac{4\pi^2}{\tan\theta}$$

Again, when $T^2 = 0$, $M = -m_0$. It means that the graph has a negative intercept on the M-axis which is equal to m_0].

Method—

(i) Suspend from a rigid support (Fig.–26) a spiral spring having a scale-pan attached to its lower end. Attach a pointer as shown and adjust it against a vertically held scale in such a way that the pointer slides against the scale without touching it.

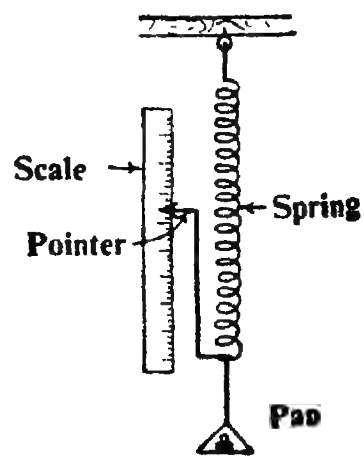


Fig.–26
Spiral spring

(ii) Now perform the statical experiment. For this purpose, put gently a load on the pan and after waiting for some time note the reading on the scale. Now gradually increase the load* in equal steps (say, half a kgm-wt.) and note the readings after each addition of the load.

Then by properly coupling the readings (as indicated in the Table) calculate† the mean extension (l) for a certain load (M) and calculate the value of the force of restitution (f) from the formula (1) given above.

(iii) Now perform the dynamical experiment. For this purpose, place gently a load on the pan and allow the combination to execute simple harmonic oscillations in the vertical direction by slightly pulling the combination downwards and then releasing it. Note the time for twenty-five oscillations‡ and by repeating the process four times calculate the time-period T_1.

Now increase the load a little and determine T_2. Taking the two loads as M_1 and M_2 calculate the restoring force‖ (f) per unit extension from the formula (2) given above and the mass of the spring (m_0) from the formula (3).

[*Note*—Make calculations graphically also.]

Do not suspend more than the maximum permissible load.
In these observations it is not necessary to know the mass of the pan, since it will cancel out during the process of calculating the extension from two readings.
In counting the oscillations, the rest position of the pointer should be taken as the point of reference.
The experiment can be repeated with different M_1 and M_2 and thus mean values for f and m_0 be determined.

ings should be noted after waiting for some time so that the spring acquires stable conditions.

(4) In no case should the load exceed the maximum permissible value otherwise Hooke's law shall not be obeyed.

(5) Mean extension should be calculated by properly coupling the readings so that no reading occurs twice in the calcuation. *Under no circumstances successive difference between consecutive readings of the pointer should be taken.*

(6) While oscillating the spring, see that the amplitude of oscillations is small and that the spring oscillates in a vertical plane.

(7) Note the times very carefully and for this purpose employ an accurate stop-watch. The time-periods occur squared in the formulae and hence any error committed in their evaluation will introduce double the error in the results.

4

Surface Tension

EXPERIMENT—14

Object—To determine the surface tension* of water with the help of Searle's Torsion Balance.

Apparatus Required—Searle's Torsion Balance, weight box, metre scale, a trough (or a dish), screw gauge, an adjustable table, and a thermometer.

Description of the Apparatus—The torsion balance devised by Dr. Searle of Cambridge is depicted in the accompanying diagram. In this apparatus there is a torsion wire which is horizontal and whose ends are fixed by adjustable clamps to the frame of the instrument. To the centre of the wire is attached a light metal beam one end of which serves as a pointer that moves over a graduated scale. The other and shorter end of the beam carries a counter-weight the position of which is adjustable. A small pan is suspended from a fixed point near the end of the longer arm, and whenever necessary, the scale can be calibrated by placing known weights in the pan. From this pan is suspended a chip which can hold a thin rectangular glass plate, which can be turned in its own plane to make its lower edge *perfectly horizontal.* The height of the frame which carries the torsion wire

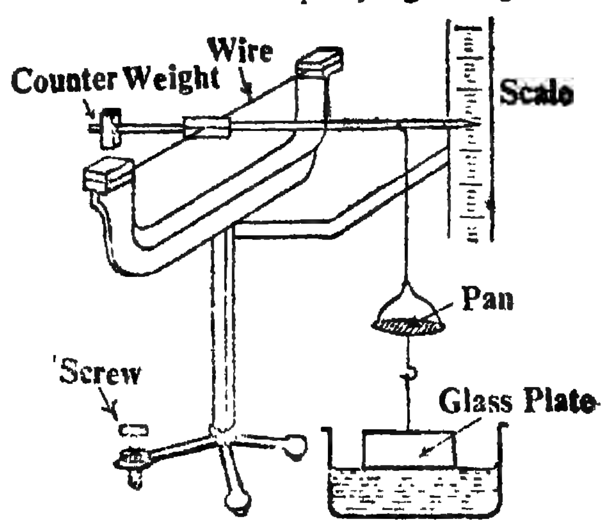


Fig.–27
Searle's Torsion Balance

* For a detailed study of Surface Tension, read author's book, Critical Study of Practical Physics and Viva-Voce".

can be adjusted by means of a vertical sliding rod, and a fine adjustment can be made by using a levelling screw provided at one corner of the base.

Formula Employed—The surface tension T of the liquid is given by the following formula :—

$$T = \frac{Mg}{2(l+t)}$$

where M = weights placed in the pan
l = length of the lower edge of the plate
t = thickness of the lower edge of the plate.

PRINCIPLE AND THEORY OF THE EXPERIMENT

The measurement of the surface tension* of a clear liquid, such as water, can be made by using a clean glass plate (such as a microscope slide) fixed in a clip and suspended from the arm of the torsion balance. *The object in view is to measure the pull on the lower horizontal edge of this plate when it is just touching the surface of the liquid.*

Let l be the length of the lower edge of the plate and t be its thickness, the pull exerted on the plate due to surface tension T is equal to $2(l+t)$ T, because the water touches the plate along a horizontal line of length $2(l+t)$ passing round the plate. Let the dish of water be then removed and the plate allowed to dry. Now, since the film of water has been broken, the downward pull exerted by water ceases to act and consequently the plate rises up and hence the pointer attached to the arm also goes up. Let the pointer be brought back to its previous position on the scale (when the lower edge of the plate was in contact with water) by placing suitable weights (of mass M) on the pan, then obviously

$$2T(l+t) = Mg$$

Hence $$T = \frac{Mg}{2(l+t)}$$

This formula† enables us to calculate the value of the surface tension T, since other quantities occurring on the right of the equation are easily measurable.

Method—

(i) Take a rectangular glass plate (e. g., a microscope slide) and carefully clean it by washing it first with caustic potash solu-

* The surface tension of a liquid is defined as "*the force exerted across unit length of any line imagined to be drawn in the surface of the liquid, the force being tangential to the surface and perpendicular to the line*". Its unit is *dynes per cm.*

† See, in this connection, the footnote given at the end of this experiment.

tion and then with clean water. This process will ensure the removal of dirt and grease. Now with the help of the metal clip suspend the plate vertically *with its longer edge horizonal* from below the pan of the torsion balance. Take a small trough or a dish (which has also been previously cleaned in the manner described above) of clean water and bring it up gradually* below the plate until *contact is just made* between the water surface and the lower edge of the glass plate.

When the edge of the plate touches the water surface, a jerk of the pointer of the balance is observed because of the downward pull due to surface tension. Note the position of the pointer† on the scale.

(ii) Now remove the trough of water and allow the plate to dry (or dry it very carefully with a filter paper). Place suitable weights on the pan of the torsion balance so as to give the same reading of the pointer as before. Then measure the length of the lower edge of the plate with a metre scale and its thickness with a screw gauge. Calculate the value of surface tension‡ of water from the formula given above.

[**Note**—Take other sets of observations with plates of different sizes.]

Observations—

[A] *Reading for the determination of M.*

S. No.	Position of the pointer	Mass put on the pan (M)	Remark
1	......	......gm.	
......			Temperature of water = ...°C
......			
	Mean	......gm.	

* A convenient way of lifting the trough is by supporting it on an adjustable table which can be raised slowly by a screw motion.

† In this operation it is just possible that the pointer may go out of the scale. To get it back on the scale, change the position of the counter-weight. The levelling screw at the base of the instrument may also be used in making the final adjustment.

‡ *Do not forget to note down the temperature of water and to mention the same with the result.*

[B] *Readings for the determination of t.*

S. No.	Thickness of the plate (observed)	Thickness of the plate (corrected)	Remarks
1	.........cm.	.........cm.	Pitch of the screw gauge= ...cm. No. of divs. on the screw head = ... ∴ L.C. =..... cm. Zero error =...cm. Length of the lower edge of the plate =........cm.
......			
......			
......			
10			
	Mean	.........cm.	

Calculations—

$$T = \frac{Mg}{2\,(l + t)}$$

dynes per cm.

Result—The surface tension of water at ...°C as found out by Searle's torsion balance = ...dynes per cm.

[Standard value = ...dynes/cm. ; Error = ... %].

Precautions and Sources of Error—

(1) Before making use of either the plate or the dish it should be very carefully cleaned first with caustic potash solution and then with clean water. This will ensure the removal of dirt and grease, the presence of even minute traces of which shall spoil the whole experiment.

For the same reason, distilled water should not be employed in the experiment.

(2) The plate should be suspended vertically *with its lower edge horizontal* so that a film of water of uniform thickness is formed and effects of the vertical sides due to surface tension are also eliminated.

(3) The reading of the pointer should be carefully taken by avoiding the error due to parallax.*

(4) The lower edge of the plate should be adjusted on the surface of water *so that it just touches the latter.* This adjustment will not necessitate the correction due to the upthrust of water on the plate.

(5) As the surface tension of a liquid varies with its temperature* the latter should invariably be recorded and also reported with the result.

(6) The main source of error arises from the fact that it is extremely difficult to adjust the plate in such a way as may ensure its just touching the free surface of the liquid. If the edge is slightly below the surface of the liquid, there will be lot of uncertainty in the value of the upthrust which will consequently limit the accuracy‡ of the result obtained with this experiment.

EXPERIMENT—15

Object—To determine the surface tension of water by measuring its rise in a capillary tube.

Apparatus Required—Glass-tubes, beaker, glass plate, a pin bent twice at right angles, thin rubber bands, plumb line, miscroscope with a mircometer eye-piece and a thermometer.

Formula Employed—The surface tension T of a liquid is given by the formula** :—

$$T = \frac{r\, d\, g\, (h + r/3)}{}$$

* A greater accuracy can be achieved in this observation if a fine pin is fixed to the end of the pointer which moves over a thin plane glass strip.

‡ In this connection, see the Note given at the end of Expt.-16.

† The degree of inaccuracy introduced in the result due to buoyancy of the liquid can be estimated by an easy calculation. If x be the depth to which the lower edge of the plate is immersed below the surface of water, then the mass of water displaced is equal to $l\,t\,x$ gm. or the upthrust is equal to $l\,t\,x\,g$ dynes. Thus the formula given above reduces to

$$2\,(l + t)\,T - l\,t\,x\,g = Mg$$

$$T = \frac{Mg}{2\,(l + t)} + \frac{l\,t\,x\,g}{2\,(l + t)}$$

The second term occurring on the right is the correction term. If $l = 10$ cm., $t = 0{\cdot}1$ cm. and $x = 0{\cdot}1$ cm. then the second term is equal to $4{\cdot}9$ and if $T = 70$ dynes per cm., the error in the result due to this cause is 7%.

** The second term inside the brackets is a correction term, which, in actual practice, is very small and hence it can be neglected.

where r = radius of the capillary tube *at the liquid meniscus.*
d = density* of the given liquid
h = height of the liquid column in the capillary tube above the free surface of the liquid in the beaker.

PRINCIPLE AND THEORY OF THE EXPERIMENT

When a capillary tube is held vertically with its one end dipped in a liquid in a large vessel, the liquid rises in the tube and a column of measurable height remains projecting above the level of the liquid in the outer vessel.

This column of the liquid is supported by the tube as a result of surface tension of the liquid, and the surface tension can be determined from the height of the liquid column and the dimensions of the tube.

Let the radius of the tube *at the place of the liquid meniscus* be r cm. and the surface tension be dynes per cm. At the line where the liquid surface and the tube meet there is exerted at right angles to their line of contact a force of T dynes on each cm. of that line. This force is exerted on the wall of the tube by the surface of the liquid and acts in the liquid surface at right angles to the line of contact. Thus, if the tangent to the liquid surface at this line is at an angle† θ with the side of the tube, we have a force acting at an angle θ to the vertical, the magnitude of this force being T dynes per cm. (Fig.–28)

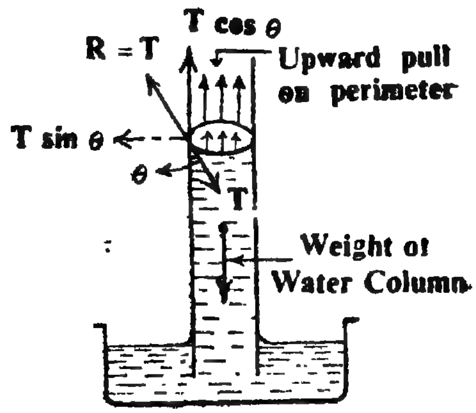


Fig.–28
Forces on a liquid column in a capillary tube.

This force which is acting at all points at an angle θ to the vertical may be resolved into vertical and horizontal components. We are concerned with the vertical components only, since the horizontal components acting along the whole meniscus cancel out. There will thus be a total vertical force equal to $2\pi r T \cos\theta$ exerted by the liquid on the tube across the line of contact, this force being exerted downwards by the liquid on the tube. Now since action and reaction are equal and opposite,

* In the case of water d = 1. For other liquids d has to be determined separately, *e. g.* with an R. D. bottle.

† This angle is known as the *Angle of Contact* which may be defined as "the angle between the tangent plan to the liquid surface and the tangent plan to the solid surface *in the liquid* at any point on the line of contact."

the tube exerts forces on the liquid across the line of contact, such that the total upward force exerted on the liquid by the tube is $2\ \pi r\ T.\cos\theta$ dynes. This force supports the column of liquid raised above the level outside, hence, if we can find the weight of this liquid column, its weight must be equal to the above force.

Weight of the liquid column raised—Let h be the height of the liquid column measured vertically from the bottom of the meniscus to the horizontal part of the free surface in the outside vessel. Then the volume of the liquid contained in this cylindrical column is $\pi r^2 h$. Now, to calculate the volume of the liquid contained in the meniscus, let us assume that the meniscus is hemispherical* in shape. Then the volume of the liquid contained above the horizontal plane to the meniscus is equal to the difference of the volume of the cylinder ABCD (Fig.–29) of radius r and height r, and the volume of the hemisphere AOD. Thus, the required volume of the liquid.

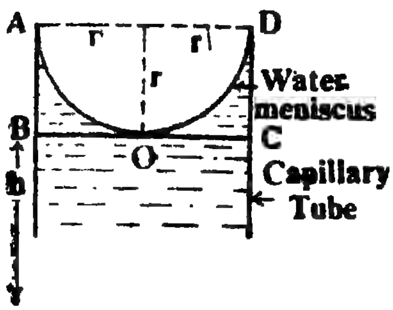


Fig.–29
Water meniscus
in a cappillary tube.

$$= \pi r^2.r - \tfrac{2}{3}\pi r^3 = \tfrac{1}{3}\pi r^3$$

Thus, the total volume of the liquid column supported inside the capillary tube

$$= \pi r^2 h + \tfrac{1}{3}\pi r^3 = \pi r^2 (h + r/3)$$

If the density of the liquid be d, then the weight of the liquid column

$$= \pi r^2 (h + r/3)\ dg$$

Equating this to the resultant force exerted by the tube on the liquid, we have

$$2\ \pi r\ T\cos\theta = \pi r^2 (h + r/3)\ dg$$

$$\therefore T = \frac{r\ d\ g\ (h + r/3)}{2\cos\theta}$$

For water which wets† the glass surface $\theta = 0$, hence

$$T = \frac{r\ d\ g\ (h + r/3)}{2}$$

which is the required formula.

---

* This will be so when the bore of the capillary tube is *very fine*. Thus, this assumption is only partially true in actual paractice.

† Angles of contact change largely with the freshness of the surface in contact. If the glass surface is *quite clean*, the angle of contact for water is 0° ; if not, the angle may vary between 8° — 9°.

Method—

(i) Take several pieces of glass tubing and clean them carefully with caustic soda and then with nitric acid,* washing out the nitric acid with considerable quantities of water.

Then dry the tubes with dry air forced through them. Draw capillary tubes with the help of a Bunsen or blowpipe flame. Select capillaries of uniform bore for your experiment.

(ii) Now fix the capillary tube parallel to the length of a clean glass plate by means of two thin rubber bands which also support a pin bent twice at right angles. Hold the plate vertically† in such a way that the capillary tube dips in the liquid and the pin is just not touching the free surface. The liquid rises in the capillary tube. Raise the beaker a little. The liquid column will be seen to rise, and it will fall‡ as soon as the beaker is lowered.

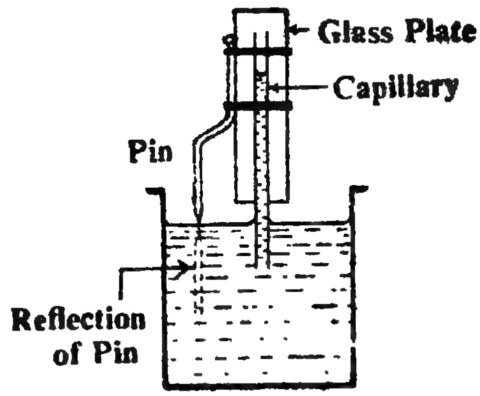


Fig.-30
Surface tension by capillary tube.

This process of raising and lowering of the beaker shall also ensure that the bore of the capillary in the neighbourhood of the meniscus is wetted, thus making the angle θ occurring in the formula equal to zero.

(iii) Measure the height h of the liquid column with the help of a travelling microscope. For this purpose, focus the microscope first on the meniscus, then on the point in such a way that when the images of the pin and its reflection are viewed through it, the cross-wire is exactly between them. The vertical distance through which the microscope has to be displaced between these two positions is measured on the microscope stand and h is thus obtained very accurately.‡‡

* Caustic soda is used first to remove any grease in the tube ; it is used before the acid because the latter can be washed out more easily with water.

† Test the verticality of the plate with a plumb line.

‡ If the liquid column does not fall back readily, it means that the tube or the liquid or both are not free from contamination. In that case they should be replaced by cleaner ones.

‡‡ A simplified but less accurate method of measuring h is with the help of dividers, setting the dividers so that when one point is just at the surface of the liquid in the beaker, the other end is at the level of the meniscus in the tube. Alternatively, the capillary tube may be attached on to a *clean* scale made of stainless steel or celluloid, and h measured directly from it. In these cases the bent pin is not used with the apparatus.

(iv) Now measure the radius of the tube with a travelling microscope or with a microscope having a micrometer eye-piece. This is the most important part of the experiment and consequently it should be done very carefully.

For this purpose, break the capillary at the position of the meniscus which should be marked with ink on the outside of the tube.*

Keeping the broken end uppermost, fix one piece with red wax on a glass plate and measure the diameter along two mutually perpendicular directions by setting the cross-wire *tangential to the inner boundary line of the section* in each adjustment. Repeat the process for the other broken piece also. The mean of the values of the four diameters† gives 2r.

(v) Then calculate the value of the surface tension by making use of the formula given above.‡ Lastly, note the temperature of water and report the result accordingly.

Observations—

[A] *Readings for the determination of h.*

No. of capillary tube	Reading of the meniscus	Reading of the level outside	h	Remarks
1	.........cm. ....cm. ....cm.	.........cm. ....cm. ....cm.	...cm.	L. C. of the microscope scale =......cm.
2				Temp. of water = ...°C
⋮				

* Under no circumstance should the tube be broken by bending it, otherwise the fracture will be uneven. To get a clear fracture perpendicular to the length of the capillary, make a scratch on the tube at the ink mark with a tiny file and apply the tension *along* the length of the tube.

† The four readings should vary (if at all) amongst themselves by a very small amount.

‡ (1) The experiment can be performed with four or five tubes of different bores, and h can be shown to be inversely proportional to r.

(2) If any liquid other than water be used, its density must be determined (*e. g.*, with a relative density bottle) before T can be calculated.

[B] *Readings for the determination of diameter.*

(i) Readings for one broken end of the tube.

S. No.	Reading along any diameter	Reading along a perp. diameter	Diameter (Mean)	Remark
1	......cm.	......cm.		
2	......cm.	......cm.	......cm.	L. C. of the microscope scale=...cm.
3	......cm.	......cm.		

(ii) Readings for the other broken end of the tube.
[Note—Make a similar table.]

Calculations—

Mean diameter of the capillary tube =...cm.

∴ Mean radius of the capillary tube =...cm.

Now*
$$T = \frac{rdg\,(h + r/3)}{2}$$
=......dynes per cm.

Result—The surface tension of water at...°C = ...dynes/cm.
[Standard value = ...dynes/cm.
Error = ...dynes/cm=...%]

Precautions and Sources of Error—

(1) The diameter of the tube should be measured *at the meniscus,* because the rise of the liquid in the capillary tube depends upon the value of the radius at this place.

(2) The capillary tube selected for the purpose should have a uniform bore throughout, otherwise the measurement of the diameter even slightly below or above the meniscus may cause considerable error in the result.

(3) The diameter of the bore should be measured in two mutually perpendicular directions. While taking readings for the diameter, the cross-wire of the microscope should be adjusted *tangential to the inner* edge of the section of the bore. Moreover,

* If h has been measured with an ordinary scale, the correction term (r/3) can be neglected.

both the broken pieces of the tube should be employed for this determination.

(4) The back-lash error with the microscope should be avoided by always turning the screw in the same direction.

(5) Great care should be observed in cutting the tube, otherwise the section will present a very rugged and uneven appearance and the measurement of diameter will be unsatisfactory.

(6) The verticality of the capillary tube in the beaker should be tested with a plumb line.

(7) As every type of contamination is detrimental for surface tnsion experiments, special care should be observed in cleaning the tube, the beaker and the plate. For the same reason, tap water should always be preferred to distilled water since the latter is likely to have traces of grease.

(8) Temperature of water should always be recorded since the value of the surface tension of a liquid depends upon its temperature.*

(9) One of the sources of error in this experiment is the probable contamination of the liquid surface as also the contamination of the capillary tube, the cleaning of which is tedious.

(10) The chief source of error, however, is in the determination of the radius of the bore. According to theoretical consideration the radius of the tube should be measured at the place of the meniscus. A great accuracy in the measurement of the radius is possible only when the tube has a uniform bore†. If the tube happens to be of an ununiform bore, the measurement of diameter even slightly below or above the meniscus shall obviously result in considerable error in the value of the surface tension. To get a capillary tube of absolutely uniform bore throughout is extremely difficult in practice.

(11) The glass tubes available in the laboratory are not generally clean inside. If before drawing the capillaries, they have not been properly cleaned, then due to the presence of contamination the result will be adversely effected. Similarly, if the water surface is dirty or the beaker has not been properly cleaned, this will also constitute a source of error.

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* In this connection, see Note given at the end of Exp.–16.

† The uniformity of the bore can, however, be tested by introducing a thread of mercury in the tube and measuring the length of the thread in different positions which, if the bore is uniform, will be the same everywhere. If the capillary tube is found to be uniform throughout, this procedure can be adopted for determining the value of the radius accurately by weighing the mercury introduced in the tube. However, this procedure cannot be employed in a laboratory practice.

EXPERIMENT—16

Object—To determine the surface tenison of a liquid (water) by Jaeger's method.

Apparatus Required—Jaeger's apparatus, glass tube, a scale, thermometer, and a microscope having a micrometer eye-piece, or a travelling microscope.

Description of the Apparatus—The apparatus consists of a thin glass tube drawn out at its lower end to a fine capillary C (of radius 0·2 mm. to 0·5 mm.) fixed vertically in the experimental liquid. The capillary tube is connected to a manometer M and also to a bottle B, which in turn is connected to a water reservoir R. By means of the stop-cocks S_1 and S_2, bubbles of air are allowed to form at C. This is accomplished by allowing the water from R to flow out in B at a slow and uniform rate, thereby forcing out air from B through the connecting tube to C where it comes out in the form of bubbles*.

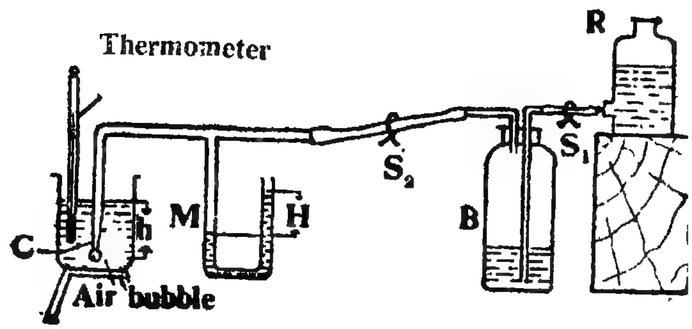

Fig.–31. Jaegar's apparatus.

Formula Employed—The surface tension T of the liquid is given by the formula :—

$$T = \frac{Rg}{2}(Hd_1 - hd_2)$$

where R = the radius of the orifice of the capillary tube.

H = maximum reading of the manometer just before the air bubble breaks away.

d_1 = density of the liquid† in the manometer.

h = depth of the tip of the capillary tube below the surface of the liquid.

d_2 = density of the experimental liquid (in the case of water $d_2 = 1$).

* For studying the variation of surface tension with temperature, the experimental liquid can be heated and its temperature can be recorded by a thermometer immersed in it.

† Xylol (density = 0·86 gm. per c. c.) is a suitable liquid for manometric purposes.

PRINCIPLE AND THEORY OF THE EXPERIMENT

On account of surface tension the free surface of a liquid always tries to become plane. If, under certain circumstances, the face is maintained curved, then an excess pressure (equal to 2T/R for spherical surfaces) has to be maintained on the concave side. In the above formula R is the radius of the spherical surface.

In the Jaeger's method, air bubbles are formed in the experimental liquid and when the bubble is just detached from the tube the excess pressure is observed. Thus, by knowing the value of the excess pressure and the radius of the bubble, the surface tension of the liquid can be calculated.

When the capillary tube in the Jaeger's apparatus is dipped in the liquid, liquid rises in it and the meniscus is approximately hemispherical. If the pressure inside is increased (by dropping water from R into B), it pushes the water column in the capillary tube lower and lower and ultimately an air bubble begins to form at the orifice of the tube. The curvature goes on increasing until finally a hemispherical bubble of radius equal to that of the orifice protrudes into the liquid. At this stage, the bubble gets unstable because, if it grows further its curvature decreases (i. e., its radius of curvature increases) and hence the excess pressure required at this stage falls below the excess pressure present. The bubble, therefore, gets detached from the tube. The pressure falls a little and again begins to build up as the next bubble is forming.

To calculate this excess pressure let us consider a soap bubble which is assumed to be spherical and of radius R. Considering the equilibrium of the upper hemisphere (Fig.—32) this excess pressure p acts on the upper hemisphere and exerts on it an upward resultant force whose magnitude is given by p. πR^2. This force tends to blow the two hemispheres apart. However, this disruptive tendency of the bubble is counter-balanced by forces due to surface tension which acts in the *two surface* of the film round the line of contact. Since the film has two surfaces, the total force trying to keep the hemispheres together is equal to 2 (2π R.T). Thus

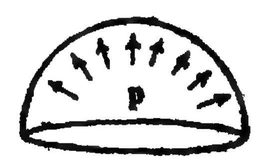

Fig.-32
Excess pressure in a bubble.

$$4\pi RT = p.\pi R^2 \qquad \text{or} \qquad T = \frac{pR}{4}$$

If, instead of a soap film, we consider the drop of a liquid or an air bubble formed in a liquid (as in the present experiment), the force due to surface tension will act only upon one surface and hence in this case

$$2\pi RT = p.\pi R^2 \qquad \text{or} \qquad p = \frac{2T}{R}$$

Now, in the Jaeger's experiment, if H be the maximum pressure difference as recorded by the manometer, then the air pressure inside the bubble, when it is just on the point of being detached from the orifice, is $P+Hg.\,d_1$ where P is the atmospheric pressure and d_1 is the density of the liquid in the manometer. Similarly, the pressure outside the bubble is $P + hg.\,d_2$ where h is the depth of the tip of the capillary tube below the free surface of the liquid and d_2 is the density of the liquid. Thus, the excess pressure inside the bubble is

$$(P + Hg.\,d_1)-(P + hg.\,d_2) = g\,(Hd_1 - hd_2)$$

and this must be equal to $\frac{2T}{R}$. Thus

$$\frac{2T}{R} = g\,(Hd_1-hd_2)$$

$$\text{or} \qquad T = \frac{Rg}{2}\,(Hd_1-hd_2)$$

If the experimental liquid is water, $d_2 = 1$, then putting $d_1=d$, we have for the surface tension* of water—

$$T = \frac{Rg}{2}\,(Hd-h).$$

Method—

(i) Take a glass tube and clean it carefully with caustic soda and then with nitric acid, washing out the nitric acid with considerable quantities of water. Then dry the tube with dry air forced through it. Draw at its one end a capillary of about 0·2 mm. to 0·5 mm. diameter. Cut the capillary square, so that when viewed under a microscope the edges present an extremely clear and well-defined circular appearance. Clamp the tube† in a vertical position below the surface of the experimental liquid contained in a beaker. Then arrange the apparatus as shown in the figure.

(ii) Now allow water to drop in the bottle B from the reservoir R in such a way that successive bubbles are formed at the orifice of the capillary tube at intervals of ten seconds. In order to get bubbles singly and at large intervals, it is advisable to keep the volume of

* If the manometer contains water, d in the formula should be put equal to 1.

† To ensure that the bubbles are formed at the same depth h and also to facilitate its measurement, a scratch may be made on the body of the tube which should be submerged in the liquid upto this mark.

air in the bottle B small. The pressure indicated by the manometer rises until the bubble becomes unstable, and suddenly falls as the bubble is detached from the tube. Note down the maximum difference in the liquid columns in the two limbs of the manometer. The difference of the readings for the two limbs repeated a number of times gives the value of H.

(iii) Now by keeping the capillary orifice underneath a vernier or micrometer microscope determine its diameter* in two mutually perpendicular directions.

Measure h (the distance of the orifice from the scratch) carefully with the microscope. Then determine the density † of the manometer liquid with the help of a specific gravity bottle. Then calculate the value of surface tension of the liquid from the formula given above. Note the temperature of the liquid in the beaker.

Note—The experiment can be repeated if other jets of different orifices are available. If it is not possible, then different sets of readings can be taken by changing the depth of immersion (h) of the jet.]

Observations—

[A] *Readings for the determination of H.*

Manometer Readings		H	Remarks
Upper level of the liquid	Lower level of the liquid		
......cm.	......cm.	......cm.	(1) Temp. of water = ... °C (2) R. D. of manometer liquid (given) =......
	Mean	......cm.	

* In this determination, adjust the cross-wire *tangentially* to the *inner* edge of the orifice.

† If the manometer contains water, $d_1 = 1$, or if it contains a given light oil, its density may be known from the Table of Physical Constants. In that case, this determination is unnecessary.

[B] *Readings for the diameter of the orifice.*

Serial No	Reading along any direction			Reading along perp. direction			Mean	Remark
	I End	II End	Diameter	I End	II End	Diameter		
1	..cm.	...cm.	...cm.	...cm.	...cm.	...cm.		L. C. of the microscope =......cm.
⋮								
⋮								
5								
						Mean	.....	

[C] *Readings for the determination of h.*

S. No.	Reading at the end of jet	Reading at the scratch	h	Remark
1	.........cm.	.........cm.	...cm.	L. C. of the microscope =...cm.
⋮				
⋮				
⋮				
		Mean	...cm.	

Calculations—

Mean radius of the orifice = ...cm.

Now $\frac{Rg}{\ } (Hd_1 - hd_2)$

......dynes per cm.

Result—The surface tension of water at......°C = ...dynes per cm.

[Standard value = ...dynes/cm.
Error = ...dynes/cm. = ...%].

Precautions and Sources of Error—

(1) The capillary tube employed in this experiment should be scrupulously clean as even traces of grease are extrermely detrimental to surface tension experiments.

(2) In the theoretical considerations of the formula it has been assumed that the maximum pressure in the bubble occurs when it is hemispherical. To satisfy this condition the orifice of the capillary tube should be circular and small (say, of the order of 0·03 cm.)

(3) The apparatus should be perfectly air-tight. It is, therefore, advantageous to have the whole apparatus in one single price avoiding the use of rubber joints.

(4) The maximum pressure recorded by the manometer should be independent of the formation of bubbles. For this purpose their formation should be so regulated that successive bubbles appear at intervals of nearly ten seconds.

(5) The manometer should contain a liquid of low density*.

(6) Temperature† of water should invariably be recorded and reported with the result.

(7) While taking readings with the microscope, back-lash error should be avoided. The diameter of the orifice should be measured in two mutually perpendicular directions.

(8) The chief source of error in this experiment lies in the assumption that the maximum pressure occurs in the bubble when it is hemispherical (and thus its radius is equal to the radius of the orifice of the capillary tube). Now this is true only when the diameter of the orifice is infinitely small—a fact which cannot obviously be realised in practice‡.

(9) Secondly, the pressure difference H is a small quantity

* For this purpose, xylol (density = 0·86 gm./c.c.) is preferable since its density is less than that of water.

† In this connection, see the Note given at the end of this expt.

‡ An accurate formula for the calculation of surface tension without making use of the assumptions involved in the derivation of the above formula is the following :—

$$T = \frac{Rg}{2}\left[Hd_1 - \left(h + \frac{2r}{3}\right)d_2\right]$$

and is measured by a liquid manometer which is not susceptible of yielding very great accuracy.

(10) The orifice of the tube, according to theory, should be exactly circular and it should be held so that it is horizontal. Now these two conditions cannot be fully realised in practice.

(11) Greasy matter, even in traces, considerably affects the result.

VARIATION OF SURFACE TENSION WITH TEMPERATURE

For small ranges of temperature, the surface tension of a liquid varies linearly with temperature. If T_0 be the surface tension at 0°C, then at t°C its value is given by

$$Tt = T_0 (1 - a\, t)$$

where a is a constant known as temperature coefficient of surface tension.

However, the most comprehensive relation representing the variation of surface tension with temperature is the Eotvos—Ramsay—Shields formula :

$$T\,(Mvx)^{2/3} = k\,(\theta_c - \theta - d)$$

where T = Surface tension at θ°K

M = Molecular weight to the unassociated liquid.

v = Specific volume of the liquid

x = degree of association of the liquid

θ_c = Critical temperature of the liquid

and k, d are constants.

From this formula it is clear that T vanishes when $\theta = \theta_c - d$, *i. e.*, the surface tension of a liquid vanishes at a temperature a few degrees below the critical temperature.

5

Viscosity

EXPERIMENT—17

Object—to determine the coefficient of viscosity* of water by observing its flow through a capillary tube.

Apparatus Required—Viscosity apparatus, constant level water tank, a capillary tube, a beaker, a graduated cylinder, a stop-watch, a travelling microscope and a thermometer.

Description of the Apparatus—A suitable form of apparatus to use in an experimental determination of the coefficient of viscosity by this method is depicted in Fig.—33.

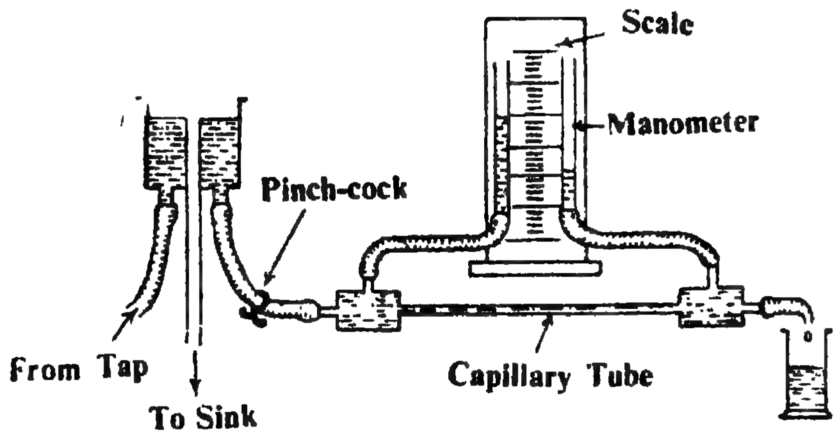

Fig.–33. Viscosity Apparatus.

AB is the capillary tube through which water is allowed to flow from a constant level water tank R. The tank is provided at the bottom with inlet and outlet tubes near the sides and with a constant level overflow tube in the middle. From the unions, provided at the ends of the capillary tube, two lengths of India-rubber tube make connection with the manometer M. The difference in

* For a detailed study of viscosity read author's book "A Critical Study of Practical Physics and Viva-voce."

the levels between X and Y gives the value of the pressure difference between the ends of the experimental tube in cm. of water. A pinch-cock C enables the flow of water to be regulated.

Formula Employed—The coefficient of viscosity η of a liquid is given by the formula :—

$$\eta = \frac{\pi P R^4}{8 V l} = \frac{\pi (hdg) R^4}{8 V l}$$

where P = difference of pressure at the two ends of the capillary tube

R = radius of the capillary tube

V = volume of water collected per second

l = length of the capillary tube

h = reading of the manometer

d = density of the liquid (for water d = 1).

PRINCIPLE AND THEORY OF THE EXPERIMENT

When adjacent layers of a liquid move with a relative velocity, forces (known as viscous forces) are brought into play which tend to reduce this relative movement of the layers.

If we consider a liquid whose upper layer CD (Fig.-34) is moving with a velocity v, while the lower layer AB is at rest, then the intermediate layers will have the velocity distribation as depicted in the diagram by the length of the arrows.

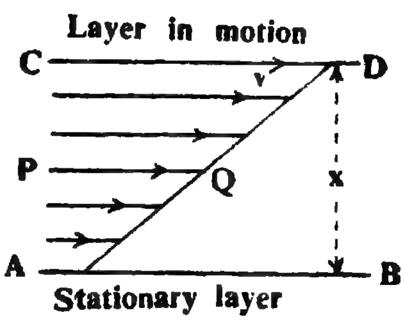

Fig.-34
Velocity distribution in a liquid.

The force acting on any area in a plane at right angles to the diagram, and parallel to PQ, is proportional to the area A, and to the velocity gradient v/x.

Thus

$$\text{Force} \propto \text{Area} \times \text{Velocity gradient} \propto A.\frac{v}{x}$$

or, if we take two layers separated by a distance dx and moving with a relative velocity dv, then

$$F \propto A.\frac{dv}{dx} \qquad \text{or} \qquad F = -\eta A \frac{dv}{dx}$$

where η is a constant for the liquid and is called the *coefficient of the viscosity.* Thus

*"The coefficient of viscosity of a liquid is defined as the tangential force per unit area required to maintain a velocity gradient of unity between two parallel layers in the liquid."**

If we consider the flow of a liquid down a tube, the axial layer moves with a definite velocity and the layer in contact with the wall is at rest. If the pressure difference causing the flow is not too great, the result is the regular type of motion (known as *stream-line motion*). This occurs for small rates of flow only, i. e., when the velocity of flow is below a certain limiting value called the *critical velocity*. If the pressure acquires a value which results in the liquid reaching a velocity beyond the critical value, the liquid no longer proceeds in this stream-line flow. The result in this case is called *turbulent motion.*

We will assume, in the discussion that follows, that the pressure applied at the end of the tube is below this critical pressure, and that the motion of the liquid in the tube is therefore, regular or stream-lined.

Now the value of the co-efficient of viscosity of a liquid can be obtained by measuring the quantity of liquid passing in a known time through a capillary of uniform bore, when a definite pressure difference exists between the ends of the tube.

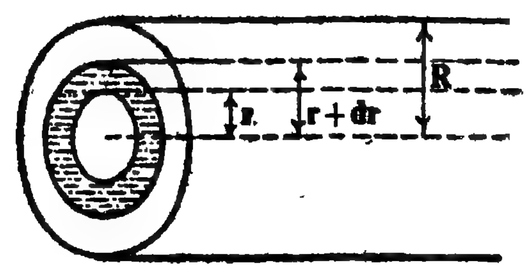

Fig.–35
Section of a liquid cylinder.

Let us consider the flow of the liquid through a cylinder of radius R (Fig.–35). The velocity of the liquid along the axis of the cylinder will be maximum and it will go on decreasing as we move towards the wall where the velocity will be reduced to zero. Let us consider the cylindrical shell of radius r and r + dr. When steady conditions are reached, let the velocity at a distance r from the axis be u. Then the velocity gradient is du/dr and the viscous drag per sp cm. of the inner cylinder is η.du/dr. But the area of the inner cylinder is $2\pi r l$, where l is the length of the cylinder. Thus, the total drag on the inner cylinder is given by

$$F = -2\pi r l.\eta \frac{du}{dr} \quad \dots \quad \dots \quad (1)$$

This drag or viscous force acts in a direction opposed to the pressure gradient. The force due to the pressure P existent at the

* The unit of coefficient of viscosity *"dynes per sq. cm. per unit velocity gradient"* is also called *"poise"* in honour of Poiseuille.

ends of the liquid cylinder tending to accelerate it is $P\pi r^2$, and for steady conditions the two forces be equal. Thus, we have

$$-2\pi r l.\eta \frac{du}{dr} = \pi r^2 P$$

or
$$\frac{2\eta l}{P} du = -r \cdot dr \tag{2}$$

Now integrating this expression we have

$$\frac{2\eta l}{P} . u = -\frac{r^2}{2} + C \quad \dots \tag{3}$$

where C is a constant whose value can be found out from the boundary conditions. Now, at the wall of the tube the liquid is at rest, *i. e.*, when $r=R$, the velocity $u=0$. Substituting these values in (3) we have

$$0 = -R^2/2 + C \qquad \therefore C = R^2/2$$

Substituting this value of C in (3) we have

$$\frac{2\eta l}{P} . u = \frac{1}{2}(R^2 - r^2)$$

or
$$u = \frac{P}{4\eta l} \times (R^2 - r^2) \quad \dots \quad \dots \tag{4}$$

This gives the velocity (u) of the liquid at any distance (r from the axis of the tube.

The volume dV of liquid which flows through the tube per second between the radii r and $r + dr$ is given by

$$dV = 2\pi r.\ dr.\ u = \frac{\pi P}{2\eta l}(R^2 - r^2)\ r.\ dr$$

and the total volume V flowing through the tube per second is obtained by integrating this expression between the limits $r=0$ to $r=R$. Thus

$$V = \int_0^R \frac{\pi P}{2\eta l}(R^2 - r^2)\ r.\ dr.$$

$$= \frac{\pi P R^4}{8\eta l} \quad \dots \quad \dots \tag{5}$$

which is the required relation. This relation is known as *Poiseuille's formula* for the flow of a liquid through a capillary tube. As we have already seen above, *this formula holds good only when the motion of the liquid in the tube is regular or stream-lined.* This stream-line motion occurs so long as the liquid does not move with a velocity greater than the critical velocity.

Now, this critical velocity v is given by the formula, $v=k\eta/\rho R$, where ρ is the density of the liquid and k is a constant which is known as Reynold's number. Thus critical velocity is inversely proportional to the radius of the tube. It is for this reason that Poiseuille's formula holds good for capillary tubes over a wide range of pressures applied ot the ends of the tube.

Method—

(i) Set up the apparatus as shown in the figure. Allow the water from the tap to run into the tank and adjust its flow at such a rate through the inlet tube of the tank that the level of water in the tank is maintained constant. Arrange the flow through the capillary tube such that the emergent water issues out as *a slow trickle or succession of drops*, so that the kinetic energy ot the liquid is small.

(ii) When the flow is regulated and a steady flow takes place in the tube, insert the end of the exit rubber tube in a graduated cylinder and collect water for a known interval of time and find its volume. From this calculate V, the volume of water flowing per second through the capillary tube. Measure the pressure difference h as shown by the manometer. Repeat the readings for V and h.

(iii) Now alter the pressure difference by raising or lowering the water tank and thus obtain several values of V corresponding to different values of h and calculate a mean value* of h/V. Also note the temperature of water.

(iv) Measure the length of the capillary tube. Then measure the diameter of the bore of tube with a vernier or micrometer microscope†. For this purpose, clamp the tube in a horizontal position and focus the microscope in such a way that *cross-wire is tangential to the inner rim of the circular section.* Take the reading of the microscope for this setting. Then set the cross-wire tangential to the inner edge of the opposite side of the above and take the reading of the microscope. Difference between the two readings given the diameter of the capillary tube. Turning the

* The value of h/V can also be determined graphically. The graph of V against h should be linear. For large values of h the departure from the straight line in the graph indicates the presence of turbulent motion in the tube. From the linear part of the graph obtain the value of h/V.

† The bore of the tube can also be measured (and this method should be adopted whenever a greater accuracy is required) by drawing a thread of mercury into it, and measuring the length of the thread with a vernier microscope. The thread is then run out into a weighed watch-glass and its mass determined. Now mass of the mercury thread $= \pi R^2 l' d'$, where l' is the length of the thread and d' is the density of mercury.

capillary through a right angle measure, in a similar manner, the *diameter in a mutually perpendicular direction.* Repeat these observations several times for this end, and then again for the other end of the tube.

(v) Having thus determined h/V, *l*, and R, calculate the value of the coefficient of viscosity of the liquid from the formula given above.

Observations—

[A] *Readings for the determination of h and V.*

S. No.	Manometer reading (h)	Volume of water collected	Time taken	Volume flowing per sec. (V)	$\frac{h}{V}$	Remark
1	......cm.	...... c. c. ...... c. c. Mean=... c. c.	...sec ...sec ...sec	—...c. c.		Temperature of water =...°C
2						
3						
				Mean		

[B] *Readings for the determination of the diameter of the tube.*

(i) Readings for one end of the tube.

S. No.	Reading along any direction			Reading along a perp. direction			Remark
	I End	II End	Dia-meter	I End	II End	Dia-meter	
1	...cm.	...cm.	...cm.	...cm.	...cm.	...cm.	L. C. of the microscope =...cm.
:							
:							
		Mean	...cm.		Mean	......cm.	

(ii) Readings for the other end of the tube.
[Note—Prepare a similar table].

Calculations—

Mean radius of the tube = ... cm.

Now $\eta = \dfrac{\pi (h\ d\ g) R^4}{8 V l}$

$= \dfrac{\pi g R^4}{8l} \times \left(\dfrac{h}{V} \right)$ (since d = 1)

= ...... poise

Result—The coefficient of viscosity of water at .... °C
= ...... poise

[Standard value = ... poise
Error = ... poise, or = ... %]

Precautions and Sources of Error—

1. The capillary tube chosen for the experiment should have a uniform bore as far as possible. The bore should not exceed 0·5 mm. in diameter since the formula holds good in the case of narrow tubes. For wider tubes the flow becomes turbulent.

2. The horizontality of the tube should be tested with a spirit level.

3. The pressure difference between the ends of the capillary tube should be so adjusted that the liquid leaves the tube in a trickle. If, before trickling, the water runs back along the side of the capillary tube and then trickles, apply a little grease at the end of the tube, but take care that the bore is left perfectly clear of the grease.

4. Since the radius of the tube is a small quantity and it occurs as R^4, the diameter should be measured very carefully. Diameter should be recorded for both the ends of the tube and in two mutually perpendicular directions.

5. Since the viscosity of a liquid depends upon its temperature the latter should invariably be recorded and reported with the result.

6. Poissuille's formula holds for stream-line and steady flow which is obtainable with narrow tubes of uniform circular bore and when a small pressure difference exists at the two ends. Now, if the bore of the capillary tube is not uniform and is not exactly circular, the flow under this condition will not be stream-lined and the very conditions of the formula will change. This will obviously constitute a serious source of error.

VARIATION OF THE VISCOSITY OF LIQUIDS

(i) **With temperature**—The viscosity of liquids is dependent on temperature (it diminishes with rise in temperature) to a very

marked degree, but although the relationship has been the subject of many investigations, no satisfactory simple formula has been suggested to express this connection with any great degree of accuracy. In the table given below the coefficient of viscosity for water is given at various temperatures.

Viscosity of water at different temperatures.

Temp. °C	Viscosity (C.G.S. units)	Temp. °C	Viscosity (C.G.S. units)
0	0·01793	40	0·00657
5	0·01522	50	0·00550
10	0·01311	60	0·00469
15	0·01142	70	0·00406
20	0·01006	80	0·00356
25	0·00893	90	0·00316
30	0·00800	100	0·00284

(ii) **With pressure**—With fairly mobile liquids the effect of pressure on viscosity is not very marked. For instance, when ether at 20°C is subjected to an increase of pressure by 500 atmospheres, its viscosity is *raised* by about sixty per cent. With water, however, the, viscosity *decreases* for the first few hundred atmospheres. With some liquids the effect of pressure is extremely marked, for instance, for liquids of large viscosity, the ratio of the coefficients for 1 and 100 atmosphere pressure is of the order 1 to 10, and with all liquids (except water) the effect of pressure increases at high pressures.

EXPERIMENT—18

Object—To determine the coefficient of viscosity of a transparent various liquid (glycerine) by Stokes's method.

Apparatus Required—A tall wide-mouthed glass jar, glycerine, steel ball-bearings (of different sizes, if available), metre scale, vernier callipers, screw gauge, stop-watch, watch-glass and a sensitive thermometer.

Formula Employed—The coefficient of viscosity (η) of a liquid is given by the formula :—

$$\eta = \frac{2}{9} \times \frac{r^2 (\rho - \delta) \cdot g}{v (1 + 2{\cdot}4\, r/R)}$$

where r = radius of the steel ball-bearing falling through the liquid.

ρ — density of the steel ball (7·72 gm./c.c.)

δ = density of the experimental liquid (for glycerine, δ = 1·26 gm./c.c.)

v = terminal velocity of the ball.

R = radius of the glass jar.

PRINCIPLE AND THEORY OF THE EXPERIMENT

This method is based on the application of *Stokes's Law* to the fall of spheres through a liquid. It was shown by Stokes that if a sphere of radius r is allowed to fall through a viscous liquid it soon acquires a constant velocity v_0 which is given by

$$F = 6\pi n r v_0 \quad \ldots \quad (1)$$

where F is the viscous drag tending to resist the motion of the sphere.

In the steady state F is equal to the downward force which, in this case, is the apparent weight of the sphere in the liquid. Thus

$$6\pi \eta r v_\phi = \frac{4}{3}\pi r^3 (\rho - \delta) g \quad \ldots \quad (2)$$

where ρ is the density of the material of the sphere and δ is the density of the liquid.

If the liquid is contained in a vessel, the velocity of fall is influenced by the proximity of its walls. A relation between v', the observed velocity in a vessel of radius R, and v_ϕ, the velocity in a vessel of infinite radius has been given viz.,

$$v_\phi = v\left(1 + 2{\cdot}4\frac{r}{R}\right) \quad \ldots \quad (3)$$

This correction is called the Ladenburg correction.* Applying this correction to equation–(2) we at once have

$$\eta = \frac{2}{9} \times \frac{r^2 g(\rho - \delta)}{v(1 \times 2{\cdot}4 \times r/R)} \quad \ldots \quad (4)$$

which is the required relation.

* This correction may be put to test by allowing spheres to fall down glass tubes of different radii placed in the containing vessel.

Method—

(i) Take a long wide-mouthed cylindrical glass jar and fill it with the exprimental liquid. Slip some thin paper collars (such as P, Q, R) round the glass jar.

(ii) Now take some steel ball-bearings and measure their diameter with a screw gauge. Place them in a small amount of the experimental liquid in a watch-glass. Roll them in the liquid till their surfaces are thoroughly wetted.

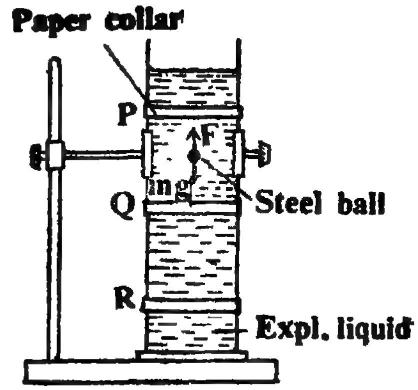

Fig.- 36
Viscosity by Stokes's method.

(iii) Drop* a ball centrally in the jar and when it has dropped a few centimetres, start a stop-watch to obtain its terminal velocity by timing a measured fall. To avoid error due to parallax, use upper edges of the paper collars as points of reference for this determination. Use two or three balls of each size available and allow them to drop through the liquid one after the other in quick succession to ensure identical temperature conditions.

(iv) Measure with a metre scale the distance between successive paper collars and thus knowing the time of fall for a measured distance calculate the terminal velocity. Then obtain the average velocity from the different readings.

(v) With the help of a vernier callipers measure the internal diameter of the glass jar, then knowing the densities of steel and glycerine from the Table of Physical Constants, calculate the coefficient of viscosity of glycerine with the help of the above formula†.

---

* Do not pick up the ball with the hand. It should preferably be done with the help of the blade of an ordinary knife.

† (i) The densities can be found experimentally also by using a specific gravity bottle.

(ii) If many sizes of ball-bearings are available, a graph between v and r^2 can be plotted and η can be calculated by taking the value of r^2/v from this straight line-graph.

Observations—

[A] *Readings for the determination of velocity.*

S No.	Distance between two paper collars	Time of fall	Terminal Velocity	Remarks
1	.........cm.	......sec.	...cm/sec.	(1) Temp. of the liquid = ...°C
⋮				(ii) Density of the liquid = ...gm/c.
⋮				(iii) Density of steel = ...gm/c.
⋮				
		Mean	.. cm/sec.	

[B] *Readings for the determination of R and r.*

S. No.	Observed internal diameter of the jar (2R)	Observed diameter of the ball-bearings (2r)	Remarks
1	......cm.	......cm.	(i) L. C. of the vernier callipers = ...cm.
⋮			(ii) Zero error of the callipers =......cm.
⋮			(iii) L. C. of the screw gauge=......cm.
⋮			(iv) Zero error of the screw gauge =.....cm.
	Mean=...cm.	Mean=...cm.	

Calculations—

(i) Mean corrected diameter of the jar = ...cm.

∴ Mean corrected radius (R) of the jar = ...cm.

(ii) **Mean corrected diameter of the balls = ...cm.**

∴ **Mean corrected radius (r) of the balls = ...cm.**

Now, $$\eta = \frac{2}{9} \times \frac{r^2 (\rho - \delta) g}{v (1 + 2{\cdot}4\ r/R)}$$

=.........poise.

Result—The coefficient of viscosity of glycerine at......°C as determined by Stokes's method = ...poise.

[Standard value = ...poise.

Error = ...poise = ...%].

Precautions and Sources of Error—

(1) The radius of the ball-bearing is a very small quantity hence it should be measured very accurately with a fine micrometer screw. During this determination the back-lash error of the screw should be avoided.

(2) The ball-bearing should be dropped *centrally* in the glass jar, so that they are far removed from the walls.

(3) Before being dropped, the balls should be completely wetted in the experimental liquid and they should not be picked up with hand but on the blade of a knife.

(4) The terminal velocity of the sphere becomes constant only after it has traversed some distance, hence the stop-watch should be started only after it has had a descent of say, 10 cm. Thus the upper edge of the first paper collar should be located at this position.

(5) To remove the error due to parallax, do not fail to use paper collars wrapped round the jar. The stop-watch should be started just at the instant when the ball just touches the upper edge of the paper collar and should be stopped when it just touches the upper edge of the other one.

(6) Since the viscosity of liquids changes rapidly with temperature, the temperature of the liquid should be measured with a sensitive thermometer, and the fall experiment with different balls should be performed one after the other in rapid sequence to ensure identical temparature conditions.

(7) The chief source of error in the experiment is the lack of arrangement to ensure perfect constancy of temparature* of the liquid throughout the experiment.

(8) The terminal constant velocity is attained quickly by those balls only which are very small in size, but in this case the proportionate error in radius is large. For large balls this proportion

error can be reduced, but in this case the terminal velocity may not be attained.

(9) Time is recorded in this experiment by quickly shifting the eye from one plane to the other between the starting* and stopping of the stop-watch. On the whole this process involves some error in the measurement of time. This then constitutes a source of error.

* The experiment can be slightly modified by surrounding the jar in a wide vessel containing water whose temperature can be read twice or thrice with a sensitive thermometer during the course of the experiment. In that case paper collars can be replaced by coloured cord bands.

SOUND

6

Frequency Determinations

EXPERIMENT—19

Object—To determine the frequency* of a tuning fork with a sonometer.

Apparatus Required—A sonometer, the given tuning fork a pad, chemical balance with weight box, a meter scale, and slotted kgm-weights.

Description of the Apparatus—A sonometer (Fig.-37) consists of a firm wooden frame carrying two fixed bridges over which one or more strings or wires can be stretched, One end of the string is fixed over one of the fixed bridges, while the other end, passing over the other bridge carries a hanger. The Tension in the string is adjusted by suspending masses on the hanger hanging from it, and for this reason a vertical pattern of the instrument is always preferable over a horizontal one. If a horizontal pattern is to be used the string has to pass over a pulley so that the weights can hang downwards. There is usually considerable friction at the pulley, so that the tension on the string is not necessarily the same as the weight hanging from the end. A movable bridge is also supplied, by moving which along the string the sounding length of the wire can be altered at will, and thus its pitch is changed as a result of the altered length.

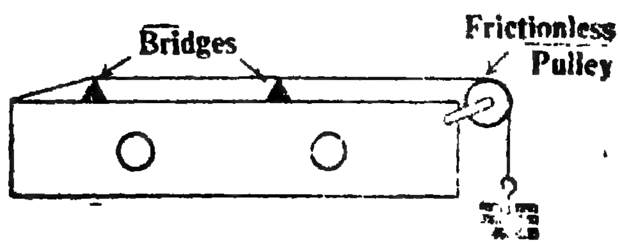

Fig.-37. Sonometer.

When the wire is plucked in the middle it is thrown into stationary vibration having nodes at the two edges of the bridges and

*For a detailed study of Frequency Determinations, read author's book "A Critical Study of Practical Physics and Viva-voce."

an antinode in the middle. The volume of the note emitted by the wire is considerably increased with the help of the sounding board due to the forced vibrations imposed by the vibrating wire upon it and the air contained therein.

Formula Employed—When the note emitted by the sounding length of the wire is in unison with the tuning fork, the frequency n of the latter is given by the following formula :—

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

where l = length of the vibrating segment of the sonometer wire.

T = tension applied to the wire.

[= Mg : M = total mass suspended from the wire].

m = mass of the wire per unit length.

PRINCIPLE AND THEORY OF THE EXPERIMENT

Let us consider the string OP (Fig.–38) of mass m per unit length and under a tension T. Let its initial direction coincide with the x-axis, while its vibrations take place along the y-axis. Thus OA′B′P is the displaced position of the string. Let us consider the motion of an element AB of the string. The length of the element is δx and its distance from the origin is x_2. In the displaced condition the element acquires the position A′B′ whose length, if the flexure of the string is small, can be assumed equal to δx.

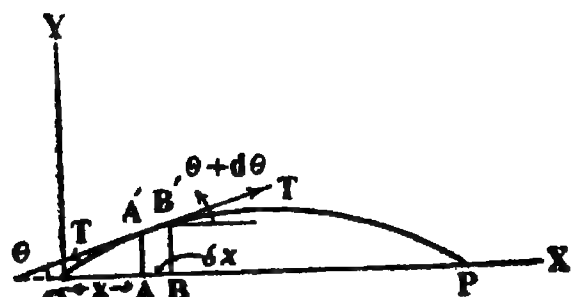

Fig.–38
Forces on a vibrating string

If the tensions acting at the extremities of the element make angles θ and (θ + dθ) with the x-axis, the resultant force acting in the direction of the y-axis is given by

$$T\,[\sin\,(\theta + d\theta) - \sin\theta]$$
$$= T.\cos\theta.\,d\theta$$
$$= T.\delta.\,(\sin\theta)$$
$$= T.\delta\,(\tan\theta)$$

since, when θ is small, sin θ = tan θ.

Now, since the mass of the string is m per unit length, the mass of the element is $m.\delta x$, and hence the force acting on it in the direction of the y-axis is $m.\delta x.\ \frac{d^2y}{dt^2}$. Hence equating the two forces we have

$$m.\delta x.\ \frac{d^2y}{dt^2} = T.\delta\ (\tan\theta) = T\ \frac{d^2y}{dx^2}$$

since $\tan\theta = \frac{dy}{dx}$. Thus

$$\frac{d^2v}{dt^2} = \frac{T}{m} \times \frac{d^2y}{dx^2}$$

which represents a wave disturbance travelling along the length of the string and its velocity v is given by

$$v = \sqrt{\frac{T}{m}}$$

Now, consider the two ends of the string of length l to be fixed. A wave travelling along it is reflected at the two ends in succession, thus giving rise to two waves travelling in opposite directions. These give rise to stationary waves, and, for the fundamental mode of vibration, an antinode is formed in the middle of the string, the two ends always acting as nodes. Thus, if λ be the wave-length of these vibrations,

$$l = \frac{\lambda}{2} \qquad \text{or} \qquad \lambda = 2l$$

If n be the frequency of vibration of the string, we have

$$n = \frac{v}{\lambda} = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

This expression enables us to calculate the frequency of a string if the various quantities l, T, and m are known. If the length and tension be so adjusted that the string is in unison with a fork, then the frequency of the latter is equal to that of the former and hence can be known.

Method—

(i) Stretch the sonometer wire with a suitable known tension by placing weights on the hanger. Sound the tuning

fork by gently striking it at the end of a prong against a soft pad and place it on the top of the box. A loud sound will be heard. Now, *starting with a small length of the wire* between the fixed and the movable bridges, pluck* it in the middle so as to excite its fundamental mode of vibration and compare its note with that of the fork.

(iii) Now, shift the position of the movable bridge so as to increase the length of the sounding wire till the frequency of the fork is very nearly equal to that of the wire. This will be apparent from the appearance of beats, when the two are sounded together. Shift the movable bridge slowly till the beats appear to be drawn out, meaning thereby that the number of beats diminishes and the frequency of the wire approaches that of the fork. Finally, move the bridge further till the beats disappear and the two are in unison†.

If in this position a paper rider be placed on the middle of the wire, it will be energetically thrown off. But the rider method of adjustment is not an accurate one and hence should be avoided as far as possible.

(iii) Now, measure carefully the length of the wire between the bridges. Repeat the observation a few times and take the mean of the vibrating lengths of the wire. Then take similar readings by changing the tension†† of the string.

Next cut off a length (preferably one metre) of the wire, weight it in an analytical balance, and thus determine its mass per unit length. Then calculate** n from the formula given above.

* Avoid the use fo finger nails in this process.

† This can be further tested by placing the vibrating fork on the box when the wire will be thrown in strong *sympathetic vibrations*. This can be easily visualised by the haziness of the contour of the wire, as also by gently placing the thumb nail underneath the wire when its vibrations can actually be felt.

†† In the process of increasing the load on the hanger, be careful not to stretch the wire beyond the elastic limit.

** For calculating n, find the mean of T and l^2 and with this value of T/l^2 find the value of n, or draw a graph between T and l^2 and from this graph, which is a straight line, read off T and l^2 for any point lying on it and calculate the frequency.

Observations—

[A] *Readings for the determination of l.*

S. No.	Load applied (M)	Length of the wire in unison	Mean l	l^2	Remark
1	......kgm	(a)...cm (b)...cm (c) ..cm	...cm	...cm^2	Mass of 1 metre long wire=...gm $\therefore$ m =...gm/cm
:					
:					
:					
:					
Mean	......kgm		Mean	...cm^2	

[**Caution**—After completing the experiment revenue weights from the hanger. Under no circumstances should the wire be left in a stretched condition.]

Calculations—

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

$$= \frac{1}{2l}\sqrt{\frac{Mg}{m}}$$

Hence, $$n^2 = \frac{g}{4m} \times \left(\frac{M}{l^2}\right) = \ldots\ldots$$

$$\therefore \; n = \ldots\ldots \text{ vibs/sec}$$

Result—The frequency of the fork—

(i) as determined experimentally = ... vibs/sec

(ii) as given on the fork itself = ... vibs/sec

[Error = ... vibs/sec = ...%].

Precautions and Sources of Error—

(1) The sonometer wire should be uniform and free from kinks.

(2) The wire should always be plucked in the middle to excite its fundamental mode of vibration. The displacement imparted to it in the middle should be small and the use of finger nails should be avoided.

(3) For bringing the wire in unison with the fork, start with a small length of the wire and alter the length in small increments.

(4) Unison should always be tested by the method of "*removal of beats*".

(5) While finding out the tension of the wire, *do not forget to add the mass of the hanger.* If a sonometer employs a spring balance note down the zero error, if any.

(6) While increasing the tension of the wire, be careful that the wire is not stretched beyond elastic limit. For this purpose, before starting the experiment have an idea of the magnitude of the breaking load of the given wire with the help of the Table* of Physical Constants.

(7) In the derivation of the formula $v = \sqrt{T/m}$ it has been assumed that the wire is perfectly flexible. Hence, due to the rigidity of the experimental wire an error shall creep in the result.

(8) If the wire is not uniform or if its composition is variable, then also the result will be erroneous.

(9) In this horizontal pattern of the sonometer there is always present some friction at the pulley, hence the value of tension is less than that actually applied. This consequently affects the value of the frequency. Moreover, the tension on the two sides of the bridges may not be the same.

(10) There is always some practical difficulty for the ear (specially when it is untrained) to establish perfect unison in two musical sounds.

ADDITIONAL EXPERIMENTS

No.—19 (a)

(1) *Determination of the density of the material of a wire using a sonometer*—Take a fork of known pitch and tune a length of the sonometer wire, under a known tension, in unison with the given fork.

* See Table—4 given at the end of the book. Breaking load is given by the product of *breaking stress* and *area of cross-section* of the wire.

In the equation $$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

n is given, T is known, and l is measured. Hence, m can be calculated.

Now, by definition, m is the mass of a cylinder of unit length and of diameter equal to that of the wire, so that

$$m = \pi r^2 \rho$$

where r is the radius of the wire, and ρ its density. Hence, ρ can be determined from the calculated value of m, if the radius is measured with a micrometer screw.

[**Note**—In this determination the wire should not be removed from the sonometer board. The experiment should be conducted exactly in the same way as described above and the readings for the radius of the wire should be taken at several places in two mutually perpendicular directions and they should be entered in a separate tabular form.]

No.—19 (b)

(2) *Determination of the mass of a load by a sonometer*—Take a fork of known frequency and after carefully suspending the given load from the hanger, tune a length l of the wire in unison with the fork. Then determine m, mass per unit length of the wire, by weighing a sufficiently long wire (say, one metre) with an analytical balance. Thus, in the equation

$$\frac{1}{2l}\sqrt{\frac{T}{m}}$$

n, l, and m can all be determined, hence calculate T in dynes. Now if M be the mass of the given load, T = Mg dynes, hence M = T/g and the unknown mass can thus be found.

[**Note**—If the given load is not sufficient to impart a suitable tension to the string, it may be suspended along with a known mass M′. In this case, T = (M + M′)g. Thus, knowing T and M′ the value of M can be calculated out.]

No.—19 (e)

(3) *To study the variation of pitch with length (or to verify the law of length for a vibrating string)*—Set up the sonometer and adjust the tension of the wire so that on plucking it, a musical note is produced. Take several forks of known frequency, and alter the length of the wire by shifting the bridge, so as to give unison with each fork in turn, the tension of the wire being kept unaltered throughout. Determine the lengths l_1, l_2, l_3, etc., corresponding with the frequencies n_1, n_2, n_3, etc., of the forks (and of the

vibrating wire also since it is in unison with them). It will be found that $n_1 l_1 = n_2 l_2 = n_3 l_3 =$ etc. Showing thereby that

"*The frequency of the wire under constant tension is inversely proportional to its length.* Draw also a graph between n and $1/l$ which will be found to be a straight line.

[**Note**—This result can also be employed to find the frequency of a fork either graphically or by calculation. Thus, tune the wire first in unison with a known fork, and afterwards with the unknown fork. Hence

$$\frac{n_1}{n_2} = \frac{l_2}{l_1} \quad \text{or} \quad n_1 \text{ (unknown)} = n_2 . \frac{l_2}{l_1} \text{ (all known).}$$

From the graph the value of n_1 can be read off corresponding to $1/l_1$].

No.—19 (d)

(*4*) *To study the variation of pitch with tension.* (Or, *To verify the law of tension for a vibrating string*).

[**Note**—In the following procedure an indirect method is employed, the wire being turned by altering the length as well as the tension. The effect of the alteration of length can be allowed for by a simple calculation involving the "*Law of Length*".]

Stretch two wires of the same material and mass per unit length of the sonometer. Apply different tension T_1, T_2, T_3, etc. to one of the wires and find the lengths l_1, l_2, l_3, etc. of this wire which vibrate in unison with a certain length of the other wire which is being kept under constant tension.

In order to find how the pitch of a constant length of the wire varies with the tension acting in the wire apply the law of length. Let the frequency of the wire of the length l_1, when pulled with a tension T_1, be n_1. A length l_2 of the same wire had the same frequency n_1 when pulled with a force T_2. If we had used the same length of wire as at first (l_1), the frequency under tension T_2 would have been given by

$$\frac{n_2}{n_1} = \frac{l_1}{l_2} \qquad \text{or} \qquad n_2 = \frac{l_1}{l_2} \times n_1$$

and we can therefore, calculate the pitch n_2 of a length of wire l_1 when under a tension T_2.

Similarly the length l_1 would have had a frequency n_3 given by the formula—

$$n_3 = \frac{l_1}{l_3} \times n_1$$

if it had been vibrating under a tension T_3.

In this way calculate n_2, n_3, etc., and show that n is proportional to $\sqrt{T}$. Arrange the results of your observation in the tabular form given below :—

S. No.	Tension applied (T)	Length of the wire giving frequency n_1 (l)	Calculated frequency for a length l_1 $n = n_1 \frac{l_1}{l_2}$ etc.	$\sqrt{\frac{T}{n}}$
1	T_1	l_1	n_1	
2	T_2	l_2	$n_2 = n_1 \frac{l_1}{l_2}$	
3	T_3	l_3	$n_3 = n_1 \frac{l_1}{l_3}$	

It will be found that the last column of the table is a constant quant ty, showing that the frequency (n) of the wire is proportional to the square root of the tension ($\sqrt{T}$) to which it is subjected. This verifies the Law of Tension for the transverse vibration of stretched strings.

[**Note**—The above conclusion arrived at by calculation can also be tested by plotting a graph between n and $\sqrt{T}$ which will be found to be a straight line.]

EXPERIMENT—20

Object—To determine the frequency of A. C. mains with the help of a sonometer.

Apparatus Required—A sonometer, a hanger, half kgm. weights, horse-shoe magnet, step-down transformer, meter scale and a screw gauge.

Formula Employed—The frequency of the A. C. mains is given by the following formula :—

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

where l = length of the sonometer wire between the two bridges when it is thrown into resonant vibration.

T = tension applied to the wire.

[= Mg, where M is the total mass (*i. e.* weights + hanger) suspended from the wire].

m = mass per unit length of the wire.

PRINCIPLE AND THEORY OF THE EXPERIMENT

Suppose a sonometer wire is stretched with a constant load and the wire is placed between the poles of a strong horse-shoe magnet which is kept in such a way that the magnetic field is applied in a horizontal plane and at right angles to the length of the wire. Now suppose an alternating current of low voltage (as obtained with the help of a step-down transformer) is passed through the sonometer wire. Due to the interaction between the magnetic field and the current flowing in the wire, the wire will be deflected and the direction of deflection of the wire will be up and down, *i. e.*, it will be perpendicular* to both the direction of the field and the direction of the current. Now, because the current is alternating, the wire will move upwards for half the cycle, while for the next half-cycle it will move downwards. In other words, we can say that under the influence of the periodic vibrations of the alternating current the wire will execute forced variations. If now the length of the vibrating wire will be so adjusted that the natural frequency of vibration of the wire becomes equal to that of the current, then there will be resonance and the wire will vibrate with a large amplitude and a loud sound will be produced. At this stage, the frequency of the A C. mains will be given by the well-known formula :—

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

where m (the mass of the wire per unit length) is given by the formula—

$$m = \pi r^2 \rho$$

where r (radius) can be determined with a screw gauge and ρ (density) can be known from the Table of Physical Constants. Alternatively, the value of m can be found out by weighing a known length of the wire in a chemical balance.

* The direction of deflection of the wire is in accordance with the well-known Fleming's left-hand rule in electricity. The deflection of the wire is similar to one observed in the coil of a galvanometer when a current is passed through it.

Method—

(i) Before starting the actual experiment, have an idea of the breaking stress for the material of the wire from the Table* of Physical Constants. From this calculate the breaking tension (= breaking stress × area of cross-section of the wire) for the experimental wire. During subsequent experiment, *the load suspended from the wire should not exceed half the breaking tension.*

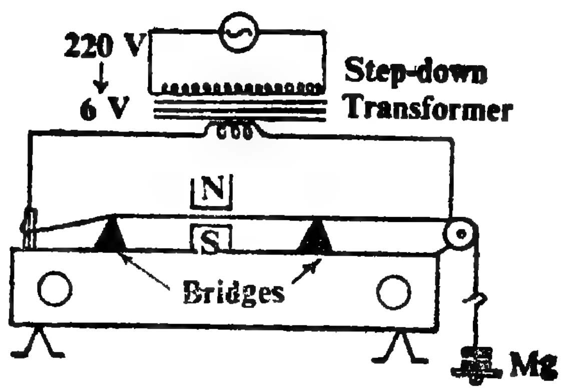

Fig.–39
Frequency of A.C. mains.

(ii) Apply a suitable tension to the wire. Mount the horse-shoe magnet NS vertically *at the centre of the wire* in such a way that the wire passes freely between its pole-pieces and the faces of the magnet are perpendicular to the wire.

(iii) Connect the secondary terminals of the step-down transformer to the two ends of the sonometer wire (Fig—39), and connect the primary of the transformer to the A.C. mains. Now, adjust the positions† of the bridges in such a way that resonance occurs and maximum loudness of sound is heard. Measure the distance between the bridges. Repeat this process at least twice more and determine the mean value of l. Record the tension, *which should include the mass of the hanger also.*

(iv) Next altering the values of tension take several sets of readings for T and l. Calculate for each set the value of T/l^2. If the experiment has been successfully performed, the value of T/l^2 will be the same for each set. Finally, calculate the mean value of T/l^2.

(v) Now determine the diameter of the wire with a screw gauge at several points along the length of the wire and at each point along to mutually perpendicular direction. From these readings determine the mean radius of the wire. From the Table of Physical Constants find out the density of the material of the wire.

(vi) Finally calculate the frequency of A.C. mains as shown below.

* For this see Table—4 given at the end of the book.

† When the bridges are shifted to increase the vibrating length of the wire, *the magnet should always be kept in the middle of the wire.*

Observations—

[A] *Readings for the determination of T and l.*

S. No.	Tension applied to the wire (T) (dynes)	Length of the resonating wire (l)	Maen l	l^2 (cm^2)	$\frac{T}{l^2}$ (dynes/cm^2)
1.	500 × 981	(a) 51·5 cm (b) 51·4 cm (c) 51·6 cm	51·4 cm	2642	185·7
2.	1000 × 981	—	72·8 cm	5300	185·1
3.	1500 × 981	—	89·2 cm	7957	184·9
				Mean	185·2

[B] *Readings for the determination of the diameter of the wire.*

S. No.	Reading along any dimeter	Reading along perp. diameter	Mean observed diameter	Remarks
1.				L.C. of the screw gauge =.. cm Zero error =cm Density of the wire (brass) =8·6 gm/c.c.
⋮				
10				

Calculations—

Mean corrected diameter of the wire = 0·0524 cm.

∴ Mean radius of the wire = 0·0262 cm.

Now $$n^2 = \frac{1}{4l^2} \times \frac{T}{m} = \frac{1}{4.\pi r^2 \rho} \times \frac{T}{l^2}$$

or $$n^2 = \frac{185{\cdot}2}{4 \times (3{\cdot}14)^2 \times ({\cdot}0262)^2 \times 8{\cdot}6}$$

$$\therefore\ n^2 = 49{\cdot}98 = 50 \text{ cycles/sec (nearly)}$$

[Calculation work—

Numerator

log 185·2 = 2·2676

$\bar{2}$·8701

Diff. = 3·3975

∴ ½(Diff.) = 1·6988

Denominator

log 4 — 0·6021

log 3·14 = 0·4969

2 log ·0262 = $\bar{4}$·8366

log 8·6 = 0·9345

Sum = $\bar{2}$·8701

log ·0262 = $\bar{2}$·4183

Antilog 1·6988 = 49·98]

Result—The frequency of A.C. mains = 50 cycles/sec

[Actual value = 50 cycle/sec

∴, Error = 0·0]

Precautions and Sources of Error—

(1) The sonometer wire should be uniform and free from kinks.

(2) The horse-shoe magnet should be placed vertically with its face normal to the wire, which should pass freely in between the poles of the magnet.

(3) The sonometer wire as well as the clamp (used to hold the magnet) should be of non-magnetic material.

(4) For bringing the wire in resonant vibration, start with a small length of the wire and increase the length in small steps. The magnet should be adjusted close to the middle of the wire.

(5) While finding out the tension of the wire, do not forget to add the mass of the hanger.

(6) The tension applied to the wire should not stretch it beyond elastic limit.

(7) The diameter of the wire should be measured at several points along its length and at each point along two mutually perpendicular directions.

(8) The chief source of error in this experiment is the presence of friction at the pulley, hence the tension is less than that actually applied.

EXPERIMENT—21

Object—To determine the frequency of an electrically maintained tuning fork by Melde's method using (i) transverse arrangement, and (ii) longitudinal arrangement.

Apparatus Required—Electrically maintained tuning fork, an accumulator, a rheostat, clamp stand, pulley (on ball-bearing), chemical balance, weight-box, thread of uniform thickness, and a meter scale.

Description of the Apparatus—An electrically maintained tuning fork furnishes an interesting example of the case where the force which supplies the energy for maintenance of vibrations has no periodicity of its own, but the periodicity is imposed by the system which is maintained vibrating.

The chief parts (Fig.–40) of the electrically maintained tuning fork are (i) the fork F, (ii) the electromagnet M, (iii) the steel spring S and the platinum tip P, at which contact is made. The vibration of the fork is maintained by the intermittent current that flows through the electromagnet placed between the prongs of the tuning fork. When current flows in the circuit the electromagnet is energised and the prongs are pulled inwards, and then the steel spring S fails to make any contact with P. Thus the circuit is broken* and consequently, the electromagnet fails to offer any pull upon the prong, which fails back and the contact is re-made. In this way, the fork is kept vibrating.

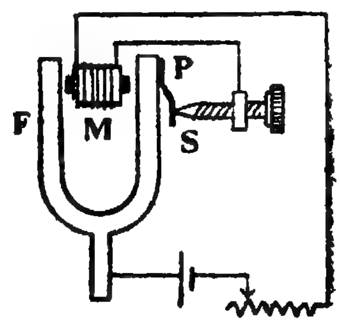

Fig.–40
Electrically maintained tuning fork.

Formula Employed—(i) For the **transverse arrangement** the frequency (n) of the fork is given by the formula :—

When the circuit breaks, a spark is produced here. In order to reduce this sparking, and thus to increase the efficiency of the maintained vibrations of the fork, a condenser of suitable capacity can be placed as in an induction coil, in parallel with the spark gap.

$$= \frac{1}{2l}\sqrt{\frac{T}{m}} = \frac{1}{2l}\sqrt{\frac{Mg}{m}}$$

where l = length of the string fundamental mode of vibration.

T = tension applied to the string.

[M = total mass suspended from the string].

m = mass per unit length of the string.

(ii) **For the longitudinal arrangement** the frequency of the fork is given by the following formula :—

$$n = \frac{1}{l}\sqrt{\frac{T}{m}} = \frac{1}{l}\sqrt{\frac{Mg}{m}}$$

where the symbols have the above significance.

PRINCIPLE AND THEORY OF THE EXPERIMENT

(1) *When the thread is stretched out in line with the length of the prong so that it executes oscillations perpendicular to this line, the arrangement is called a transverse arrangement.* Stationary waves are formed in the usual manner, and for the fundamental mode of vibration an antinode will appear in the middle. The end of the string where it is attached to the prong and the position where it touches the pulley, will be the places where nodes will be formed. For the first harmonic to appear there will be an additional node formed in the middle of the string (see Fig.-41). Thus, for the formation of stationary waves a long the string its length should be a multiple of half the wave length of the waves travelling along it. Now the frequency n of the fork is equal to that of the string, and hence its value is given by :—

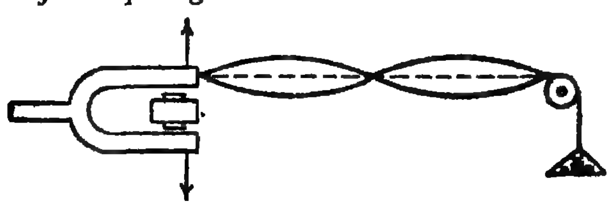

Fig.-41
Melde's expt.
(Transverse arrangement)

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}} \quad \ldots \quad \ldots \quad (1)$$

where l is the length of the string giving fundamental note, T is the tension and m is the mass per unit length of the string.

(2) *In the longitudinal arrangement the path of the tip of the prong is parallel to that of the string*. The stationary waves are formed due to the superposition of the direct and reflected waves along the string. With a given load the length of the string can be so adjusted that we get a large amplitude of vibration of the string with one ventral segment (Fig.–42). Keeping the length of the string constant and the load can be so varied that the number of well defined loops formed on the string is changed to two, three etc.*

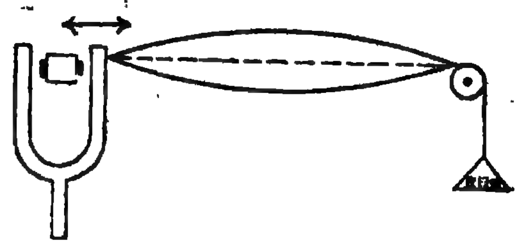

Fig.–42
Melde's expt.
(Longitudinal arrangement)

To follow the mechanism of the maintenance of vibrations in this case, let us refer to the figures given here. Fig. (i) depicts the initial position of rest of the fork, while the other figures represent the various stages of its vibration. At the outermost position of the prong (Fig. (ii)) the string sags, and the particles of the string are at rest. As the prong moves inwards, the particles of the string are drawn upwards, and reach the mean position when the prong has reached the innermost extremity (Fig. (iii)). In this case, the particles are passing through their mean position with maximum velocity in the upward direction. As the prong moves outward, the particles of the string continue their upward motion when the prong is again at the outer extremity (Fig. (iv)) and has then completed one vibration. It is evident that the period of vibration of the string is double that of the fork, since in the time the fork completes one vibration, the string has performed only half a vibration. Hence the frequency of the fork

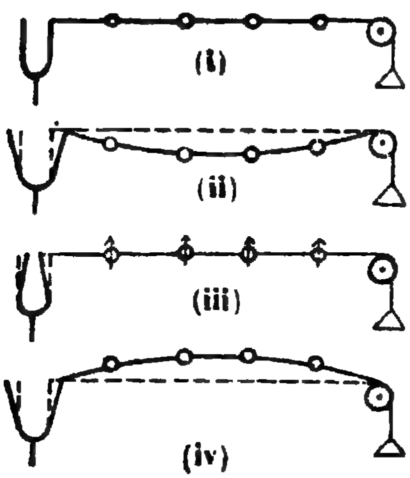

Fig.–43
Explanation of longitudinal arrangement.

$$n = 2 \times \text{the frequency of the string.}$$

$$= 2 \times \frac{1}{2l}\sqrt{\frac{T}{m}}$$

If the string vibrates in p segments under a tension T_p, the relation between this tension and the tension T for the fundamental vibration is given by $T_p = T/p^2$. A similar formula holds good for the transverse arrangement also.

$$= \frac{1}{l}\sqrt{\frac{T}{m}} \quad \ldots \quad \ldots \quad (2)^*$$

Method—(*a*) *Transverse Arrangement.*

(i) First of all, prepare the electric circuit of the electromagnet by putting in series with it an accumulator, a rheostat and a plug-key. With the help of the screw provided on one prongs of the fork tie one end of the thread, and after stretching it in line with the prong, pass it over the pulley clamped to a stand. Attach a light pan at the other end of the thread. Now by adjusting the contact points set the fork in vibration.

(ii) Place suitable weights in the pan and change slowly the length of the string till the thread is found to break up in a number of segments forming well-defined nodes and antinodes. By adding slowly some sand in the pan reduce the nodes to points. Mark their positions with ink and after measuring their distance apart with a metre scale calculate the mean distance which gives l of the formula.

(iii) Find out the weight of the pan with its contents. This gives the tension applied to the string. Measure a considerable length of the thread and weigh it in a chemical balance and thus calculate m Then evaluate the frequency of the fork with the help of the formula given above.

(iv) Keeping the length of the thread unaltered, repeat the experiment by changing the tension, thereby altering the number of loops in which the wire is set vibrating. From each set calculate* n and then take the mean.

[*b*] *Longitudinal Arrangement.*

For this procedure, stretch the thread at right angles to the length of the prong so that the thread vibrates in the same line in which the tip of the prong vibrates. Then carry out the experiment exactly in the manner in which the above experiment has been conducted.

(a) For calculating n, another procedure can be to find the mean value of T and the mean value l^2, thereby evaluating the frequency only once.
(b) The experiment can also be conducted by keeping T constant and varying the length of the string and thereby altering the number of segments in which the string is split up.

From equations (1) and (2) it is clear that the same length of the string under the same tension will vibrate with different number of loops in these two modes of vibration. *If the number of loops in the longitudinal mode be 1, 2, 3,...their corresponding number in the transverse mode shall be 2, 4, 6,...*

Observations—

[A] *Readings for Transverse Arrangement.*

S.No.	No. of Loops	Distance between consecutive nodes	Mean l	l^2	M	Remarks
1	...	......cm ......cm	...cm	...cm^2	...cm	(1) Length of the string taken for weighing =...cm (2) Mass of this string =...gm
...						
...						
...						∴ m =...gm/cm
...						
			Mean	...cm.^{2}	...gm.	

Calculations—

$$n^2 = \frac{1}{4m} \times \left(\frac{T}{l^2}\right) = \frac{g}{4m} \times \left(\frac{M}{l^2}\right)$$

$= ....$

$\therefore$ n = ... vibs/sec

Result—The frequency of the electrically maintained tuning fork (vibrating in the transverse mode)

(i) as determined experimentally = ... vibs/sec

(ii) as given by the makers = ... vibs/sec

(Error = ... vibs/sec = ... %)

[B] *Readings for Longitudinal Arrangement.*

[**Note**—Prepare a similar table for the longitudinal mode of vibration of the fork and carry out the calculations by making use of the corresponding formula given above.]

Precautions and Sources of Error—

(1) The string* used in this experiment should have a uniform linear density and should be inextensible.

(2) For the longitudinal mode of vibration, the thread should be so arranged that it is in line with that line in which the tip of

*A fishing cord is quite suitable for this purpose, as it is sufficiently uniform.

the prong vibrates. For the transverse mode, the thread should be stretched in line with the length of the prong so that the vibrations of its tip are at right angles to the thread.

(3) A pulley on ball-bearing should be employed so that the tension of the string does not appreciably differ from the weight hanging from its end.

(4) Readings for the length between the nodes should be taken only when the nodes have been reduced to points. This should be achieved by slowly adding sand in the pan.

(5) One of the sources of the error in the experiment arises due to friction at the pulley. Due to its presence, the tension of the string is less than that actually applied.

(6) The chief source of error* is the lowering in the frequency of the fork due to the presence of the clamping screw on its prong.

EXPERIMENT—22

Object—To determine the frequency of a tuning fork by the "Falling Plate Method."

Apparatus Required—Falling plate appratus, tuning fork with a light style attached to one of its prongs, a violin bow or a padded hammer, a travelling microscope, and a turpentine oil lamp.

Description of the Apparatus—The 'Falling Plate Apparatus' (Fig. 44) consists of a smoke-blackened sheet of glass hanging by a thread which passes over two pins in a heavy frame. A light style attached to one of the prongs of the fork gently touches the plate near its bottom. The tuning fork is clamped at its stem in such a way that its inclination with the plate can be suitably adjusted. When the plate is allowed to fall under gravity, the style traces a wavy curve in the soot deposited on the surface of glass. A padded stand is provided at the base of the apparatus to prevent the plate from breakage.

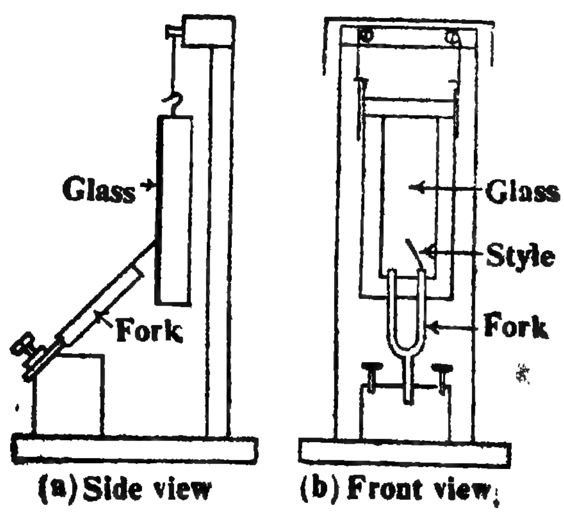

Fig.–44
Falling Plate Apparatus.

* This can, however, be eliminated by employing a fork whose frequency, with the screw on, is given by the manufacturers.

Formula Employed—The frequency N of the fork is given by the following formula :—

$$N = n\sqrt{\frac{g}{s_2 - s_1}}$$

where n = number of waves counted on the plate.
g = acceleration due to gravity.
s_1, s_2 = two consecutive distances on the plate which cover the same number of waves n.

PRINCIPLE AND THEORY OF THE EXPERIMENT

Fig.–45 represents a wavy curve traced by the style of the vibrating tuning fork on the sooty glass surface as the plate falls freely under gravity.

Let AB ($= s_1$) contain n waves. Let BC be the next distance ($= s_2$) which also contains an equal number of waves ($= n$).

If the velocity of the plate when passing the first crest (at A) is v_0, and the time to taken no trace n waves is t, we have

$$s_1 = v_0t + \tfrac{1}{2}\, g\, t^2$$

When the second of the marked crests (at B) was being passed, the velocity is

$$v_1 = v_0 + g\, t$$

The distances s_2 is given by

$$s_2 = v_1t + \tfrac{1}{2}\, g\, t^2$$

or
$$s_2 = (v_0 + g\, t)\, t + \tfrac{1}{2}\, g\, t^2$$
$$= (v_0t + g\, t^2 + \tfrac{1}{2}\, g\, t^2$$

Thus
$$s_2 - s_1 = g\, t^2$$

or
$$t = \sqrt{\frac{s_2 - s_1}{g}}$$

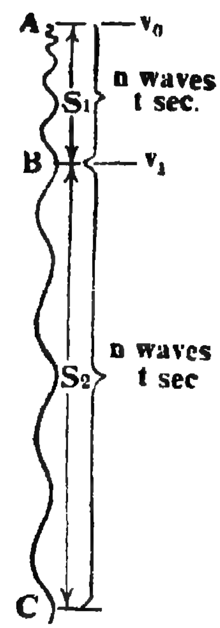

Fig.–45
Trace on the falling plate.

In this time n vibrations are executed by the fork, hence its vibration frequency N is given by—

$$N = \frac{n}{t} = n.\sqrt{\frac{g}{s_2 - s_1}}$$

which is the required formula. Taking g as known at the place of experiment and measuring s_2, s_1 and counting n, the frequency of the fork can be determined.

Method—

(i) Blacken the surface of the glass plate with soot coming from a turpentine oil flame, and mount it in its frame and suspend it at the proper height by means of the thread passing over the two nails fixed in the stand for this purpose.

(ii) Mount the fork in its stand and adjust its inclination in such a way that the style attached to one of its prongs *just touches* the plate near its bottom.

(iii) Set the fork vibrating by bowing it with a violin bow (or, by gently striking it with a padded hammer) and release the plate by burning the thread.

(iv) Take out the plate and examine the wavy curve traced out in the soot deposited on the plate. Starting at the first clearly defined crest on the trace, count off a certain number of waves n and mark the n^{th} crest. Count further beyond this point a further set of n waves, and mark the $(2n)^{th}$ crest. Now, with the travelling microscope measure the distances between the two sets of waves. They give s_1 and s_2 of the formula. Calculate N, the frequency of the fork, from the formula given above.

(v) By re-blackening the plate repeat the observations and get the mean value of the frequency of the fork.

Observations—

S.No.	No. of waves (n)	Position of the microscope			s_1	s_2	Remarks
		at A	at B	at C			
1	...	...cm	...cm	...cm	...cm	...cm	L. C. of the microscope = ... cm
⋮							Value of 'g' at... = ...cm/sec²
⋮							

Calculations—

I Set

$$N = n\sqrt{\frac{g}{s_2 - s_1}}$$

$$= \ldots \ldots \text{ vibs/sec}$$

[**Note :—Calculate N similarly for the other sets**]

$\therefore$ Mean value of N = ... vibs/sec

Result—The frequency of the fork as determined by the 'falling plate method' = ... vibs/sec.

[Value marked on the fork = ... vibs/sec

Error = ... vibs/sec = ... %].

Precautions and Sources of Error—

(1) The inclination of the fork and the position of the style should be so adjusted that the latter *just* touches the plate near its bottom.

(2) The points marked A, B, C should be carefully chosen so that they are situated on the corresponding points of the wave. For instance, if A lies on a crest of the wave, B should lie on a crest n waves apart, and C should lie on another crest 2n waves apart.

(3) While using the travelling microscope, avoid back-lash error.

(4) One of the sources of error in the experiment lies in the fact that the friction of the style against the plate retards its motion which consequently results in a lowering* of the frequency.

(5) The chief source of error is due to the presence of the style on the prong of the fork, which results in a lowering of the frequency of the fork†.

(6) In the arrangement, where the plate is dropped in a grove, the friction in the grove can be very serious. The fall of the plate is then not free. This constitutes a source of error.

* This can, however, be corrected by taking a fork of known frequency and calculating the value of g from the formula, and then by substituting this value of g in subsequent experiments.

† This error can be eliminated by following the procedure of beats. For this purpose, another fork of a slightly higher frequency than the one under experiment is taken. By properly loading the former fork, the two forks are tuned in unison. Then the style is attached to the prong of the experimental fork and the apparatus adjusted so that the tip of the style just touches the glass plate. This fork is set in vibration. The frequency of this fork is obviously lowered and hence when the two forks are sounded together, beats are produced. The number of beats (say, x) is counted. This number is added to the experimentally determined frequency of the fork and thus corrected value of the frequency is obtained. Thus we have—

$$N = x + n\sqrt{\frac{g}{s_2 - s_1}}$$

EXPERIMENT—23

Object—To show that the frequency of a resonator varies inversely as the square root of its volume and to determine the neck correction of the resonator.

Apparatus Required—A resonator, rubber tubing, pinch-cock, clamp stand, a set of tuning forks, and a graduated cylinder.

Description of the Apparatus—The accompanying figure depicts a globular type of resonator known as Helmholtz resonator. It is generally made of brass provided with a wide mouth S at one end to receive the exciting waves and a narrow end E which can be inserted in the ear to find out if the resonator responds to a certain note or not. The air cavity is thus almost completely enclosed with the result that only a small portion of the energy is radiated into the medium. The damping is, therefore, small and the tuning sharp. For this reason, it is specially suitable for detecting sounds of a definite pitch and feeble intensity. If a complex note is sounded near the mouth of such a resonator and if this note contains the component to which the resonator can resound, the latter will pick out and magnify the component so that it is clearly audible to the ear. Such resonators are made in sets which are employed in the analysis of complex musical sounds for the study of their quality.

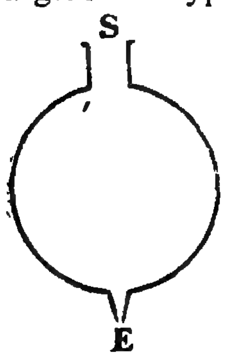

Fig -46 Helmholtz resonator.

Formula Employed—The frequency n of a resonator is given by the following formula :—

$$n = \frac{V}{2\pi}\sqrt{\frac{a}{lv}}$$

where V = velocity of sound in air.

a = area of cross-section of the neck.

l = length of the neck.

v = volume of the resonator.

Thus n^2v = constant.

Hence, if a curve is drawn between n^2 and v, it should be a straight line passing through the origin. Actually the straight line cuts an intercept c on the y-axis. Thus, the equation to the straight line is given by—

$$n^2 (v+c) = \text{constant.}$$

c is known as the "*neck correction*" (see Fig.–48).

PRINCIPLE AND THEORY OF THE EXPERIMENT

The theory of the Helmholtz resonator is simple on account of the fact that the volume of the resonator is large in comparison to that of neck with the result that hardly any vibratory motion is produced in the air chamber inside, the contained air is either compressed or rarefied at any instant. Since the wave-length of sound in free air is large compared to the dimensions of the resonator, the air particles in the neck vibrate to and fro together like one solid mass, thus acting like a reciprocating piston alternately compressing and rarefying the air contained in the resonator.

Let a be the area of cross-section, and l the length of the neck. The mass (m) of this air which is responsible for the periodic movements of the air inside is a $l\ \rho$, where ρ is the density of air.

Now, for adiabatic changes in the cavity we have

$$pv^{\gamma} = \text{const.} \quad \ldots \quad \ldots \quad (1)$$

Differentiating this equation we have

$$\gamma p v^{\gamma-1}\ dv + v^{\gamma}\ dp = 0$$

or

$$dp = -\gamma p.\ \frac{dv}{v} \quad \ldots \quad (2)$$

For a small change in the volume the total force on the aerial piston is (dp. a), and the change in volume dv per unit displacement of the piston is (a × 1) = a. Thus the restoring force (f) per unit displacement of the piston is given by

$$f = dp.\ a^2$$

$$= -\gamma p.\ \frac{a^2}{v} \quad \ldots \quad (3)$$

from (2), since dv = 1

The differential equation of motion is

$$m\ \frac{d^2x}{dt^2} + f\,x = 0$$

which indicates a simple harmonic motion whose period is given by

$$T = 2\pi\sqrt{\frac{m}{f}}$$

or the frequency is given by

$$n = \frac{1}{2\pi}\sqrt{\frac{f}{m}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{\gamma pa^2/v}{al\rho}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{\gamma p}{\rho} \cdot \frac{a}{lv}}$$

$$= \frac{V}{2\pi}\sqrt{\frac{a}{lv}} \qquad \dots \qquad (4)$$

where V is the velocity of sound in air and is given by $V = \sqrt{\frac{\gamma p}{\rho}}$

Thus, equation (4) indicates that *"the frequency of the resonator is inversely proportional to the square root of its volume."*

Method—

(i) Set up the apparatus as shown in the accompanying diagram. At the narrower end of the resonator attach a piece of tightly fitting rubber tubing having a pinch-cock (P). Fill the resonator with water upto the base of the neck. Place the graduated cylinder underneath the resonator.

(ii) Hold a vibrating tuning fork close to the neck of the resonator and allow the water to run out slowly, till a position is attained when the resonator responds to the frequency of the vibrating fork. Measure the volume of water that has flown into the cylinder. This also gives the volume (v) of the air cavity of the resonator, which is its resonance with the fork. After refilling the resonator as before, repeat the above observation and calculate the mean value of v.

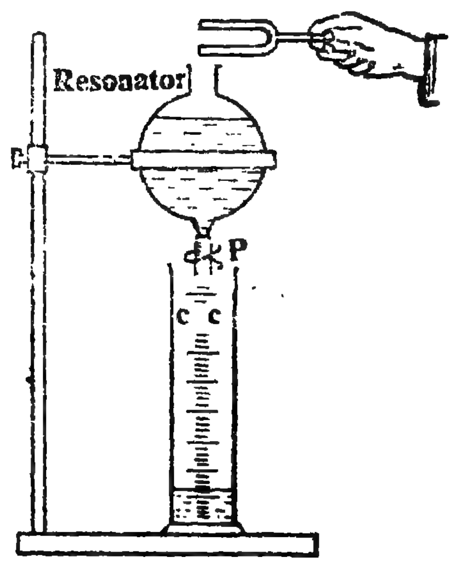

Fig.–47
Resonance with a resonator.

(iii) In this way, repeat the experiment with other forks of the given set and find the corresponding volume of the cavity of the resonator in resonance. Then draw a graph (Fig.–48) between v and $1/n^2$, representing the former along the y-axis and the latter along the x-axis. The graph will be a straight line, thus verifying the first part of the experiment. Produce this line so that it cuts the volume axis. Measure this intercept which gives the magnitude of the neck correction.

(iv) Record the temperature of the air cavity twice or thrice during the course of experiment.

Observations—

Temp. of the air cavity : (1)...°C ; (2)...°C ; (3)...°C.

S. No.	Frequency of the fork (n)	Volume of the resonator (v)	Mean v	$\frac{1}{n^2}$	n^2v	Remark
1	...vib/sec	...c.c. ...c.c. ...c.c.	.. c c	...	...	The graph showing the relation between v and $1/n^2$ is drawn in Fig.-48.
2						
⋮						

Calculations—

From the graph, the intercept on the volume axis is.........c. c.

Hence neck correction = ... c. c.

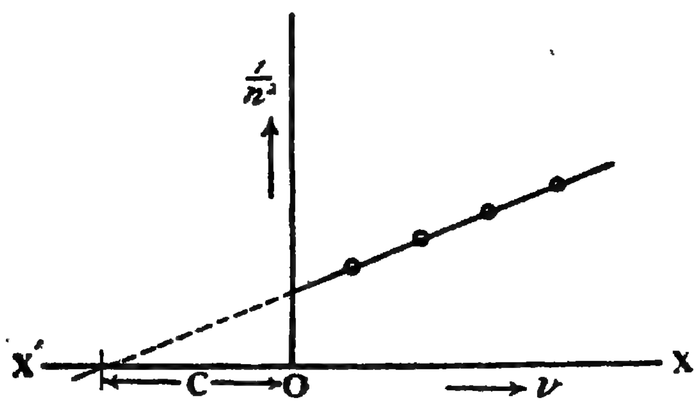

Fig.–48
v—$1/n^2$ graph for a resonator

Result—The graph between $1/n^2$ and v is a straight line and the product n^2v is, within the limits of experimental error, fairly constant.

The neck correction as estimated from the graph = ...c. c.

Precautions and Sources of Error—

(1) The point of maximum resonance of the air cavity with the fork should be carefully noted. If this position is not clearly discernible, the mean of those volumes should be determined where the resonance appears to be maximum.

(2) The experiment should be conducted in a calm atmosphere away from noise.

(3) For measuring the volume of the water flowing out of the resonator a graduated cylinder reading upto half a c. c. should be employed.

(4) For each fork, the observations should always be repeated.

(5) The graph should be smoothly drawn and if some points happen to lie off the graph, the straight line should be so drawn that equal number of points lie on each side of it.

(6) The chief source of error in this experiment lies in the difficulty of attaining the position of maximum resonance.

(7) Secondly, if the termperature of the air cavity changes during the course of the experiment, an error shall creep in.

7

Velocity Determination

EXPERIMENT—24

Object—To determine the velocity of sound* in air at the temperature with the help of a Kundt's tube.

Apparatus Required—Kundt's tube, resined leather, lycopodium powder, and a metre scale.

Description of the Apparatus—Kundt's apparatus (Fig -49) consists of a glass tube AB about a metre long and of diameter about 3 cm. provided with a adjustable piston P near one end. Near the

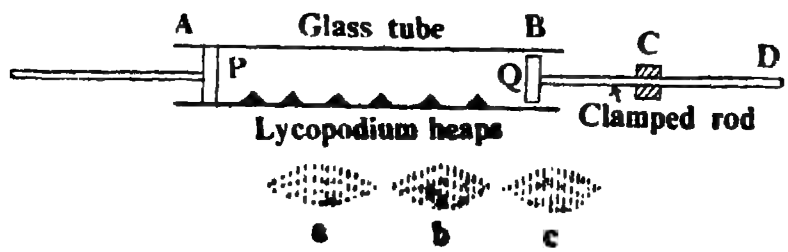

Fig.-49
Kundt's Tube.

other end B of this tube is a second piston Q, attached to the end of a metal rod DQ. This rod is clamped at its middle point C. It is the column of air in between the pistons P and Q that is set into stationary vibration by moving the piston P in and out thereby altering the length of the column of resonant air. The metal bar, which may be of brass with a diameter of 0·5 cm., can be set into longitudinal vibration by stroking it along CD with a piece of resined leather. For a rod of glass as the driver, a piece of cloth

*For a detailed study of Velocity of Sound, read author's book "A Critical Study of Practical Physics and Viva-Voce".

moistened with alcohol may be used for the purpose of exciting vibrations in the air column of the tube. A light powder, such as lycopodium powder, is placed in a line at the bottom of the tube extending along its length between the pistons. This powder helps us in locating the positions of the nodes formed in the air column. To locate the positions of these nodes, a paper scale (in the form of a long strip) can be conveniently parted to the glass tube.

Formula Employed—The velocity of sound in air (V_a) is given by the formula :—

$$V_a = \frac{l_a}{l_r}\sqrt{\frac{Y}{\rho}}$$

where l_a = the average distance between the nodes formed in the resonant column of air.

l_r = length of the brass rod.

Y = Young's modulus for the material (brass) of the rod.

ρ = density of the material (brass) of the rod.

PRINCIPLE AND THEORY OF THE EXPERIMENT

The metallic rod clamped in the mildle is set into longitudinal vibration and the air column is varied by pulling out or pushing in the other piston till the air column contained between the two pistons is thrown into violent resonant vibrations. Stationary waves formed in this column with nodes at the two discs and more in between as revealed by the heaping to the lycopodium powder in those positions. Let the mean distance between two consecutive nodes be l_a, then

$$l_a = \lambda_a/2 \qquad \text{or} \qquad \lambda_a = 2l_a$$

where λ_a is the wave-length of sound waves in air. If V_a be the velocity of sound in air, then

$$V_a = n\lambda_a = 2nl_a \qquad \dots \qquad (1)$$

where n is the frequency of vibration of the note emitted.

Now the vibrations produced in the rod are also of the stationary type and for the fundamental mode of vibration, there is a node in the middle where the end is clamped) and an antinode at either end of the rod. If l_r be the length of the rod, then

$$l_r = \lambda_r/2 \qquad \text{or} \qquad \lambda_r = 2l_r$$

where λ_r is the wave-length of sound waves in the rod. If V_r be the velocity of sound waves in the rod, we have

$$V_r = n\,\lambda_r = 2n\,2l_r \qquad \dots \qquad (2)$$

From equations (1) and (2), we have

$$\frac{V_a}{V_r} = \frac{l_a}{l_r}$$

or

$$V_a = \frac{l_a}{l_r} \cdot V_r \quad \ldots \quad (3)$$

Equation (3) can be employed to evaluate V_a provided that V_r be known. Alternatively, we know that the velocity of sound for longitudinal vibrations in a solid rod is given by the following formula :—

$$V_r = \sqrt{\frac{Y}{\rho}} \quad \ldots \quad (4)$$

where Y is the Young's modulus and ρ is the density of the material of the rod. Substituting this value of V_r in (3) we have

$$V_a = \frac{l_a}{l_r}\sqrt{\frac{Y}{\rho}} \quad \ldots \quad (5)$$

Thus knowing Y and ρ from the Table of Physical Constants and measuring l_a and l_r experimentally, we can determine the velocity of sound in air at the temperature at which the experiment has been considered.

Velocity of Longitudinal Waves in a Medium—Let us consider a cylinder A B C D (Fig.-50) of a medium. The cylinder is confined between two layers A B and C D whose distances from the origin are x and x + dx respectively, *i. e.*, the length of the cylinder is dx. Let the area of cross-section of the cylinder be 'a', so that the volume of the cylinder is a . dx.

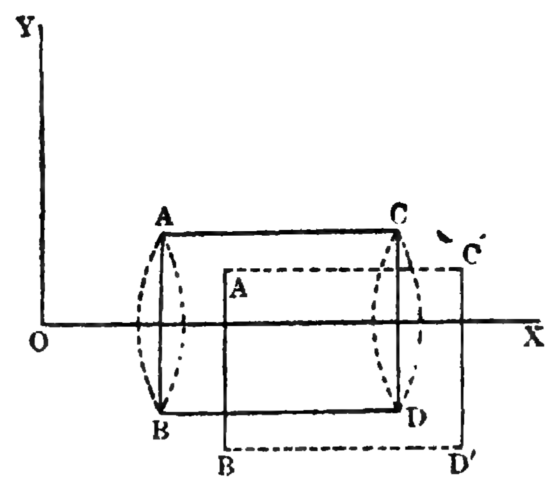

Fig.-50. Calculation of longitudinal wave velocity.

Now suppose a longitudinal wave is travelling in the positive direction of the x-axis. Due to this the layer A B is displaced to the position A′ B′ and the layer C D is displaced to the position C′ D′ so that the cylinder occupies the new position* A′ B′ D′ C′. Let the displacement of

* For the sake of clarity in the diagram, the cylinder has been shown slightly displaced along the y-axis also.

the A B layer of the cylinder along the x–axis be y and that of the C D layer be y + dy. Thus, the distance of the layer A′ B′ from the origin is (x + y), while that of C′ D′ is (x + dx + y + dy), so that the length of the cylinder is now (dx + dy). Thus, the volume of the cylinder in the displaced position is equal to a (dx + dy), and the change in the volume of the cylinder is a. dy. The strain produced in the medium in this process is given by

$$\text{Strain} = \frac{a \,.\, dy}{a \,.\, dx} = \frac{dy}{dx}$$

If E be the elasticity of the medium, we have

$$\text{Stress} = E \times \text{strain}$$

$$= E \times \frac{dy}{dx} = F\,(x)\ (\text{say})$$

This is the value of the stress at a distance x from the origin. Thus, the stress at a distance (x + dx) is given by

$$F\,(x + dx) = E\,\frac{dy}{dx} + E\,\frac{d^2y}{dx^2}\,dx \quad \text{(from Taylor's theorem)}$$

Thus, the force acting on unit area of the cylinder is equal to $E\,\frac{d^2y}{dx^2}\,dx$, which is the difference of the two stress. Hence, the total force F acting over the active cross-section of the cylinder is given by

$$F = E\,\frac{d^2y}{dx^2}\,dx \times a \qquad \dots (1)$$

Now, if ρ be the density of the medium, the mass of the cylinder will be equal to a dx ρ, and hence the force acting on it is given by

$$F = a \,.\, dx\,\rho\,\frac{d^2y}{dt^2} \qquad \dots (2)$$

where d^2y/dt^2 is the acceleration of the cylinder. Thus, from (1) and (2), we have

$$\frac{d^2y}{dt^2} = \frac{E}{\rho} \times \frac{d^2y}{dx^2} \qquad \dots (3)$$

This is the differential equation for a wave propagation whose velocity v is given by

$$v = \sqrt{E/\rho} \qquad \dots (4)$$

When waves are excited in a rod the modulus of elasticity concerned is Young's modulus. The velocity of sound is then given by the formula

$$v = \sqrt{\gamma/\rho} \quad \text{... (5)}$$

When sound waves travel in a gaseous medium, the modulus of elasticity concerned is the adiabatic elasticity whose value is given by γ P, where γ is the ratio of the two specific heats of the gas and P is its pressure. Thus, for a gas

$$v = \sqrt{\gamma P/\rho} \quad \text{... (6)}$$

[**Note**—The original formula, $v = \sqrt{P/\rho}$, as given by Newton, was found to be erroneous, since it gave the value of the velocity of sound in air as 280 metres/sec, a value much below the experimental value of 330 metres/sec. The discrepancy was satisfactorily explained by Laplace, who argued that the compressions and rarefactions, which take place in a gas when sound waves are transmitted through it, take place so quickly that heat has no chance to leave or enter the system. Thus, E in formula—(4) has to be replaced by adiabatic elasticity (= 7P) and not by isothermal elasticity (= P) as suggested by Newton. Laplace's fomula (6) given above yields, on calculation, a value of 331 metres/sec for air, which is in close agreement with experiment.]

Method—

(i) For the purpose of the experiment dry* the tube first, and place the lycopodium powder in a line at the bottom of the tube extending along its length between the two pistons. A convenient way of inserting the power in the tube is to spread it along a metre scale, place the scale inside the tube, and turn it upside down. Rotate the tube so that the powder is just on the point of falling down.

(ii) Excite the rod to emit longitudinal vibrations by stroking it along its length with a piece of resined leather. Adjust the piston on the other end by moving it in or out very slowly so that the air column confined in between the two pistons is thrown in resonant vibrations. This will be revealed by the existence of the agitation of the powder which will settle down at and near the nodes where the air is least in motion.

(iii) Choose as carefully as possible the position† of a node at one end of the tube and locate the other node nearest the other

If the tube is not dry it must be warmed above a Bunsen flame, and a current of air blown through it.

In actual practice, the pattern of the deposit of the powder will be somewhat similar to that given separately in the figure above (Fig.–49), and the longest line (like a, b, c) of each set marks the position of the nodal point.

end. Measure the length between these two points, and divide this length by the number of spaces to get the average value of half the wave-length (which is l_a of the formula).

(iv) Take out the rod and measure its length to give l_r of the formula. Obtain the values of the Young's modulus and the density of the material of the rod from the Tables of Physical Constants. Calculate the velocity of sound in air with the help of the formula given above.

(v) Record the temperature of the air in the tube and report it with the result.

Observations—

S. No.	Distance between two nodes (l)	No. of spaces in between (n)	l_a (l/n)	Remarks
	......cm	...	...cm	(i) Length of the rod = ...cm (ii) Density of the material of the rod = ...gm./c.c. (iii) Young's modulus of the material of the rod = ...dynes/cm^2 (iv) Tem. of air = ...°C

Calculations—

$$V_a = \frac{l_a}{l_r} \times \sqrt{\frac{Y}{\rho}}$$

$= ......$cms/sec

$= ......$metres/sec

[**Note**—The above is the value of velocity of sound at the room temperature, say, at t°C. Now, the velocity of sound at 0°C can be calculated with the help of the formula :—

$$V_o = V_t - 0{\cdot}6\,t$$

This value of V_o can then be compared with the standard value.]

Result—The velocity of sound in air at...°C = ... metres per second, and the velocity of sound at 0°C is found by calculation to be ...... metres/sec.

[Standard value at 0°C = 331·1 metres/sec

Error = ...metres/sec, or = ...%].

Precautions and Sources of Error—

(1) As the presence of water vapour affects the velocity of sound, the tube and the powder should be *perfectly dry.* For this purpose, tha tube should be warmed above a Bunsen flame and a current of air drawn through it.

(2) The rod should be rigidly clamped *exactly at the middle point* and its axis should coincide with that of the tube.

(3) The powder should be sprinkled in the tube *in a thin layer*, then the pattern of the powder deposit at the nodal points will be of a suitable nature for satisfactory measurement.

(4) The longest line in the deposited heaps of the powder marks the nodal position. This should be employed for measurement.

(5) It may probably happen that there is not exact resonance at first. For attaining the exact position the movable piston should be very slowly moved forward or backward.

(6) Temperature of the air column should be recorded and reported with the result.

(7) An error in the result shall creep in if no arrangement has been adopted for perfectly drying the experimental air. *Velocity of sound in moist air is mere than that in dry air.*

(8) The chief source of error, however, lies in the fact that *the velocity of sound in a limited space is not the some as in an extended etmosphere.**

ADDITIONAL EXPERIMENTS WITH KUNDT'S TUBE

No.—24 (a)

(*1*) *The velocity of torsional vibrations in a rod.* If instead of stroking the rod QD (Fig.–49) longitudinally with the resined leather, it is held near the end D, and the leather turned so that it slips over the surface in a direction that would cause the rod to

* If the radius of the tube be r, then the velocity of sound V_a is connected with V, the velocity in an extended atmosphere, by the formula

$$V_a = V(1 - r/c)$$

where c is a constant. This error, therefore, can be eliminated by using two tubes of radii r_1 and r_2 and determining the velocity of sound in them separately. Thus, if V_1 and V_2 be respectively the experimentally observed values, we have

$$V_1 = V(1 - r_1/c)$$
$$V_2 = V(1 - r_2/c)$$

Eliminating c we have

$$V = \frac{V_2 r_2 - V_1 r_1}{r_2 - r_1}$$

By employing this formula, this error can be eliminated.

rotate round its axis, a note shall be emitted. This note corresponds to torsional vibrations and its frequency is different from that of the note given by the longitudinal mode of vibration of the rod. The air column can be set to resonance by adjusting the movable piston as before, and by applying similar calculations the velocity (V_t) of torsional oscillations in the rod can be determined. Thus

$$V_t = \frac{lr}{l'_a} \times V_a$$

where l'_a is the mean distance between two consecutive lycopodium heaps in this case.

[**Note**—To produce a note by this process is a bit difficult. A little practice, however, shall meet with success. But it is not easy to obtain a loud note by this method, hence no attempt should be made to attain greater loudness by increasing the force applied on the rod.]

No.—24 (b)

(2) *Determination of Young's modulus and modulus of rigidity*—These constants can be easily evaluated by determining experimentally the velocity (V_1) of longitudinal vibrations and the velocity (V_t) of torsional vibrations in the rod. Now, these velocities are given by the formulae—

$$V_1 = \sqrt{\frac{Y}{\rho}}$$

and

$$V_1 = \sqrt{\frac{n}{\rho}}$$

where Y is the Young's modulus, n is the modulus of rigidity and ρ is the density of the material of the rod. Thus, by measuring these velocities with a Kundt's tube and taking the value of ρ from the Table of Physical Constants, the values of Y and n can be calculated.

No.—24 (c)

(3) *The velocity of sound in a gas (say, carbon di-oxide)*—For conducting this experiment, the apparatus is slightly modified as given in the following diagram :—

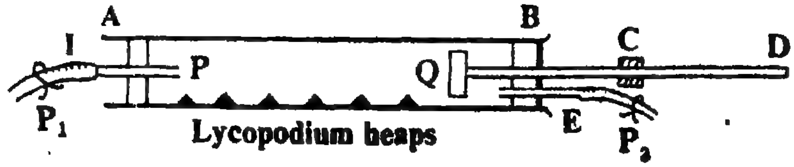

Fig -51
Velocity of Sound in Carbon di-oxide.

The adjustable piston P in this case is made perfectly air-tight by employing a rubber band round the outside of the disc. The former handle of the piston is dispensed with in this case, and in its place the disc carries centrally a tube, provided at its outer end,

with a rubber tube. This is the inlet tube (I) to admit the gas and the piston can be adjusted with the help of this tube. The rubber tube is provided with a pinch-cock P_1. On the other end B the metal rod passes through a tightly fitting cork, through which also passes a tube E. This is the exit tube and can be operated by the pinch-cock P_2

To start with the experiment, the inlet tube is connected to the source of the gas and the tube E is put in communication with the outside atmosphere by opening the pinch-cock P_2. The gas is allowed to flow in a steady stream so that it does not disturb the lycopodium powder spread along the bottom of the tube. The gas will displace the air in the tube which will flow out at E. The flow of the gas is continued for a sufficient time to ensure that the whole air is driven out and the tube contains only carbon di-oxide. The pinch-cocks are then closed and the apparatus allowed to stand for sometime to acquire the room temperature. Now the experiment is conducted in the usual manner.

Let the velocity of sound in the gas be V_g and let the corresponding wave-length be λ_g. The frequency (n) of sound is the same in the gas as well as in air, hence

$$n = \frac{V_g}{\lambda_g} = \frac{V_a}{\lambda_a}$$

$$\therefore \quad \frac{V_g}{V_a} = \frac{\lambda_g}{\lambda_a} = \frac{\text{Distance between two nodes in the gas}}{\text{Distance between two nodes in air}}$$

Thus, to calculate V_g, first the resonance is obtained between the air column and the rod and the mean distance between the nodes is found out. Then the air is drived out and the experimental gas filled in. Resonance is again obtained and the mean distance between the nodes formed in this case is measured.

No.—24 (d)

(4) *To calculate the ratio of the specific heats of a gas (say, carbon di-oxide)*—The velocity of sound in the gas at 0°C is given by the formula :—

$$V_o = \sqrt{\frac{\gamma P}{\rho_0}}$$

where γ is the required ratio. P is the pressure of the gas and may be determined by the barometer since the experimental tube has been filled at atmospheric pressure. ρ_0 is the density of carbon di-oxide at 0°C and can be taken equal to 0·001974 gm/c.c. Thus, by measuring the velocity of sound (V_t) at the room temperature and then calculating its value (V_o) at 0°C with the help of the formula—

$$V_0 = V_t - 0{\cdot}47\ t \text{ metres/sec}$$

we can calculate the value of γ with the help of the formula given above. For this purpose, express P in dynes/cm², and take density of mercury as 13·6 gm/c. c.

LIGHT

8

Reflection and Refraction of Light

EXPERIMENT—25

Object—To determine the height of a tower with the help of a sextant.

Apparatus Required—A sextant, and a 50 ft. measuring type.

Description of the Apparatus—The instrument consists of a graduated arc, SS (Fig.-52) connected to two radial arms A and B. A third arm C is movable about the centre of the arc. It is fitted with a clamp and tangent screw, so that it can be accurately adjusted, and it carries a vernier V at its end which moves over the graduated scale of SS. At the other end of this arm is attached a plane mirror (M_1) known as the *index glass*, the plane of which is perpendicular to the plane of the arc. This radial arm can be clamped in position with the clamping screw and then it can be slowly moved with the help of the tangent screw.

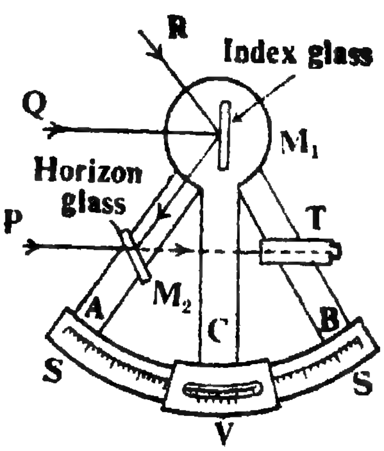

Fig.-52
Sextant

Fixed to the radial arm A in the second mirror M_2, known as the *horizon glass*, whose plane is also perpendicular to the circular arc. It consists of a plate of glass of which only the lower half is silvered, the upper half being transparent.

On the arm B is fixed a telescope T whose axis is parallel to the plane of the arc and passes through the centre of M_2. This telescope receives the direct rays through the transparent portion of M_2, and the twice reflected rays from M_1 and M_2.

Coloured glasses are also provided with the instrument for diminishing the brightness of any object such as the sun. These can be made to intercept the light immediately before falling on the mirrors.

Formula Employed—The height h of the tower is given by the following formula :—

$$h = \frac{d}{\cot \alpha - \cot \beta}$$

where α = the angular elevation of the tower from the more distant point of observation.

β = the angular elevation after walking a distance d towards the tower.

d = distance between the two points of observation.

PRINCIPLE AND THEORY OF THE EXPERIMENT

When the index glass is exactly parallel to the horizon glass, rays from a distant object can reach the telescope by two separate paths. One set of rays passes through the clear portion of the horizon glass and enters the telescope without deviation. Another set of rays is reflected from the index glass to the silvered portion of the horizon glass, and enters the telescope in the same direction as the first set. The parallel rays P and Q (Fig.–52) are brought to a focus in the focal plane of the objective lens of the telescope, and give rise to a single image of the distant object. When this is the case the movable arm C should be at the zero mark of the graduated arc. If it is not so, the reading is noted ; it is called the *zero reading*. If now the movable arm be turned through a small angle, the rays reflected by the mirror M_1 will enter the telescope at a different angle, and the image formed by these rays will be displaced with respect to the image seen directly.

Now, suppose it is required to measure the angle between two objects situated in the directions M_1R and M_2P respectively. The sextant is supported so that the telescope is pointed directly towards the object in the direction M_2P, the rays passing through the transparent portion of M_2. The mirror M_1 is rotated till the rays coming in the direction RM_1 are reflected along M_1M_2, and fall on the silvered portion of M_2 which reflects them into the telescope. Then the angle between the directions of the two objects is the angle RM_1Q which is twice* the angle BM_1C, through which the movable arm has turned from the zero position.

This is so because the beam of light on reflection turns through twice the angle of rotation of the reflecting mirror.
For a detailed study, read author's book "A Critical Study of Practical Physics and Viva-Voce".

Thus to save calculations, the arc SS is usually graduated so that each degree is numbered as two degrees ; and hence the required angle can be directly read off from the graduations. *The algebraic difference between the present reading and the zero reading gives the required angle.*

Now, let an object AB (Fig.-53) of height h subtend un angle α at the first point of observation P, and let the same object subtend an angle equal to β at the second point of observation Q, then

$$h \cot \alpha = (x + d)$$

and $$h \cot \beta = x$$

whence $$h = \frac{d}{\cot \alpha - \cot \beta}$$

Thus, the height of the given object, say a tower, can be evaluated.

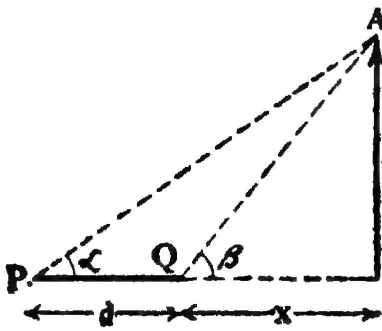

Fig.-53
To measure the height of a tower.

Method—

In order to obtain accurate results the following conditions must be satisfied by the instrument :—

(a) The plane of the index glass must be perpendicular to the plane of the graduated circular scale.

(b) The plane of the horizon glass must also be perpendicular to the plane of the graduated circular scale, and the horizon glass and the index glass must be parallel to each other when the zero of the vernier falls against the zero of the main scale.*

(c) The axis of the telescope must be parallel to the plane of the graduated circular scale and must pass through the centre of the horizon glass.†

(i) First of all, determine the vernier constant of the scale provided with the instrument. Then draw a short horizontal line with a piece of chalk-stick on the tower in level with the height of your eye. Let this line be called B. Now move to a sufficient distance from the tower and mark your position on the ground. This is the first point of observation. Let it be called P. *Stand erect at this point* and hold the sextant with its plane vertical. From this point of observation, point the telescope towards B such that part of the

If this condition is not satisfied, the instrument has a zero error which should be recorded for each set of observation ; *it varies when the distance of the object from the sextant is altered.*

Adjusting screws are provided for making the necessary adjustments, but we shall assume that these adjustments have already been made by the instrument-makers.

horizontal line is seen through the transparent portion of the horizon glass. The other part of the line B is seen through the reflecting part of this mirror at some vertically shifted position. Now, turn the movable arm (which, in turn, rotates the index glass) *till both parts of the horizontal line are seen as one continuous line across the two halves of the horizon glass.* Clamp the arm, and use the tangent screw for finer adjustment. In this position take the readings of the main scale and the vernier scale. The total reading gives the zero error corresponding to this point of observation. *Give a proper sign* (positive or negative according as the zero of the vernier falls to the right or to the left of the zero of the main scale) *to this zero reading.*

(ii) Now, hold the sextant in your hand in such a way that the plane of the graduated arc is vertical and the telescope is directed towards the chalk mark. Look at the mark through the unsilvered portion of the horizon glass as before. Rotate the movable arm slowly. By this process it will be seen that the image of the top of the tower (as seen by double reflection at the mirrors) moves downwards. Go on moving this arm till this image coincides* with the directly seen mark. Record, as before, the readings of the main scale and the vernier scale. Compute the total reading, which gives the *observed* angular elevation of the tower. Calculate the true angular elevation α by applying the zero correction.

(iii) From this point of observation P move a known distance, say 20 or 25 feet, (as measured with the tape) to another point Q towards (or away) from the knot† of the tower. *Siaad erect again* and hold tho sextant in the same manner as above.

Again record the zero-error (with proper sign) and the angular elevation as before. Then evaluate the true angle of elevation β.

(iv) Then calculate the height†† (AB) of the tower with the help of the formula given above. Repeat your measurements, changing α each time and in this way get the mean value of h.

Observations—

(a) Value of one main scale division = ...

(b) No. of divisions on the vernier = ...

(c) $\therefore$ Least count of the instrument = ...

* After clamping the arm, use the tangent screw for securing the exact coincides of the two images.

† The foot of the tower and the two points of observation P and Q *should be in the same straight line.*

†† Obviously, the result obtained is the height of the tower excluding the height of the mark from the foot of the tower. Hence, to get the total height of the tower add this value to the observed height.

[A] *Angular elevation from the first point of observation.*

S. No.	Readings for Zero Error			Readings for observed Angular Elevation		
	Main scale reading	Vernier reading	Total reading	Main scale reading	Vernier reading	Total reading
1						
2						
3						
		Mean			Mean	

[B] *Angular elevation from the second point of observation.*

[**Note**—Draw a similar table]

[C] Distance between the two points of observation (d)=...ft.

Calculations—

(i) Corrected elevation from I point of observation (α) = ...

(ii) Corrected elevation from II point of observation (β)= ...

Now $$h = \frac{d}{\cot \alpha - \cot \beta}$$

$= ... ...$ft.

Now, height of the chalk mark from the foot of the tower= ...ft.

$\therefore$ Total height of the tower = ...ft.

Result—The height of the tower as measured with the help of a sextant = ... ft.

Precautions and Sources of Error—

(1) The plane of the index glass and the plane of the horizon glass should be normal to the plane of the graduated circular arc, and when the glasses are exactly parallel, the zero of the vernier must coincide with the zero of the main scale.

(2) The axis of rotation of the index glass should coincide with the centre of the graduated circular scale.

(3) The axis of the telescope should be parallel to the plane of the circular scale.

(4) The foot of the tower and the two points of observation for the angular elevation should be in the same straight line.

(5) *The zero error* should be determined for each point of observation and applied to the corresponding reading.*

ADDITIONAL EXPERIMENTS

No—25 (a)

Experiment—To measure the distance between two objects in the same horizontal plane with a sextant.

For this purpose, two cross-marks, made in the same horizontal line on a wall, can be employed. The sextant must be held in the same horizontal plane as the two marks. In the accompanying figure C_1 and C_2 are the positions of the two marks and S is the position of the sextant. Measure the angle θ subtended by the two marks at S. Measure also the distance b and c and thus calculate the distance (= a) between the objects with the help of the formula :—

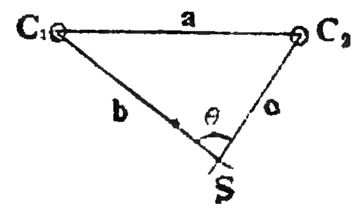

Fig.–54
To measure the distance between two objects.

$$a^2 = b^2 + c^2 - 2\,b\,c\cos\theta$$

Check the result by measuring this distance directly.

No.—25 (b)

Experiment—To determine the elevation of the sun with the help of a sextant and an artificial horizon.

An artificial horizon is a horizontal reflecting surface. It can be the surface of mercury in a small trough or it may be a carefully levelled plane mirror consisting of black glass to avoid reflection form the lower surface. (see Fig.–55).

First of all determine the zero error of the sextant by coinciding the image of the sun as seen directly† through the unsilvered portion of the horizon glass with the image of the sun obtained by double reflection at the two mirrors.

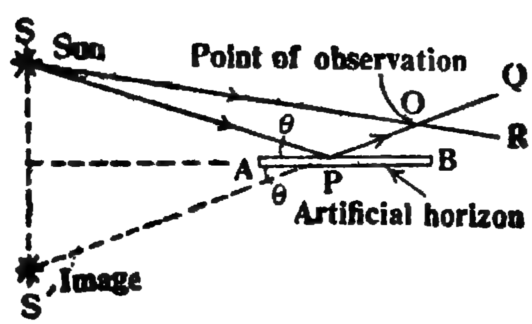

Fig.–55
To determine the elevation of the sun.

* The zero error (†) is inversely proportional to the distance (x) of the point of observation from the object. Thus, if a graph is plotted between (θ) and $1/x$, a straight line is obtained. *Perform this experiment in the laboratory.*

† Use coloured glasses, provided with the instrument, to reduce the intense glare of the sun.

Now let the artificial horizon AB be placed in a convenient position so that the image S′ of the sun can be seen directly, as well as by reflection, and measure the angle subtended at the point of observation O by the sun and its image by holding the sextant vertically. Apply zero correction to this reading. *The angular elevation of the sun is half of the corrected value of this angle.*

From the diagram it is clear that we are measuring the angle QOR since the instrument should necessarily be above the artificial horizon. Actually, we require the angle SPS′ ($=2\theta$), but since we are using a distant object, the two angles do not differ appreciably. Thus the elevation may be measured by half the angle QOR. This then measures the angular elevation of the sun.

To determine the height of the sun in linear units, the elevation should again be measured at a second point P′ (as described in the main experiment above). Now the distance PP′ being known, the actual height of the sun can be calculated by the method already described.

No.—25 (c)

Experiment—To measure the angular diameter of the sun with **the help of a sextant.**

For conducting this experiment, hold the sextant vertically and look at the sun through the transparent portion of the horizon glass. Use sun-glasses to reduce the intense glare of the sun. In Fig-55, S is the direct image of the sun. Now turn the movable arm of the sextant such that the image S′, obtained by double reflection from the two mirrors, *just touches* the rim of S Note this reading.

Now, turn the movable arm further till the image S′, afther overlapping S, crosses on the other side. Adjust the arm till this image S″ just touches the direct image S on the opposite rim. Note down this reading of the sexatnt as well.

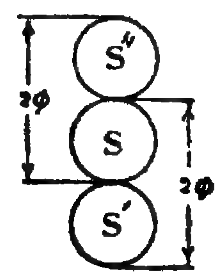

Fig.–56
To measure the angular diameter of the sun.

It is obvious from the figure that the difference between the two readings gives 2ϕ where ϕ is the angular diameter of the sun.

EXPERIMENT—26

Object—To determine the refractive index of water by total internal reflection using Searle's method.

Apparatus Required—Searle's apparatus*, glass container for the experimental liquid, and a green glass plate.

For accurate determination of refractive index by this method, Wollaston's apparatus consisting of a spectrometer, an air-film and a sodium lamp is employed, but the advantage of Searle's method is that the apparatus is simple and much less costly.

Description of the Apparatus—A sectional diagram of the actual apparatus employed in this experiment is illustrated in Fig.-57.

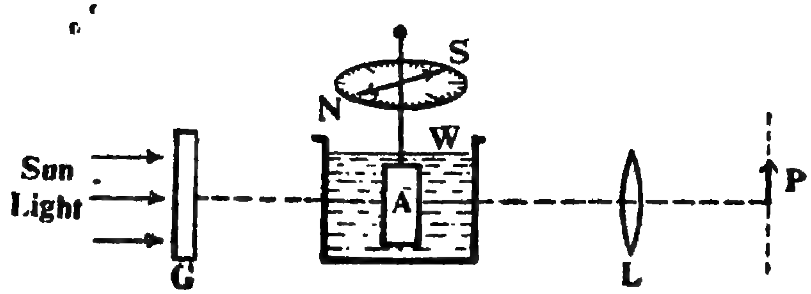

Fig.–57
Searle's apparatus for total reflection.

The apparatus consists of a rectangular glass trough whose vertical sides are plane parallel. The trough contains the experimental liquid (water in this case) in which is suspended the air-film cell. The cell consists of a thin film of air enclosed between two thin glass plates having optically plane surfaces. The glass plates are cemented together along the edges so that no liquid can enter into the air-film and thus spoil it. The air-film can be rotated about a vertical axis, its rotation being given with the help of a pointer moving over a graduated circular scale. A convex lens and a pin are also provided with the instrument as shown. Sometimes, a green glass plate known as filter, is placed outside the vessel in order to provide nearly monochromatic light.

Formula Employed—The refractive index of the liquid (water in this case) is given by the formula—

$$_a\mu_w = \text{cosec}\ \theta$$

where $_a\mu_w$ is the refractive index of water and θ is the critical angle for water-air interface.

PRINCIPLE AND THEORY OF THE EXPERIMENT

When a ray of light passes from a medium of refractive index μ_1 to another of refractive index μ_2, with an angle of incidence i and angle of refraction r, we have

$$_1\mu_2 = \frac{\mu_2}{\mu_1} = \frac{\sin i}{\sin r}$$

In the case when $\frac{\mu_1}{\mu_2} > 1$, the value $\frac{\mu_1}{\mu_2}.\sin i$ must not exceed unity. In the limiting case when $\sin i = \mu_2/\mu_1$, the corres-

ponding value of r is 90°, and i then measures what is known as the *critical angle*. Let it be denoted by θ. For values of i greater than θ the surface of separation of the two media acts as a perfect reflector.

If i is slowly increased a value is finally attained when the refracted ray suddenly disappears. In this case, if the second medium is air so that $\mu_2 = 1$, and if the first medium be water, we have

$$1/\mu_1 = \sin i = \sin \theta$$

or $${}_a\mu_w = \operatorname{cosec} \theta.$$

This is the formula* employed in the present experiment.

Method—

(i) Arrange the experiment either in an ordinary room or a dark room. When arranged in an ordinary room, a green glass plate may be employed just before the vessel of water to make the diffused sunlight nearly monochromatic. If the experiment is arranged in a dark room, a sodium lamp may be used as a source of monochromatic light.

(ii) Suspend the air-film† in water, and place your eye behind the pin P. To start with, keep the film surface in such a way that it is nearly normal to the line of sight.

(iii) Rotate the film gradually, till the dark edge appears in the field of view. Adjust the position of the pin P till there is no parallax between it and the dark edge. Read both the ends of the pointer on the graduated scale.

(iv) Now rotate the film in the opposite direction, till the dark edge coming from the opposite side, coincides with the pin. Read both the ends of the pointer again. By taking the difference of the readings of the same end of the pointer, calculate 2θ, *i. e.*, twice the value of the critical angle. From this obtain the mean value of the critical angle and evaluate the refractive index of water‡ with the help of the formula given above.

* See the note given at the end of this experiment.

† Examine carefully beforehand the-trust-worthiness (*i. e.* water-tightness) of the air-film.

‡ Record the temperature of the liquid before and after the experiment and calculate the mean temperature, which should be reported with the result.

Observations—

Readings for the determination of the critical angle.

S. No.	Rotation of the air-film				Value of 2θ	Mean value of θ	Remark
	Clockwise		Anticlockwise				
	One end of the pointer	Second end of the pointer	One end of the pointer	Second end of the pointer			
							Temp. of water (i) before the expt. = ... (ii) after the expt. = ...
							Mean temp. = ...

Calculations—

$$_a\mu_w = \text{cosec } \theta$$

$$= \ldots\ldots\ldots$$

Result—The refractive index of water at.........°C and corresponding to the green light of the solar spectrum = ...

[**Note**—When the experiment is conducted with diffused white light, it is found that the region of darkness is separated by a coloured band from the bright region. This is due to the fact that the refractive index of a substance varies with the wave-length of light. The critical angle for the violet colour which is least, hence during rotation of the air-film it is this colour which is first cut off, and hence violet light is absent from the rays nearest the dark edge. The other colours disappear later as their critical angles are successively reached. Hence, the only colour which is present before the image is completely cut off is the red one which has the largest wave-length. *Thus when the experiment is conducted in white light, the value of the refractive index is for the red and not for the mean (yellow) ray.*]

A NOTE ON THE THEORY OF THE EXPERIMENT

The accompanying figure depicts the path ABCD of a ray incident on the water-glass surface at an angle θ. The refracted ray into the glass strikes the glass-air surface at the critical angle ($= r$ in the figure). consequently the ray CD travels parallel to the surface of separation of the two media.

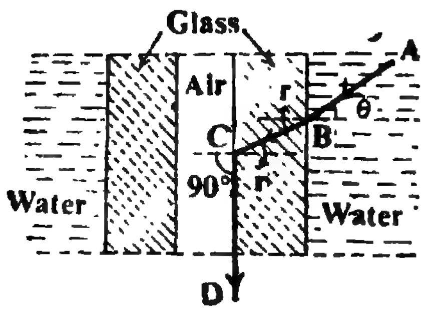

Fig.–58
Total reflection of a ray of light.

It is obvious from the figure that the critical angle is for glass-air combination, but the apparatus described above has been used to determine the refractive index of the liquid in the vessel. This will be clear from the following reasoning :—

From the figure $$_{a}\mu_{g} = \frac{\sin 90^\circ}{\sin r} = \frac{1}{\sin r}$$

Also $$_{w}\mu_{g} = \frac{\sin \theta}{\sin r}$$

But $$_{w}\mu_{g} = \frac{_{a}\mu_{g}}{_{a}\mu_{w}}$$

Hence $$_{a}\mu_{w} = \frac{_{a}\mu_{g}}{_{w}\mu_{g}} = \frac{1}{\sin r} \times \frac{\sin r}{\sin \theta} = \frac{1}{\sin \theta}$$

We have thus to measure the angle θ, and from it we deduce the value $_{a}\mu_{w}$, the refractive index from air to water.

EXPERIMENT—27

Object—To determine with the help of a Nodal Slide the focal length of a combination of two convex lenses separated by a distance and also to verify the formula—

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

where F = focal length of the combination.

f_1, f_2 = focal lengths of the given lenses.

d = distance between the two lenses.

Apparatus Required—Nodal assembly, and two convex lenses.

Description of the Apparatus—The *Nodal-Slide Assembly* consists of an optical bench carrying four uprights to hold its various

components. One upright carries a frosted electric bulb encased in a metal housing having a circular aperture, through which issues

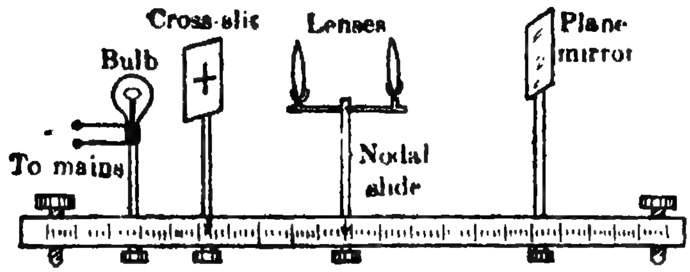

Fig.–59
Nodal-Slide Assembly.

out the light to illuminate the cross-slit cut in a metal plate carried by the adjacent upright. The third upright carries the nodal slide,* which is capable of rotation about a vertical axis, and the angle of rotation can be recorded, if so required, from its graduated circular base. The given lenses are placed in holders which are provided with the carriage of the nodal slide, and the distance between the lenses can be directly read on a linear scale provided with the carriage. The carriage can be made to slide and as such the axis of rotation of the nodal slide can be made to pass through any point on the axis of the lens system. In order to find out through what point on the axis of the lens system the axis of rotation of the nodal slide is passing, an index mark is made on the slide, past which the carriage scale moves. This index mark locates the axis of rotation of the nodal slide. The fourth upright carries a plane mirror which can be rotated about a horizontal axis perpendicular to the bed of the bench.

PRINCIPLE AND THEORY OF THE EXPERIMENT†

It is an important property of the *nodal points* of an optical system that *if a ray of light passes through one of them, its conjugate ray passes through the other nodal point, and is always parallel to the incident ray.*

Let us consider an optical system (Fig.–60) of which N_1 and N_2 are the nodal points. Let a beam of parallel rays BC, AN_1,

* This is called a *nodal slide* because by *sliding* the carriage the position of the second nodal point of a lens or of a lens system can be located.

† For a detailed study, read author's book "A Critical Study of Practical Physics and Viva-Voce".

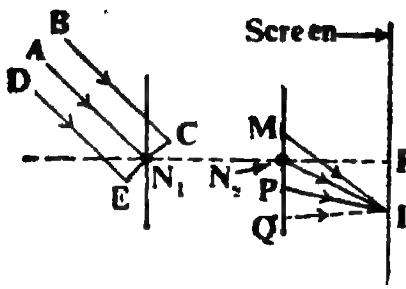

Fig.-60
Nodal points.

and DE be incident on the system, then the corresponding emergent rays MI, N_2I, and PI will meet at I in the second focal plane F_2I of the system. I is, therefore, the image formed in this focal plane. Now, suppose the optical system is such that it can be rotated about a vertical axis passing through the second nodal point N_2. As the system rotates the nodal point N_1 describes a short arc and now the ray BC begins to pass through the nodal point N_1 and hence N_2I becomes its conjugate ray. The other two rays AN_1 and DE emerge out as PI and QI respectively. It is thus obvious that even after a slight rotation of the optical system about the nodal point N_2, the emergent rays continue to pass through I, which consequently remains stationary.

Thus, if a point could be found on the axis of an optical system such that *a slight rotation of the system about a vertical axis passing through this point keeps the image stationary* on the screen, then this point corresponds to the second nodal point of the system Since the medium on both sides of the system is the same, the nodal point coincides with the principal point, and consequently the distance between the screen (which corresponds to the second focal plane) and the axis of rotation (which corresponds to the second principal plane) will give the focal length of the system.

This condition can easily be realised in practice by adopting the following procedure :—

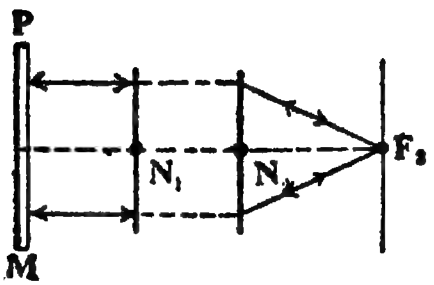

Fig.-61
Nodal points and
focal length.

Let a point source (Fig.-61) be located at F_2 and let PM be a plane mirror placed as shown. When the image of the source is formed at the same place, it follows that the pencil of rays emerging out from the system is parallel. As it strikes the plane mirror normally it retraces its path after reflection and forms the image at F_2. If the axis of rotation of the optical system is made to coincide with N_2 the image at F_2 shall be practically stationary. Then the focal length of the system is equal to N_2F_2.

By adopting this procedure the equivalent focal length F of a combination of two lenses, d cms. apart. can be determined. If the separate focal lengths of the two lenses be f_1 and f_2, the observed

value of F can be compared with the value calculated from the theoretical formula*—

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1 f_2}$$

Method—

(i) Mount the lamp, the cross-slit, one of the two given lenses and the plane mirror on the optical bench *at equal heights* and in such a manner *that their line is parallel to the bench.* Keep the plane of the lens normal to the bench, *i. e.*, its principal axis should be parallel to the bench. Let the plane mirror be placed *not far behind* the lens. Move slowly the upright carrying the cross-slit till *a clear and well-defined* image† of the cross-slit is obtained in the cross-slit plane itself. (By tilting the plane mirror suitably, the image should be brought quite close to the cross-slit). In this situation the cross-slit is in the focal plane of the lens. Note the position of the lens and the slit. The distance between the two gives the focal length of this lens.

Now turn the nodal slide through 180° so that the other face of the lens now receives the incident light. Adjust as before and determine the focal length. The mean of the two values gives the value of f_1.

(ii) Similarly, determine the focal length f_2 of the second given lens.

(iii) Now mount the two lenses on the nodal slide at a particular distance d. Keep the line joining the lenses parallel to the bench, and the planes of the lenses perpendicular to this line. Move the slit backward and forward till a clear and well-defined image of the slit is formed in the plane of the slit itself. (By tilting the mirror suitably, the image can be brought quite close to the slit). The cross-slit is now in the focal plane of the lens system.

Now rotate the lens system about its vertical axis through a small angle both ways and note carefully what happens to the image of the cross-slit. You will see that the short focus of the image does not change, but the image is displaced side ways. From this we conclude *that the axis of rotation is not passing through the first*

* For a proof of this formula consult any standard Text-book of Optics.

† Be cautious about a bogus image which is sometimes received on the screen (see precuation (4) at the end of the expt.)

‡ Unless the axis of rotation passes through the nodal point of the optical system, the image shall move on the screen when the system is rotated. If the no-shift position of the image is crossed, the direction of motion of the image shall get reversed.

nodal point of the lens system. In order to procure this, repeat the following three steps :—

(a) Shift the rod carrying the two lenses a few cms in the clamp in the upright.

(b) Adjust the cross-slit backward and forward till the focus of the image is regained.

(c) Rotate the lens system by a few degree both ways about the upright axis to see if the image remains stationary or not.

Repeat these three steps till a rotation of the lens system about the upright axis does not produce any displacement in the image. In this position, the upright, about which the lens system is rotated, gives the position of the first nodal plane, and the upright carrying the cross-slit gives the position of the first focal plane. Note the position of these uprights. Their distance apart gives the focal length of the combination.

Repeat the process by turning the nodal slide through 180°. The means of the two values of focal length gives F.

[Note—If the aim of the experiment is to verify the theoretical formula, then the turning of the nodal slide through 180° may be left out, and the experiment should be repeated for three or four values of d. However, if the aim of the experiment is to determine F for one given value of d. then the nodal slide should be turned through 180° and the readings should be repeated.]

(iii) Verify the formula* given above by calculating the value of F after substituting the values of f_1, f_2 and d.

[Note—In this experiment the distance have been measured at the bases of the uprights. *If any appreciable bench error is present, it should be taken into account.*]

* The experiment can be repeated by changing the value of d and determining each time the focal length of the combination.

Observations—

[A] *Readings for the determination of F.*

S. No.	Distance between the two lenses (d)	Position of the cross-slits (a)	Position of the nodal-slide			Focal length of the combination F =(b)—(a)
			I Setting	II Setting (after turning through 180°)	Mean (b)	

[B] *Readings for the determination of* f_1.

S. No.	Position of the cross-slit (a)	Position of the nodal-slide			Focal length of the lens $f_1 = (b) - (a)$
		I Setting	II Setting (after turning through 180°)	Mean (b)	
				Mean	

[C] *Readings for the determination of* f_2.

[Note—Make a similar table as drawn at (B)].

Calculations—

From the above the values of

$f_1 = \ldots \ldots$ cms ; $f_2 = \ldots \ldots$ cms ; $d = \ldots \ldots$ cms

$$\therefore \quad \frac{1}{F} = \frac{1}{f_1} - \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

$$= \ldots \ldots \ldots$$

or $\quad F = \ldots$ cms

Results—The focal length of the combination of the two green lenses at a distance of ... ... cms. apart

(i) as observed experimentally = ... cms.

(ii) as calculated by the formula = ... cms

Thus, within the limits of experimental error, the observed and calculated values of the focal length of the combination are approximately equal, hence the formula

$$\frac{1}{F} = \frac{1}{f_1} - \frac{1}{f_2} - \frac{d}{f_1 f_3}$$

is verified.

Precautions and Sources of Error—

(1) The height of the lenses should be so adjusted that their principal axis passes through the point of intersection of the cross-slits.

(2) To test the stationary position of the image, the nodal slide should be rotated through a small angle, say 5°.

(3) The no-shift position of the image shou'd be accurately ascertained. For this purpose, the nodal-slide should be rotated in the same direction with the axis of rotation lying on both sides of the desired nodal point. The displacement of the image should be reversed as the nodal point is crossed.

(4) The mirror used in this experiment should be *truly plane*.

(5) *Bench correction* should be taken into account, otherwise the distances of the uprights measured at their bases shall not be correct.

(6) If during the course of the experiment, no focal point can be located on one side of the lens system, then it means that it lies within the combination. In that case the whole combination should be turned through 180° and the other focal point should be located.

[**Note**—For a successful verification of the formula, d must be changed substantially (say, by several cms) with respect to $f_1 \times f_2$. However, when d becoms larger than one of the focal lengths, one of the focal points lies within the lens combination.]

(7) Sometimes, a bogus image is received on the screen. This is formed by reflection of light falling on the surface of the lens. This image should not be mistaken for the genuine one obtained by refraction through the lens and by reflection from the plane mirror. The unwanted image can be easily distinguished by slightly rotating the plane mirror when the bogus image will remain stationary while

the desired image will shift. This test should invariably be applied to discard the false image.

EXPERIMENT—28

Object*—To determine with the help of a spectrometer the refractive index of the material of a prism corresponding to the wave-length (= 5893 A. U.) of light emitted by sodium.

Apparatus Required—A spectrometer, a prism, reading lens, reading lamp, and sodium lamp.

Description of the Apparatus—The essential parts of a spectrometer are depicted sectionally in Fig-62. It consists of the following parts—

(a) *The Collimator* C which produces a parallel beam of light ;

(b) *The Prism Table* P on which is mounted the prism to produce the dispersion of the incident beam ;

(c) *The Telescope* T which is employed to observe the resulting spectrum.

(1) **The Collimator**—It consists of a long hollow tube which carries at its one end an achromatic lens L, and at the other end there is a draw-tube which can be moved in and out by means of a rack and pinion arrangement. Thus, the slit S provided at the farther end of the draw-tube can be placed in the focal plane of the lens L and a pencil of parallel rays can be secured. The slit has one jaw fixed and the other jaw is movable with the help of a screw, and thus the width of the slit can be suitably adjusted. The collimator is rigidly fixed to the main body of the instrument.

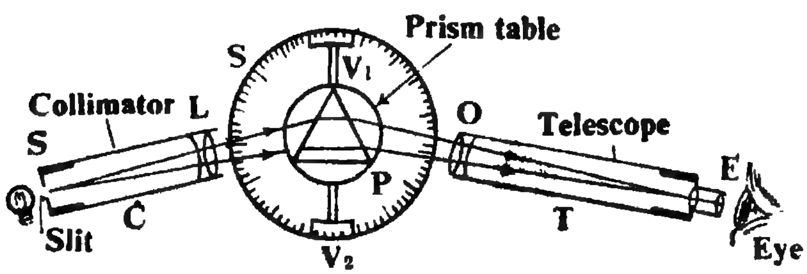

Fig.-62
Spectrometer (sectional diagram)

(2) **The Prism Table**—It is a plane circular table which can revolve about a vertical axis. It is provided usually with a clamp so that it may be fixed in any desired position, and sometimes with a tangent screw to give it a slow motion. The height of the table

*For a fuller discussion of this experiment, read author's book "A Critical Study of Practical Physics and Viva-Voce".

can be adjusted by raising or lowering the rod carrying it and then clamping it in position. The position of the table can be read with the help of two verniers carried by it and moving over a graduated circular scale carried by the telescope. The prism table can be adjusted horizontal with the help of three small levelling screws provided at its base. On the upper surface of the table are generally drawn concentric circles and straight lines parallel to the line joining any two of the levelling screws. They are helpful in securing some of the adjustments as described later.

(3) **The Telescope**—The telescope consists of a hollow tube at one end of which is fixed an achromatic objective lens O, the other end of the tube carries a draw-tube operated by a rack and pinion arrangement and carrying a Ramsden's eye-piece E. The telescope can be rotated about the same vertical axis as the prism table, and, like the latter, is usually provided with a clamp and a tangent screw.

Formula Employed—The refractive index (${}_a\mu_g$) of the material of the prism is given by the following formula :—

$$ {}_a\mu_g = \frac{\sin \frac{A + \partial_m}{2}}{\sin \frac{A}{2}} $$

where A = Angle of the prism.

∂_m = Angle of minimum deviation.*

Method—

(i) Before proceeding to make actual measurements with a spectrometer, it is essential to properly adjust its various components. The exact adjustment is a process which requires considerable care.

(a) Alignment of the Source with the Collimator—

Keep the spectrometer with its collimater slit near the window of the source. Make the slit a bit wide and look through the collimator lens. (For this purpose, the telescope may be turned aside.) The whole lens of the collimater should appear well illuminated symmetrically. If it is not so, then turn the specterometer as a whole keeping the slit always in front of the window. *When the*

* Here the angle of minimum deviation is for the spectral line employed (D. line of sodium in the present case). However, if a light consisting of several spectral lines (*e. g.*, murcury light) is employed, δ_m should be measured for each line and the corresponding refractive index calculated separately in each case.

collimater axis comes in line with the source center, the collimating lens is fully illuminated. This arrangement is called 'alignment'. This adjustment is very important.

(b) **Adjustment of the Telescope and the Collimator for Parallel Light by Schuster's Method**—Turn the telescope towards a uniformly illuminated surface, such as a white wall, and slide the eye-piece in and out of the tube until the cross-wires fixed in the tube can be seen distinctly. The eye-piece is now said to be in focus on the cross-wires. This adjustment should not be disturbed in subsequent adjustment.

Now illuminate the slit of the collimator with sodium light and without any focussing, place the prism approximately in the minimum deviation position. For this purpose put the prism on the table centrally. The ground face should be roughly parallel to the line joining the collimater and telescope lenses. By turning the telescope a little, the spectrun con easily be obtained in the field of view. Now turn the prism in such a way that the specturn *shifts towards the collimator axis.* Follow on the spectrum with the telescope and go on turning the prism slowing in that direction which brings the spectrum nearer and nearer the collimator axis. But after some time it will be seen that on turning the prism a little more, the spectrum now goes from the collimator axis. Even if the prism is turned the other way, the spectrum goes away from the collimator axis. This is the position of minimum deviation.

Turn the prism slightly away from this position, *bringing the refracting edge towards the telescope. Focus the telescope* on the image as distinctly as possible, making the slight rotation of the telescope which can be necessary to keep the image in the field of view. Rotate the prism *to the other side of the minimum deviation position* and now *focus the collimator* until on looking into the telescope the image is again seen as distinctly and as well-defined as possible‡

† It is assumed that the *mechanical adjustments* have been made by the instrument-maker, hence only the principal optical adjustments should be done.

‡ To facilitate the above focussing, the slit should be opened a bit wider and its image should be seen with clear and well-defined edges. When the adjustment is quite correct there should be no parallax between the cross-wires and the edges of the slit. The accuracy of the adjustment can be judged by the slightly moving the eye from side to side *behind the eye-piece* and nothing any relative motion of the cross-wire and the edge of the slit.

In this setting it must be clearly borne in mind that *if the prism is first turned so that its refracting edge moves towards the collimator, then the first focussing must be done by means of the collimator.* If at any stage it is revealed that the image gets more and more indistinct after each operation, it must be immediately inferred that the first focussing has been wrongly done.

Repeat this process of alternately focussing the collimator and the focus. When this rotating the prism, the image does not go out of telescope until on is achieved the rays issuing out of the collimator and entering into the telescope are parallel. This means that both collimator and telescope are now *individually* set for parallel rays. In practice only a few alternate focussings are required.

(c) **Adjustment of the Prism Table** -When a prism is used with a spectrometer it is necessary to adjust it so that its refracting edge is vertical, *i. e.*, parallel to the vertical slit of the collimator. This is achieved with the help of the three levelling screws D, E and F which are situated at the vertices of an equilateral triangle (Fig.-63). Set the prism ABC on the table in such a way that both the faces AB and AC receive the light coming from the collimator, and the face AC is normal* to one of the sides (for example, the side EF) of the traingle DEF.

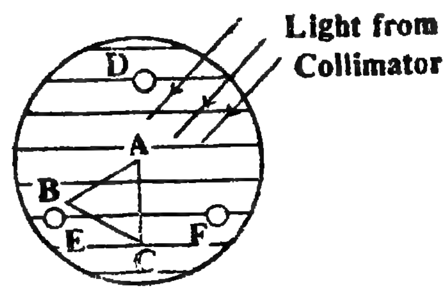

Fig.-63
Adjustment of the prism table.

Receive the light reflected from the face AC into the telescope and *with the help of the screws E and F* bring the image of the slit symmetrically in the field of view. Now rotate the telescope to receive the rays reflected by the other face (AB) bounding the refracting edge. If further adjustment of the image on the cross-wire is necessary, *it must be done by the screw D alone*, for this will not disturb the previous adjustment, since it does not turn the face perpendicular to EF out of its vertical plane

The two faces are now vertical and the instrument is ready for subsequent measurements of A and δ_m.

(ii) **Measurement of the Angle (A) of the Prism**—Open the slit fairly wide so as to allow plenty of light to pass through the collimator. Place the prism† on the table of the spectrometer with the adgle to be measured turned towards the lens of the collimator.

Some of the light falling on each face will be reflected as shown by the continuous lines drawn in Fig.-64. Move the eye

The lines ruled on the top of the table assist in the setting of the prism in this manner.

Place the prism in such a way that the refracting edge lies over the centre of the prism table. This particular mode of placing the prism is essential, since it will rectify the error which may otherwise be introduced due to want of parallelism in the emergent beam from the collimator.

in the horizontal plane of the axis of the collimator, and looking at the face AB locate the direction of the reflected beam. Now bring in the telescope just in front of the eye in this position. Then on looking through the telescope the image* (I_1) of the slit will be seen. When the image has been brought in view, reduce the width of the slit and turn the telescope, if necessary, till the intersection of the cross-wires coincides with the image of the narrow slit.

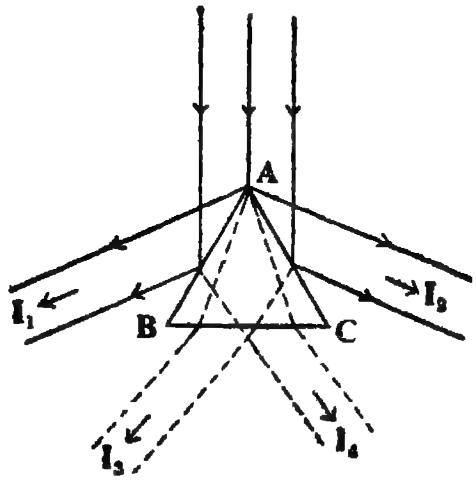

Fig.–64
Reflected & refracted images due to a prism.

Read† the position of the telescope with the help of the two verniers. Now without moving the prism or the table turn the telescope to view the second image (I_2) formed by reflection from the second face AC of the prism, and again read the position of the telescope. Deduce the angle‡ between the two positions of the telescope.

Take the mean of these two angles. The angle so obtained is double the refracting angle of the prism. Half of it shall give the required value A for the angle of the prism.

* (i) It often happens that two additional images (I_3 and I_4) come into view as the telescope is moved round in a horizontal plane. These bogus images correspond with the refracted rays shown dotted in Fig.–64. This trouble is easily avoided if the face BC is of ground glass.

(ii) Sometimes, it also happens that the required reflected images can be plainly seen with the unaided eye, but cannot be seen in the telescope. This is due to the fact that the prism table is not properly levelled, the light being reflected either upwards or downwards, so that it strikes the inside of the telescope tube It will be found in this case that when the telescope is swung into position after locating the image with the naked eye, the eye-piece of the telescope is not in level with the eye. In such a case, level the prism table with the help of the levelling screws attached to it till the eye and the eye-piece lie on the same horizontal level. Then finally adjust the level till the image of the slit occupies the same position in the telescope field when viewed by reflection on either face, and also when observed directly with the telescope and the collimator with no prism on the table.

† Use a reading lens and a reading lamp for the purpose.

‡ Find the difference of angle *between the two readings of the same vernier.*

(*iii*) **Measurement of the Angle (δ_m) of Minimum Deviation—**

Place the prism* on the table of the spectrometer so that the angle A already measured may serve as the refracting angle. Then the beam of light issuing from the collimator should fall on the face AB and emerge from the face AC to be received be the telescope.

In order to find the direction in which the telescope should be pointed in order to receive the emergent beam†, turn it to one side and using one eye only, look into the face AC of the prism, moving the eye until an image of the slit formed by refraction through the prism is found. When the proper direction has been located, turn the telescope to point in this direction, but do not move the head. On looking through the telescope, the image of the slit would now be found in the field of view.

Now, to locate the position of mimimum deviate, look through the telescope and rotate the prism table so that *the image of the slit moves towards the axis of the collimator produced.* Keep on slowly moving the telescope so as to keep the image in the field of view. Ultimately, a position will be attained when the image of the slit is as near to the axis of rotation as it can possibly be.

Adjust the telescope so that the slit is approximately in the centre of the field of view and then clamp it. Make the slit now as narrow as permissible and rotate the prism table slowly backwards and forwards several times. Move the telescope by the tangent screw until, as the prism is rotated, the slit moves up from one side until it is bisected by the vertical cross-wire. and then moves away again *to the same side without ever passing beyond this position.* Read this position‡ of the telescope.

Now, remove the prism and turn the telescope to point directly towards the collimator and set the direct image of the slit on the cross-wire. Clamp the telescope in this position and make the final adjustment with the slow motion screw. Again read the position of the telescope with the help of the two verniers. To get the angle

* In setting up the prism take care to place it in such a position that the maximum amount of light available from the collimator is utilised and enters into the telescope. This is best achieved *if the prism is placed symmetrically at the centre of the table.* Incidentally, it may be added that this mode of mounting the prism not only corrects for want of parallelism in the incident beam but also improves the resolving power of the prim.

† Have the slit wide open in looking for this image.

‡ Great care must be taken in the use of an angular scale and vernier to make quite sure of the least angle which can be determined by means of the vernier provided.

All readings should be checked before the clamp is released. While taking observations, at no time should both the prism table and the telescope be unclamped.

of minimum deviation take the *difference in the readings of the same vernier* for the two positions of the telescope.

Next turn the prism table so that the light from the collimator now falls on the face AC and the dispersed image is seen through the face AB. Repeat the experiment as above and get the value of δ_m. Finally, get the mean valve of δ_m by taking the average of the results obtained here and those obtained above.

Now, calculate the valve of μ with the help of the formula given above.

Observations—

(i) Value of one small scale division =
(ii) No. of divisions of the vernier =
(iii) ∴ Least count of the instrument =

[A] *Readings for the determination of A.*

Vernier	Reflection from I face			Reflection from II face			Difference of the two readings of the same vernier (2A)
	Main scale reading	Vernier reading	Total reading	Main scale reading	Vernier reading	Total reading	
V_1							
V_2							

Mean 2A =

[B] *Readings for the determination of δ_m.*

Vernier	For dispersed image			For direct image			Difference of the two readings of the same vernier (δ_m)
	Main scale reading	Vernier reading	Total reading	Main scale reading	Vernier reading	Total reading	
V_1							
V_2							

Mean δ_m =

Calculations—

$$\text{Mean } 2A = \dots\ \dots$$
$$\therefore\quad A = \dots\ \dots$$
$$\text{Mean } \delta_m = \dots\ \dots$$
$$\therefore\quad \mu = \frac{\sin \dfrac{A+\delta_m}{2}}{\sin \dfrac{A}{2}}$$
$$= \dots\ \dots\ \dots$$

Result—The refractive index of the material* (... ... ...) of the prism for sodium light (wave-length = 5893 A. U.) = ...

[Standard value = ... ; Error = ...%].

Precautions and Sources of Error—

(1) The *alignment* of the collimator with the source should be made to secure good illumination.

(2) The eye-piece of the telescope should be *focussed* on the cross-wire.

(3) Both the telescope and the collimator should be individually set for *parallel rays.*

(4) The prism table should be properly levelled so that its *refracting edge is parallel to the axis of rotation* of the instrument and hence to the slit.

(5) While taking measurements, the slit should be made *as narrow as permissible* and the cross-wire should be adjusted at the centre of the image.

(6) To eliminate the error due to non-coincidence of the centre of the graduated circle with the axis of rotation, both the verniers should be read. For calculating an angle the difference of the two readings *of the same vernier* should be taken.

(7) All readings should be checked before the clamp of the prism table or of the telescope is released *and at no time should both the table and the telescope be clamped simultaneously.*

(8) In order to eliminate the error due to want of parallelism in the emergent beam of light coming from the collimator, the prism

* Mention here 'crown glass', 'flint glass', or 'extra-dense flint glass', etc., according to the material of the prism used.

should be so placed* that *its refracting edge passes through the centre of the prism table.*

(9) The setting of the cross-wires on the spectral line should be checked by the no-parallax method.

ADDITIONAL EXPERIMENTS

No.—29 (a)

Experiment—To determine the refractive index fot the material of a prism for different colours of mercury light with the help of a spectrometer, and to calculate the dispersive power of the prism.

This experiment is but a repetition of the above experiment. Determine the angle of minimum deviation correspondings to each spectral line and calculate the refractive index separately in each case.

[Note—For studying the variation of refractive index with wave-length† of the spectral line employed, a graph can be drawn between the two quantities. From this study it will be seen that the value of refractive index increases as the wave-length diminishes.]

Now the dispersive power ω of the material of the prism is given by the formula—

* The reason for this particular mode of placing the prism on its table can be easily understood by referring to Fig.-65.

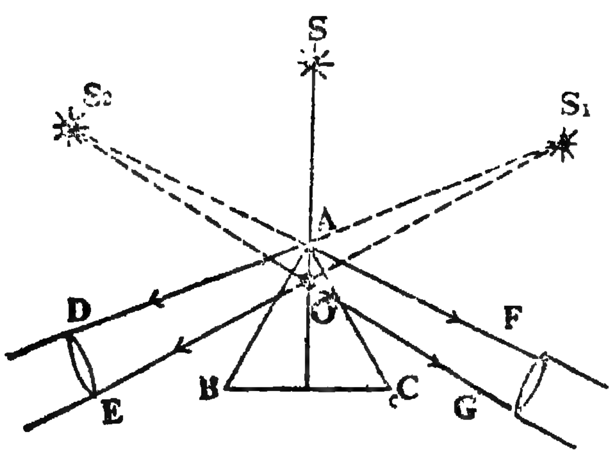

Fig.-65

The prism has been so placed that A does not lie over O, the centre of the graduated circle. If the beam incident on the prism be not parallel, the image of the slit lies, say, at S, Its images es formed by the faces of the prism lie at S_1 and S_2. Now

$$\angle S_1AS_2 = \angle DAF = 2A$$

Now the angle actually measured is EOG which is obviously less than DAF. To correct for this discrepancy, either the image S should be formed at infinity, *i. e.*, the rays issuing out of the collimating lens should be exactly parallel, or the centre of the prism table O should coincide with A. The latter condition is clearly much easier to attain.

† For the wave-length of mercury spectral lines, see the table given for Expt.-33.

$$\omega = \frac{\mu_v - \mu_r}{\mu - 1}$$

where μ_v μ_r and μ are the refractive indices of the three chosen wave-ledgths in the violet, red and yellow region respectively.

No.—29 (b)

Experiment—To determine the refractive index of a liquid* for a given wave-length of light with a spectrometer.

For this experiment is required a thin-walled hollow glass prism, whose two surfaces bounding the refracting angle should be optically true. The method is exactly the same as described previously.

[**Note**—In case the two surfaces of the glass-plate forming a face of the prism are not plane and parallel, two reflected images, one formed by the front surface and the other by the inside surface are seen. The latter image is unwanted and can be distinguished by filling the prism with the experimental liquid, when this image will be enfeebled in brightness due to loss of light in the liquid.]

* This method is applicable only for those liquids which are available in large quantities.

9

Interference Of Light

The undulatory theory of light at once leads us to expect that class of phenomena described by the term 'Interference'.* The principle of superposition of waves, as enunciated by Thomas Young, states that the particles of a medium, when simultaneously displaced by lhe arrival of several disturbances, have a resultant displacement obtained by adding together the vectors representing principle of superposition of wsves, as enunciated by Thomas individual displacements. Thus, the displacements arriving at a particle in the same direction will move it through a distance equal to the sum of the separate displacements. On the other hand, if the particle is subjected to displacements in oppositely directed directions, its displacement shall be of a smaller magnitude than if it were subjected to either of them separately. In particular, if both the displacements are exactly equal, but are oppositely directed. the particle shall undergo no displacement at all.

Thus, it is obvious that the arrival of oscillatory disturbances at any point of a medium shall cause larger or smaller displacements than would occvr as a result of each separately. Since the intensity of illumination at any point of a medium is proportional to the square of the amplitude of the vibrations at that point, the propagation of two trains of waves of light in a medium can produce places of large or small intensity. It may even be possible that there is complete darkness at certain point since it is possible that as a result of adding the diplacements vectorially we get no net effect. However, it must be clearly understood that actually there is no destruction of energy at those points where we get complete darkness ; only a redistribution of energy takes place, this energy appearing at places of maximum intensity.

* For detailed study, read author's book "A Critical Study of Practical Physics and Viva-Voce".

The essential condition for the phenomenon of interference to be observed is the constancy of the phose difference between the wave-trains given out by the two sources. Thus, no interference effects can be produced by using two separate candle flames Each point of a candle flame is a wave-source, and there is no constant relation between the phases of the waves emitted by any two points of the same flame, or of different flames. The only way to realise these sources vibrating in the same phase, or with a constant phase difference, or vibrating in such a manner that any change of phase which occurs in one is accompanied by the same change in the other, is to make one source the image of the other (as in Llyod's single mirror experiment), or to divide a pencil of rays into two either by reflection (as in Fresnel's double mirror experiment) or by refraction (as in Fresnel's bi-prism) and then to re-unite them by superposing one pencil over the other. We thus have the equivalent of two sources* emitting vibrations in the same phase.

EXPERIMENT—30

Object—To determine the wave-length of sodium light with the help of a Fresnel's Bi-prism.

Apparatus Required—An optical bench, bi-prism, slit, sodium lamp, lens to focus light on the slit, another short focus lens to determine the distance between the two virtual sources, and a bench correction rod.

Description of the Apparatus—The bi-prism may be conceived as made up of two small prisms of very small refracting angle placed base to base. In practice it is ground from a single piece of glass and polished. The action of thv bi-prism is *to produce two coherent images of the slit separated by a small distance*. If we look through the upper half of the bi-prism we see one image, while on looking through the lower half we see the other image only. If we keep our eye in between these two positions we see both the images. In fact

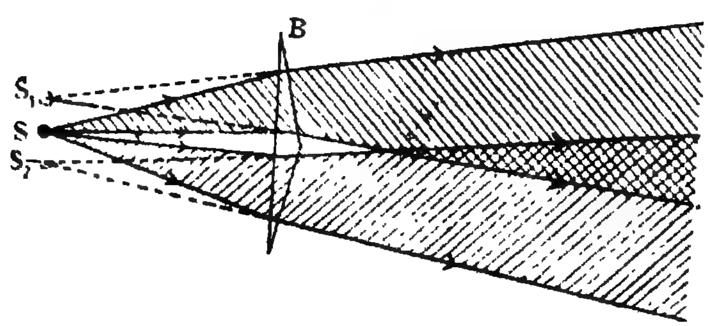

Fig.–66
Interference with a bi-prism.

* Such sources are said to be *coherent sources*.

this is the region where light if received from both the coherent sources and where interference fringes are formed (see Fig.-66, where this region is shown by double-shaded portion.)

In carrying out this experiment, accurate measurements have to be made ; for this reason an optical bench is employed (Fig.–67). It consists of a strong, rigid metal frame, provided with levelling screws on which it stands. The frame consist of two metal rails, one of which is graduated accurately in millimeters. Along the rails slide metal uprights, each of which is attached to a vernier at its lower end so that its position on the bench may be accurately read.†

The uprights serve to carry a slit micrometer eye-pieee. lens, or whatever piece of apparatus is necessary. If it is necessary to move any piece of apparatus transversely across the bench, it is placed in an upright fitted in a support fitted with a transverse micrometer screw.

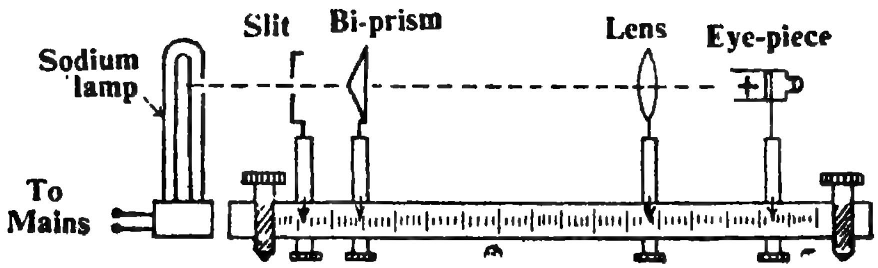

Fig.–67
λ by Bi-prism.

For mounting the bi-prism and the slit on the bench, suitable attachments are provided on the pillars. Each of these attachments is capable of rotation in its own plane and this rotation can be effected with the help of a tangent screw provided for the purpose.

One pillar carries a micrometer eye-piece of the Ramsden's type fitted with cross-wires. The eye-piece is provided with a

† Although we can read off the positions of the uprights on the bed of the frame, yet we cannot know the correct distance between say, the slit and the cross-wire, since they may not be situated exactly above the indicator marks. Hence a correction has always to be applied. For this purpose, a stand carrying a carefully measured rod (known as the bench correction rod) is placed on the rails. One end of the rod is placed in contact with the slit while the other end is viewed in the micrometer eye-piece. Let the distance between the two uprights, as measured on the scale, be '*a*' while the length of the rod be 'b'. Then to convert the readings as obtained from the uprights to the distances required we must add to the subsequently observed readings the quantity ($b—a$).

micrometer screw with a divided head, with the help of which it can be moved across the field of view at right angles to the bed of the bench.

Formula Employed—The wave-length λ of the light employed (in this case, sodium light) for the bi-prism experiment is given by the formula :—

$$\lambda = \beta . \frac{a}{D}$$

where
β = fringe width.
d = distance* between the two virtual sources of light (*i. e.* the two images of the slit).
D = distance between the slit and the screen.

PRINCIPLE AND THEORY OF THE EXPERIMENT

Let S_1 and S_2 (Fig.–68) denote two sources of monochromatic light. From each source is thus emitted a train of waves of a particular wave-length, which is the same for each of them. In addition, let us suppose that the disturbance starting from S_1 is in the same phase as that starting from S_2. These disturbances are propagated with the same velocity, and hence at those points which are equidistant from these sources the displacements will be in the same phase when they arrive there. At other points which are situated at different distances from these sources, it may happen that the displacements are in different phases on reaching there.

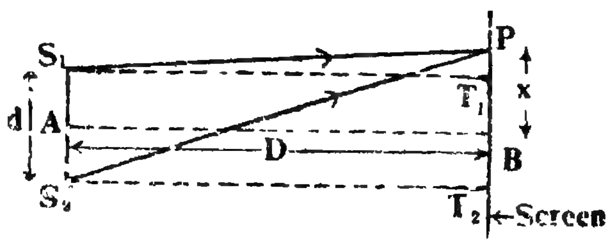

Fig.–68
Interference of light.

Let us investigate the illumination at different points on the screen placed as shown in the figure. Let AB the drawn at right angles to the middle point of $S_1 S_2$ and let it cut the screen normally at B. The point B is equidistant from the sorces, hence the disturbances will reach here in the same phases, and the displacements there being in the same direction will unite in increasing the illumination at this point, which will therefore, be always bright. Let us now consider a point P on the screen and let its distance from B be x. The waves of light from S_1 travel a distance S_1P

The formula, $d = 2a (\mu - 1)$ A, can be used to know d Here 'a' is the distance between the bi-prism and the slit, A is the angle of the prism and μ is its refractive index. Thus, knowing a, μ and A the value of d can be canculated. This is a quicker method hence it is preferable for a laboratory practice.

whereas these from S_2 a distance S_2P. Thus, those waves which arrive at P simultaneously must have started from their sources at different times. Thus, the waves on reaching P will be in different phases. If, however, the phase differencc, $(S_2P—S_1P)$, is a whole number of wave-lengths, the displacements at P will be in the same direction and the waves will thus re-inforce each other to give maximum intensity. If the path difference is equal to an odd number of half wave-lengths, the displacements of disturbances on reaching P will be directed oppositely and will tend to destroy the effect of each other giving rise to minimum intensity of light. Thus, there will be a marked falling off in brightness, and as we move out from B towards P we shall come across alternate bright and dark bands, which are known as interference bands. Thus

$$\text{for maximum brightness, } (S_2P - S_1P) = 2n.\frac{\lambda}{2} \qquad \ldots (1)$$

$$\text{for minimum brightness, } (S_2P - S_1P) = (2n+1).\frac{\lambda}{2} \qquad \ldots (2)$$

For calculating this path difference, let us drop perpendiculars from S_1, S_2 on the screen and let T_1, T_2 be the feet of these prependiculars. Now, from the right-angled triangle PS_1T_1 we have

$$\begin{aligned} S_1P &= (S_1T_1^2 + PT_1^2)^{\frac{1}{2}} \\ &= [\, D^2 + (x - b/2)^2 \,]^{\frac{1}{2}} \\ &= D\left[\, 1 + \tfrac{1}{2}.\, \frac{(x-d/2)^2}{D^2} \,\right] \qquad \ldots (3) \end{aligned}$$

expanding by binomial theorem and assuming that x and d are of small magnitude compared to D.

$$\text{Similarly,} \qquad S^2P = D\left[\, 1 + \tfrac{1}{2} \cdot \frac{(x+d/2)^2}{D^2} \,\right] \qquad \ldots (4)$$

$$\text{Hence,} \qquad S_2P - S_1P = \frac{d}{D}\, x \qquad \ldots (5)$$

Thus, from (1), (2) and (5) we have

$$\text{for maximnm brightness, } \frac{d}{D}.x = 2n.\frac{\lambda}{2} = n\lambda \qquad \ldots (6)$$

$$\text{for minimum brightness, } \frac{d}{D}\, x = (2n+1)\frac{\lambda}{2} = (n+\tfrac{1}{2})\lambda \qquad \ldots (7)$$

Now the distance between two consecutive fringes, or the fringe width as it is termed, can be obtained by giving n any two consecutive integral values and getting the difference in x. If the fringe wipth be denoted by β, we have from equation :—(6)

$$d/D \cdot x_1 \; n\lambda$$

and $$d/D \cdot x_2 = (n+1)\lambda$$

subtracting, $$d/D \cdot (x_2 - x_1) = \lambda$$

or $$\lambda = \frac{d}{D} \cdot \beta \qquad \ldots \qquad (8)$$

which is the required result.*

Equation-(8) provides us with a method for the determination of wave-length of light if we can measure the fringe width, the distance between the two coherent sources, and the distance between the plane containing the sources and the plane of observation. This is accomplished by the bi-prism method.

[**Note**—From the formula, $\beta = \lambda D/d$, it is clear that in order to get a good fringe width, D should be large and d should be small. Now D can be made small by decreasing the slit-biprism distance. But this is not always proper, since the percentage error introduced in the measurement of d is thereby increased. Hence, the correct thing to do is to increase D. Although in increasing D the intensity of light falls, yet if adjustment (c), as described below, is properly made, the intensity shall be found to be quite sufficient even for the entire length of the bench.

Further, calculation shows that *for a given fringe width*, the total number of fringes is directly proportional to the distance between the slit and the bi-prism. Hence to get a large number of fringes this distance (between the slit and the bi-prism) has to be kept large].

Method—

(i) Before starting the actual measurements with the apparatus, make the following adjustments very carefully :—

(a) With the help of a spirit level and the levelling screws (provided at the base of the instrument) **level the bed of the optical bench.**

---

* From this formula it is clear that the fringe width is a constant quantity. Further, as the fringe width is dependent on the wave-length of light, the fringes for blur rays will be narrower than those for the red rays. Thus, if the sources S_1 S_2 bie of white light, the central fringe at B will still be white snce waves of all colours reach this place in the same place of vibration. If we move up we shall come across coloured fringes, the blue colour appearing first and the red appearing last. If we move still further will be greater confusion and intermingling of colourrs and all interferece effects will be obliterated. As a consequence only a few coloured fringes can be seen with white light.

(b) **Focus the micrometer eye-piece on the cross-wires** by looking through it on a white wall. Then mounting it on its upright turn it to make one of the cross-wires vertical. Adjust the heights of the slit the bi-prism and the eye-piece to be the same. Then with the help of the tangent screw provided for rotating the slit in its own plane, **make the slit vertical.** For exact setting, get an image of the slit on the vertical cross-wire by interposing a short focus lens in between the two. Then make the slit narrow and illuminate it with sodium light focussed on it with the help of a lens.

(c) The lamp has a cover which is provided with an opening through which a divergent pencil of light issues out. Now keep your bench in such a way **that it is situated in this patch of light and its length points straight towards the lamp.** If this condition is not procured, there can be great loss of light intensity available for interference pattern. To attain this adjustment, keep the slit wide open and look through it with the eye at some distant point lying just over the bench. Move the slit end of the bench till maximum intensity is attained. Now narrow down the slit and make it vertical also.

(d) Now look at the slit through the bi-prism and move the eye sideways. From some positions of the eye you will see two images of the slit. These will not, in general, be parallel. **To make these two images parallel rotate the bi-prism in its own plane.** Now these two images should be seen when your eye is situated just above the bench. If this is not the case, **then displace the bi-prism sideways.**

(e) Now examine whether **the two images are of equal intensity** or not. In case they are not, rotate slightly the whole bench, keeping the slit end of the bench stationary. As a matter of fact, a patch of light can be clearly seen on the surface of the bi-prism. Adjustment should be so made that the middle line of the bi-prism equally divides this patch of light.

(f) Now bring the eye-piece close behind the bi-prism when a vertical patch of light will be seen in the field of view of the eye-piece Fringes can be seen in this patch of light. If they are not seen, or they are not clear, first make the slit narrower and then rotate the prism in its own plane slowly and carefully till best fringes are seen.

(g) Keeping your eye behind the eye-piece move it (the eye-piece) away from the bi-prism. The fringes become broader and broader, but their intensity goes on falling continuously. Still the fringes can be seen quite distinctly from quite a long distance D. If necessary, try to improve the fringes by slightly rotating the bi-prism in its own plane. Stray light often spoils the definition of the fringes, specially when you are working at long D, hence try to cut it off as much as you can.

Sometimes it may be necessary to slightly widen the slit in order to improve upon the fringes. Try this process also.

(h) **Adjust the line joining the slit and the edge of the bi-prism parallel to the length of the bed of the optical bench.** While moving the eye-piece away from the bi-prism you may notice that the fringes often shift sideways. This is called *lateral shift,* which can be removed by displacing the bi-prism at right angles to the length of the bench.

(ii) Now start making measurements of the various quantities involved in the formula for the evaluation of the wave-length of light.

Be careful, after arranging the apparatus to give good interference pattern, to make the determination of d ($=S_1 S_2$, the separation of the two coherent sources) before measuring the separation of fringes.

(a) **Measurement of** d—This is determined by the well-known "*displacement method*" employed with a convex lens. For this purpose, mount a short focus* convex lens on a spare upright in between the bi-prism and the eye-piece. Move the lens on the optical bench till the images of the two virtual sources are seen distinctly in the field of view. With the help of the micrometer measure the separation d_1 of the two images. Again, displace the lens to another position when the images are again seen distinctly. Measure the new separation d_2 of the images. Then†

$$S_1S_2 = d = \sqrt{d_1 d_2}$$

(b) **Measurement of** β—For this purpose it is a good plan to begin the measurement at one edge of the field of view, and to draw a table as shown below (see Table-B).

---

* The focal length of the lens should necessarily be small, since in the displacement method the distance between the object and the screen (in this case, between the slit and the eye-piece) should be greater than four times the focal length of the lens.

† Using the usual nomenclature in connection with lenses we have (taking the length of the first image as I_1)—

$$\text{Magnification, } \frac{I_1}{O} = \frac{v}{u}$$

In the displacement method v and u are intercharged, hence in the second case (taking the length of the second image as I_2)

$$\text{Magnification, } \frac{I_2}{O} = \frac{u}{v}$$

Thus, multiplying these two equations, we have

$$\frac{I_1 I_2}{O^2} = 1 \qquad \text{or} \qquad O = \sqrt{I_1 I_2}$$

Set the cross-wire accurately down the *centre* of the first bright fringe, and *move it always in one direction* by means of the screw giving the transverse motion, stopping at every two or three fringes to note the position. From these readings, deduce the distance (β) between the consecutive fringes.

(c) **Measurement of D**—This is the distance between the slit and the plane of the cross-wires. For this purpose record the positions of the uprights carrying the slit and the eye-piece. This is the observed D, which may be different from its correct value due to bench error. Apply bench correction as explained above under the heading "Description of the Apparatus".

(iii) Now calculate the wave-length of the sodium light with the help of the formula given above.

Observations—

[A] *Readings for the determination of* d.

S. No.	Micrometer reading		d_1	Micrometer reading		d_2	Remarks
	I Image	II Image		I Image	II Image		
1	...cm	...cm	.cm	...cm	...cm	.cm	(i) Pitch of the screw = ...cm
⋮							(ii) No. of divs. on the circular head =......
⋮							
⋮							(iii) ∴ L. C. of the screw = ...cm

[B] *Readings for the determination of* β.

No. of fringe	Micrometer reading (a)	No. of fringe	Micrometer reading (b)	Separation for 10 fringes (b)−(a)
1	......cm	11	......cm	......cm
3	....	13	....	....
5	⋮	15	⋮	⋮
7	⋮	17	⋮	⋮
9	⋮	19	⋮	⋮
⋮	⋮	⋮	⋮	⋮

[C] *Readings for the determination of D.*

S. No.	Position of the slit			Position of the eye-piece			Remarks
	Main-scale rdg.	Vernier-rdg.	Total rdg.	Main-scale rdg.	Vernier rdg.	Total rdg.	
1							(i) Length of bench correction rod= ... cm
⋮							(ii) Corresponding distance read on the bench = ...cm
⋮							(iii) ∴ Bench error = ...cm
⋮							(iv) Observed value of D =...... cm
		Mean	...cm		Mean	...cm	

Calculations—

(i) Mean separation for 10 fringes =...cm

∴ Mean separation for 1 fringe, β =...cm

(ii) $d = \sqrt{d_1 d_2}$ =...cm

(iii) Correct value of D =...cm

(iv) Mean fringe width β =...cm

Now $$\lambda = \frac{d}{D} \times \beta$$

= ......cm.

Result—The value of the wave-length of sodium light =...cm*

[Standard value = 5893×10^{-8} cm.

Error = ...cm. =...%]

Precautions and Sources of Error—

(1) The bed of the optical bench should be *properly levelled* with the help of levelling screws and spirit level.

*The result can also be expressed in Angstrom Units A. U.). A. U. = 10^{-8} cm.

(2) The slit should be made *narrow* within permissible limits. It should be made *parallel* to the vertical cross-wire of the eye-piece.

(3) The bi-prism should be rotated in its own plane so that its edge becomes *exactly parallel* to the slit.

(4) *The line joining the slit and the edge of the bi-prism should be parallel to the length of the optical bench.* This should be tested by displacing the eye-piece along the bench and noting that there is no lateral shift of the fringes during this process.

(5) The formula for this experiment assumes that fringe width is measured in a plane perpendicular to the length D. Hence, the screw of the eye-piece *must be perpendicular to the length of the bench.*

(6) While taking measurements for the fringe width the vertical cross-wire should be set in the centre of a bright fringe only.

(7) For determining the distance between the two vertical image of the slit, the lens should be so chosen that the distance between the slit and the cross-wire is more than four times the focal length of the lens.

If the lens does not have a stop, the images may not be short. Hence *use a stop having a central aperture.* Further, the lens axis should be kept *parallel* to the length of the bench, otherwise due to astigmatism the images will be spoiled. Moreover, in this case the formula $d = \sqrt{d_1 d_2}$ will not hold good.

(8) If the eye-piece happens to be too close to the bi-prism, it may be impossible to obtain both the images with the lens. The eye-piece should be so placed that this is possible. *Therefore this part of the experiment should be performed first and the fringe width determined afterwards.*

(9) While using the micrometer screw, pack-lash error should be scrupulously avoided *by turning the screw in the same direction.*

(10) Bench correction should be invariably employed for the determination of D, *i. e.*, the distance between the slit and the cross-wire.

(11) The chief source of error lies in a faulty adjustment of any of the components of the apparatus. Specially the adjustment setting the line joining the slit and the bi-prism edge parallel to the length of the bench should be carefully carried out. If this adjustment is slightly defective the fringe width will not be β but it will be $\beta \cos \theta$ where θ is the angle of inclination between the above two lines. This observed fringe width will always be less than the true fringe width β unless θ is equal to zero.

ADDITIONAL EXPERIMENTS

No.—30 (a)

To determine the thickness of a given mica sheet with the help of Freshnel's bi-prism.

The principle of this determination lies in the fact that when a transparent thin medium (such as a mica sheet) is interposed in the path of one of the interfering lightbeams, the resulting path difference is different from the previous value, and consequently, the fringe pattern is displaced, the magnitude of this displacement depending upon the thickness of the mica sheet.

However, it is obvious that a source of monochromatic light is useless for this experiment, since the fringe pattern in the displaced position will be indistinguishable from its previous setting, hence no measurements can be affected. For this purpose, therefore, a source with white light is employed. This gives rise to a white central fringe with coloured bands on either side of it. Thus the central fringe is easily distinguishable from its neighbours and consequently its displacement easily measured.

The accompanying diagram is self-explanatory. The mica sheet of thickness t is interposed in the path of the beam S_1P. Let the central white fringe bo displaced to the point P. Obviously, there is not path difference in the two interfering beams on reaching P. Let V_0 be the velocity of light in vacuo and V its velocity in the mica sheet.

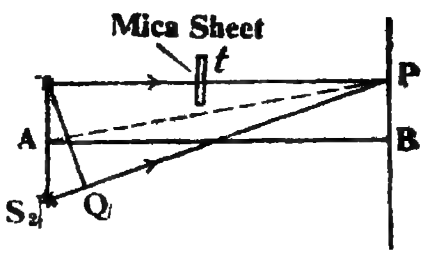

Fig.-69
To determine the thickness of a mica sheet.

Now with P as centre and PS_1 as radius draw the circular are S_1Q. If the distance S_1S_2 is very small compared to the distance S_1P. the line S_1Q will be sensibly straight and perpendicular to S_2P. Since the time-retardation at P is zero, we have

$$\frac{S_2P}{V_0} = \frac{S_1P - t}{V_0} + \frac{t}{V}$$

or
$$\frac{S_2Q + QP}{V_0} = \frac{S_1P}{V_0} + t\left(\frac{1}{V} - \frac{1}{V_0}\right)$$

or
$$\frac{S_2Q}{V_0} = t\left(\frac{1}{V} - \frac{1}{V_0}\right). \qquad \text{(since } QP = S_1P\text{)}$$

$$\therefore\ S_2Q = t\left(\frac{V_0}{V} - 1\right) = t(\mu - 1) \qquad (1)$$

since the refractive index μ of the medium is given by V_0/V.

Now from the similar triangles S_1S_2Q and ABP we have

$$\frac{S_2Q}{S_1S_2}=\frac{BP}{AB}$$

or $$S_2Q = \frac{BP}{AB} \cdot S_1S_2 = \frac{x}{D} \cdot d \quad \ldots \quad (2)$$

where x (= BP) is the displacement of central fringe, d = S_1S_2 is the distance between the two coherent sources and D = AB is the distance between the sources and the screen. Equating (1) and (2), we have

$$t\ (\mu - 1) = xd/D$$

Hence $$t = \frac{x \cdot d}{D\ (\mu - 1)} \quad \ldots \quad (3)$$

The thickness (t) of the mica sheet can be calculated with the help of equation (3).

To conduct the experiment, make all adjustments as in the previous experiment. Take a very thin mica* sheet and clamp it in a stand between the two-halves of a cork. Introduce carefully the sheet in one of the interfering beams and measure the displacement x with the help of the micrometer screw. Measure other quantities as in the previous experiment and obtain the value of refractive index† of mica from the Table of Physical Coustants and calculate the thickness of the given sheet with the help of formula (3) given above.

No.—30 (b)

Use readings of table—[B] *to draw a graph between fringe number and fringe position* (micrometer reading). The graft will be a straight line, showing that the fringes are equidistant.

Draw another graph also. Keeping positions of slit and bi-prism fixed, measure fringe width β for different values of D. *Draw a graph between* β *and* D. This will also be a straight line. If corrected values of D are used, the straight line will pass through the 0, 0 point of the graph. If uncorrected values of D are used in drawing the graph, then intersection with the D-axis at $\beta = 0$ gives bench error.

* Very thin mica layers can be easily separated from a thick piece with the sharp edge of a razor blade.

† Its value at 15°C for sodium line D⁻(λ=5893 A. U.) can be taken equal to 1·60.

EXPERIMENT—31

Object—To determine the wave-length of sodium light by Newton's rings.

Apparatus Required—Optical arrangement for Newton's rings, sodium lamp, short focus convex lens, travelling microscope, and a spherometer.

Description of the Apparatus—The accompanying diagram depicts the optical arrangement of the various components of the appratus. Light from the sodium lamp comes out through a wide a perture S and falls on a short focus convex lens F which renders it into a nearly parallel beam. The light is reflected downwards by a glass plate R which has nearly an optically plane surface and is inclined at 45° to the vertical. A plano-convex lens L, whose curved face has a large radius of curvature (of the order, say, one metre), is placed with its spherical surface on another plane plate P. Thus, the air-film for the formation of interference rings is confined between L and P. The plate P rests on a horizontal *black* surface. The microscope M is held vertically above R to view the rings.

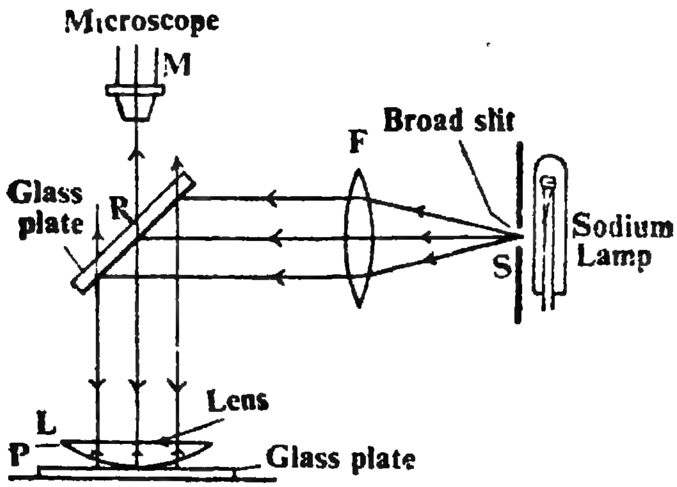

Fig.-70
Newton's rings arrangement.

Formula Employed—The wave-length (λ) of light is given by the formula :—

$$\lambda = \frac{D^2_{n+p} - D^2_n}{4pR}$$

where D_{n+p} = the diameter of the (n + p)th ring

D_n = the diameter of the nth ring

p = an integral number

R = radius of curvature of the curved face of the lens in contact with the plane surface.

PRINCIPLE AND THEORY OF THE EXPERIMENT

Newton's rings are formed as a result of interference of light reflected from the upper and the lower surface of the air-film formed between the plane glass plate and the curved face of the lens in contact with it (Fig.–71). Near the point of contact the thickness of the air-film will be very small in comparison with the wave-length of light. Consequently, when the point of contact is viewed with reflected light, it will be surrounded by a circular black spot. The thickness of the air-film increase as we proceed from the point of contact towards the periphery of the lens. Since the surface of the lens is the portion of a sphere, the thickness of the air-film will be uniform for all points lying on a circle concentric with the point of contact. Thus, when monochromatic light is employed as an illuminant, the central black spot, seen by reflected light, will be surrounded by concentric bright circles separated by dark intervals.

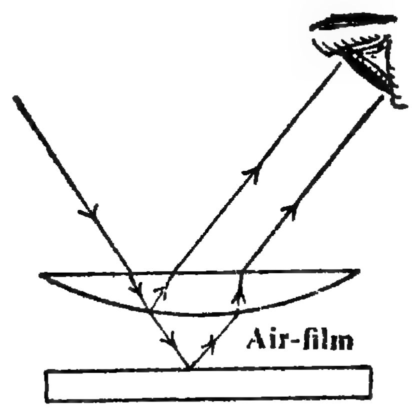

Fig.–71 Interference in air-film.

The figure given below depicts the lens L with its curved face resting on the plane glass plate P. The point of contact is O. When viewed normally by reflected light, the points A and B, equidistant from O will lie on a bright or a dark ring, according as *twice the distance AC (or BD) is equal to an odd or even number of half wave-lengths of the incident light.*

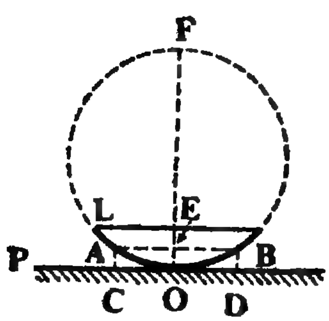

Fig.–72 Formation of Newton's rings.

From O draw diameter OF to the circle of which the curved section of the lens is an arc. Join AB, cutting OF in E. Let the diameter AB of the n^{th} ring be D_n. Then if R be the radius of curvature of the curved face of the lens—

$$(2R - t).\,t = \left(\frac{D_n}{2}\right)$$

where t is the thickness of the air-film at A or B. Since t is very small compared to R we have (neglecting t^2)—

$$2Rt = \frac{D_n^2}{4}$$

$$\therefore t = \frac{D_n^2}{8R}$$

For A and B to be situated on a bright ring

$$2t = (2n+1)\frac{\lambda}{2} \text{ or } \frac{D_n^2}{4R} = (2n+1)\frac{\lambda}{2} \quad \dots (1)^*$$

where n has the values 0, 1, 2, 3, ... for the first, second, third, fourth,...... rings respectively. Similarly, for the n^{th} dark ring we have

$$\frac{D_n^2}{4R} = 2n \times \frac{\lambda}{2} \quad (2)^*$$

Equations (1) and (2) can be employed for the determanation of λ if D_n, R, and n can be known experimentally. But there is one serious difficulty in this procedure. According to theory, as outlined above, the rings are to be counted with the central dark spot as the first order dark ring, but it is likely that due to the uncleanliness of the surfaces in contact of their deformation, the thickness at the point of contact may not be zero. Thus, the central spot may not belong to the first order and the numbering of other rings will consequently be erroneous. This uncertainty in the order of a ring is eliminated by adopting the following procedure—

The diameter of the $(n+p)^{th}$ bright ring is given by

$$\frac{D^2_{n+p}}{4R} = (2n+2p+1)\frac{\lambda}{2} \quad \dots \quad (3)$$

Subtracting (1) from (3), we have

$$\frac{D^2_{n+p} - D_n^2}{4R} = 2p \times \frac{\lambda}{2}$$

$$\therefore \lambda = \frac{D^2_{n+p} - D_n^2}{4pR} \quad \dots \quad (4)$$

Equation (4) involves the difference of the squares of the diameters of two rings, hence it is free from the objection referred to above. Consequently, this equation is invariably employed for determination of wave-length by this method.

---

* It is quite obvious from formulae (1) and (2) that the diameters of the bright rings will be proportional to the sqnare roots of the odd natural numbers ; while the diameters of the dark rings are proportional to the square roots of the natural numbers.

[**Note**—In connection with the above, the following points may be observed with interest :—

(1) If we suppose that the film separates media of different refractive indices, the refractive index of the film being less than that of the medium above it, but greater than that of the medium below it. Then the rays derived after reflection will be formed by suffering reflection at a denser medium, and consequently, a phase change of π will be introduced in each case. Thus, if the thickness of the film at the point of contact is negligible, as before, it will appear bright by reflected light (or dark by transmitted light).

An example in support of the above is when all of sassafras is enclosed between two surfaces of crown and flint glass respectively. The refractive index of this oil lies intermediate between the refractive indices of the above glasses.*

(2) The effects observed in reflected and transmitted lights are complimentary in nature. Hence, if the film is enclosed between the lens surface and a polished reflector, the reflected and transmitted system of rings will get supposed and all traces of interference pattorn shall disappear.

(3) The diameter of a ring is directly proportional to the wave-length of light employed. Hence, if white light is used as an illuminant, coloured rings will be produced, violet appearing at the inner and rod appearing at the outer edge respectively. However, with white light only a few coloured rings will be observed, since each colour will give rise to its own set of rings and after a few rings the various colours will get so much intermingled that an almost uniform illumination will result.]

Method—

(i) Before starting the actual experiment, clean thoroughly the surfaces of the lens L and the glass plates P and R. Set up the various components of the apparatus as shown in the figure given above (Fig.–70). Place the lens F in such a manner that the emergent beam is nearly parallel. For an extensive source, this lens may not be used.

(ii) Now keeping the eye vertically above the lens see whether the lens centre is well illuminated or not. In case it is not, you have to adjust the inclination of the plate R, or you may need to adjust the position of the lens or the source. When the centre as well illuminated, you can see a spot with the rings around it. These are the Newton's rings which have to be focussed in the microscope.

* Refractive index of (i) Crown glass = 1·500, (ii) Oil of sassafras = 1·525, and (iii) Flint glass = 1·560.

(iii) In order to focus the microscope, first focus the eye-piece on the cross-wire, so that it is distinctly seen in the field of view. Now place a piece of printed paper on the surface of the lens and focus the whole microscope on it. Then remove the paper and slide the microscope slowly downwards (by an amount equal to the estimated thickness of the lens) till the rings appear in the field of view. But it is possible that even now the fringes do not appear in the field of view. In that case you have to move the microscope or the whole combination containing the lens and the plates side ways or backward and forward till the fringes appear in the field of view. Now focus the microscope more accurately, *i. e.*, the fringes should be seen on the cross-wire without parallax.

[**Note.** Even after following the procedure as laid down above, the fringes do not appear, then it is very likely that the curved surface of the plano-convex lens has been, by mistake, placed up. Examine it. In the case of an equi-convex lens, however, this mistake shall not be there.

If the fringes are not quite smoothly circular, then either the lower lens surface or the upper surface of the plate (placed beneath the lens) or both are irregular. If the defect is with the plate, improvement in fringes can be had by using other portions of the plate. Of course, this has to be done by Trial. It is for this reason that this *plate must be truly plane*.]

(iv) If there is perfect contact between the lens and the plate at the centre, then the centre, according to theory, should be dark. But sometimes the centre appears to be white or hazy. This means that the contact is not perfect, either due to defect in surfaces, or more probably due to the presence of the dust particles. In that case clean the surfaces again. However, it must be emphasised that the formula involving the difference of squares of diameters of the rings will still give the value of the wave-length correctly.

(v) If good fringes have been secured in the field of view, rotate the eye-space (without changing its first adjustment) such that *one of the cross-wires has its length parallel to the direction of travel of the microscope and that it passes through the centre of the ring system.*

(vi) Now move away from the centre to one side by, say, twenty rings. From here move back again (turning the screw always in one direction to avoid back-lash error), and *set the other cross-wire tangential (Fig.–73) to successive rings.* Continue this process till you pass not only through the centre but focus on an equal number of rings on the other side. From these observations you will get the values of the diameters of the rings.

[**Note.** Enter these readings in a tabular form as given below.]

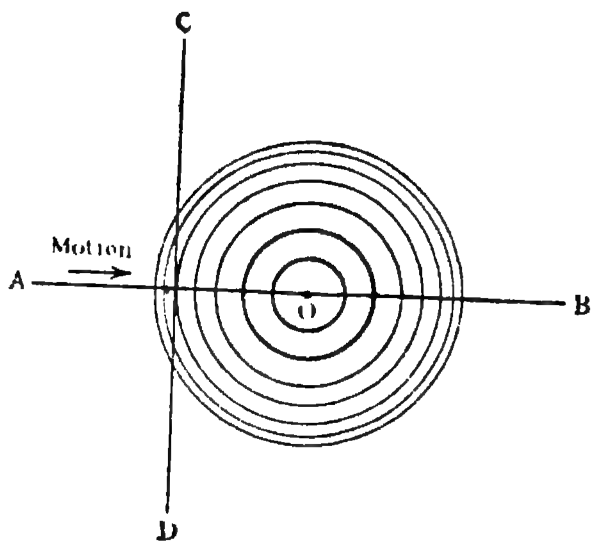

Fig.–73
Setting of the cross-wire on a ring.

(vii) Next measure the radius of curvature* of the spherical face of the lens in contact with the plate with the help of a spherometer.

(viii) With the help of the observations entered in the table, calculate the value of the diameter of each ring. Then calculate the value of the squares of the diameters of these rings and evaluate the expression $(D^2_{n+p} - D^2_n)$† choosing such a value of p as to make use of all the readings. Then obtain the mean value of this expression. Now knowing all the quantities calculate the wave-length of light.

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* The radius of curvature of the curved face is sufficiently large, hence h of the spherometer formula is extremely small. This measurement, therefore, should be carried out with extreme care. A slight error in this determination shall adversely effect the result.

† An alternative procedure for getting the value of the expression $(D^2_{n+p} - D^2_n)$ is the following :—

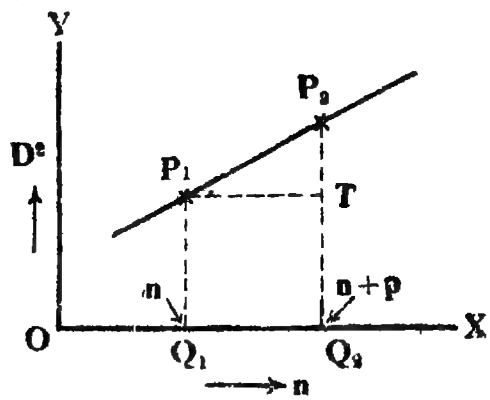

Fig.–74
Graph for Newton's rings expt.

Draw a graph with the square of the diameter as ordinate and the order of the ring as abscissa (Fig.–74). The graph shall be a straight line. Take two points P_1 and P_2 lying on this line. Drop $P_1 Q_1$ and $P_2 Q_2$ perpendiculars on the x-axis. Then

$$P_2 Q_2 = D^2_{n+p}$$
$$P_1 Q_1 = D^2_n$$
$$\therefore \quad P_2 T = D^2_{n+p} - D^2_n$$

which can easily be read off from the graph.

Observations—

[A] *Readings for the determination of* $(D^2_{n+p} - D^2_n)$.

No. of the ring	Micrometer reading (left side)	Micrometer reading (right side)	Diameter (D)	D^2	$D^2_{n+p} - D_n^2$	p
20	......cm	......cm	......cm	......cm^2	$D_{20}^2 - D_{11}^2$	
19					$D_{19}^2 - D_{10}^2$	
18					...	
......					...	
					...	9
11					...	
10					...	
9					...	
...					...	
3					$D_{12} - D_3^2$	
				Mean	......cm^2	

[B] *Reading for the determination of R.*

S. No.	Reading on the plane surface	Reading on the spherical surface	h	Remarks
1	.........cm	.........cm	...cm	(i) Pitch of the screw = ... cm
......				(ii) No. of divs. on the disc = ...
......				(iii) L.C. of the spherometer = ... cm
......				(iv) AB = ...cm BC = ...cm AC = ...cm
				Mean 'a' = ...cm
		Mean	...cm	

Calculations—

$$R = \frac{a^2}{6h} + \frac{h}{2}$$

cm

Now $$\lambda = \frac{D^2_{n+p} - D^2_n}{4pR}$$

$$= \ldots\ldots \text{ cm.}$$

[**Note**—The value of λ can also be calculated with the help of the graph (see Fig.–73). Now from the graph

$P_2T = \ldots\ldots \text{ cm}^2$

and $p = OQ_2 - OQ_1 = \ldots\ldots$

Also $R = \ldots\ldots$ cm

$$\therefore \lambda = \frac{P_2T}{4(OQ_2 - OQ_1)R}$$

$$= \ldots\ldots \text{ cm}]$$

Result—The wave-length of sodium light as determined by Newton's rings = ...cm.*

[Standard value = 5893×10^{-8} cm.
Error = ... cm. ; or = ...%]

Precautions and Sources of Error—

(1) The surfaces to be in contact with each other should be carefully cleaned with rectified spirit. Unclean surface shall give rise to elliptically shaped rings ; they may also give rise to a central bright or hazy spot. However, if the rings are circular but the central spot is not completely dark, measurements can still be taken, since the use of the expression $(D^2_{n+p} - D^2_n)$ eliminates this error.

(2) The plano-convex lens chosen for the production of rings should have a large radius of curvature for its curved face, so that the rings have a large diameter and consequently accuracy in the measurement of the ring-diameters is increased. The plate used below must be *truly plane*.

(3) The fringes must be focussed on the cross-wire by the no-parallax method, i. e., sideways motion of the eye should show no relative displacement between the cross-wire and the ring system.

* The result may also be quoted in Angstrom Units (A. U.). 1 A. U. = 10^{-8} cm.

(4) Before taking readings for the diameters, it should be ascertained beforehand that the motion of the microscope covers the entire value of the diameter.

(5) The cross-wire should be adjusted centrally on the ring and only bright rings should be employed for the measurement of diameters, as they provide a marked contrast for the setting of the cross-wires.

(6) While using the slow motion screw of the microscope, or the micrometer screw of the spherometer, care should be taken to avoid back-lash error. For this purpose *the screw should always be turned in the same direction.*

(7) Since the spherometer method* is not an accurate one for the measurement of such large radius of curvature, the result shall be adversely affected on this account.

(8) But the chief source of error lies in the fact that the rings are seen, not directly, but after refraction through the lens. Thus, the observed diameters are not correct. This difficulty can be overcome by placing the plate over the lens, but this arrangement will not be a steady one, hence it will not be convenient. The error in this case, however, is not very great because a thin lens has been employed for the purpose. Under such condition the object (in this case, the ring pattern) is at the surface of the lens and consequently at its principal plane. The image is in the second principal plane, and of the same size as the object. For a thin lens, these planes and lens surfaces are nearly coincident. In practice, we have an image of magnification slightly differing from unity, or the diameters in the formula above are to be multiplied by such a fraction. In this experiment, this facter has been omitted.

ADDITIONAL EXPERIMENT

No.--31 (a)

To determine the refractive index of a given transparent liquid by Newton's rings.

For this experiment, determine first the value of $(D^2_{n+p}—D^2_n)$ in air as described above in the main experiment Now introduce a small quantity of the given liquid in between the lens and the plate when *the rings will be found to shrink in diameter.* Measure the

* As a matter of fact, Newton's rings method is employed experimentally for the measurement of such large radii of curvature, when the wave-length of light is given.

Based on this principle the Andhra Scientific Company, Masulipatam (South India) manufacture an apparatus known as a *Guild Spherometer*, with the help of which the radius of curvature of spherical surfaces can be determined very accurately.

diameters of the rings again, and evaluate the quantity $(D^2_{n+p}-D^2_n)$ in the liquid. Now, as proved above, with the air-film enclosed between the two surfaces we have

$$\left[D^2_{n+p}-D_n^2\right]_{air} = 4\,p\,\lambda\,R \qquad \dots \qquad (1)$$

Similarly, when the liquid is interposed between the two surfaces, we have

$$\left[D^2_{n+p}-D^2_n\right]_{liquid} = \frac{4p\,\lambda\,R}{\mu} \qquad \dots \qquad (2)$$

where μ is the refractive index of the liquid. Thus

$$\eta = \frac{\left[D^2_{n+p}-D^2_n\right]_{air}}{\left[D^2_{n+p}-D^2_n\right]_{liquid}}$$

[**Note**—Since μ is a quantity greater than unity, the numerator of the above expression should be greater than the denominator, i. e. *the diameter of a ring formed in air should be greater than that in the liquid.* It is due to this fact that the rings shrink in diameter as soon as the liquid is introduced between the two surfaces. With the liquid the ring diameters contract in the ratio $\sqrt{\mu}$.]

10

Diffraction of Light

Everybody knows that the path of light entering a dark room through a hole in the wall of the room illuminated with sun-light is straight. Light travels in straight lines. The same conclusion is drawn from the formation of shadows of opaque obstacles. The rectilinear propagation of light, which can be so easily explained by Newton's corpuscular theory, remained a great stumbling-block in the way of the wave theory for a number of years. If light is a wave motion we should expect bending of light round corners of obstacles as in the case of water waves or sound waves. The failure to see any encroachment of light in the geometrical shadow of an opaque obstacle was really due to enormous size of the obstacles employed in the early experiments conducted for the purpose. The wave-length of light is of the order 5×10^{-5} cm. and obstacles used in ordinary experiments on rectilinear propagation of light were some 1,000,000 times bigger than the wave-length of the waves. It would obviously, be most awkward to do similar experiments with water waves to see their bending effects.

Careful observations on the propagation of light at the edges of opaque obstacles have, however, now revealed the fact that there is an encroachment of light on their geometrical shadows. Not only that the illumination outside the geometrical shadow is not uniform but shows alternate variations in intensity. These alternate variations in intensity and the encroachment of light on the geometrical shadow of an opaque obstacle constitute a class of phenomena known as *Diffraction* of light*. These phenomena are now so common and so closely support the theoretical expectations that they form the strong points of the wave theory of light rather than the stumbling-blocks in the way of its adoption. The correct explanation of this class of phenomena was given by Fresnel who developed the theory to a high degree of perfection. According to

* For detailed study, read author's book "A Critical Study of Practical Physics and Viva-Voce".

him the diffraction phenomena are attributable to the interference of secondary wave-lengths which originate from a single wave-front. The interference effects, on the other hand are due to the superposition of the two wave-fronts coming from two coherent sources.

EXPERIMENT—32

Object—To determine the wave-lengths of different spectural lines emitted by mercury light with the help of a plane transmission grating.

Apparatus Required—A diffraction grating, a spectrometer, a prism, mercury lamp, and a reading lens.

Description of the Appratus—A diffraction grating is made by ruling a large number of equidistant parallel straight lines* on glass. The lines are ruled by a diamond point moved by an automatic dividing engine containing a very fine micrometer screw. A photographic replica of a plate made in this way is often used in its place for laboratory practice. Gratings are either of the transmission type or of the reflection type. The transmission gratings are essentially plane, whereas reflection ones may be either plane or concave, the lines in this case are ruled on speculum† metal. The advantage of concave gratings over the plane ones lies in the fact that in their case sharp spectural images are obtained directly by reflection, and the use of lenses for focussing the spectrum is dispensed with.

Formula Employed—The wave-length λ of any spectral line is calculated from the formula—

$$(a + b) \sin \theta = n \lambda$$

where $(a + b)$ = the grating element‡.
θ = the angle of diffraction.
n = the order of the spectrum.

PRINCIPLE AND THEORY OF THE EXPERIMENT

The above formula is obtained by considering rays in pairs passing through adjacent clear spaces of grating (see Fig.–75). For instance, let us consider the two clear spaces AB and CD. We may consider the rays passing through them in pairs, taking together those rays which are symmetrically situated. Two such rays shown in the figure are QLQ′ and TMT′. These rays reach the grating in the same phase, and the wave-front incident on the grating is represented by the line ABCD......

* The number of lines ruled on the grating usually varies from 12000 to 30000 per inch.

† This is an alloy of copper and tin.

‡ If the number of lines ruled on the grating be N per inch, the grating element = 2·54/N cm.

Let us now consider a wave-front ww at a later interval and suppose that it makes an angle θ with the plane of the grating. Let us draw the rays AP′, BR′, etc. perpendicular to this wave-front and making an angle θ with the normal to the grating. Suppose, these rays are all received by a convex lens which unites them in its focal plane. Their paths from the grating to the lens are obviously different, and hence they reach the focal plane of the lens with different phases This phase difference is due to the difference of path traversed by the rays after leaving the grating.

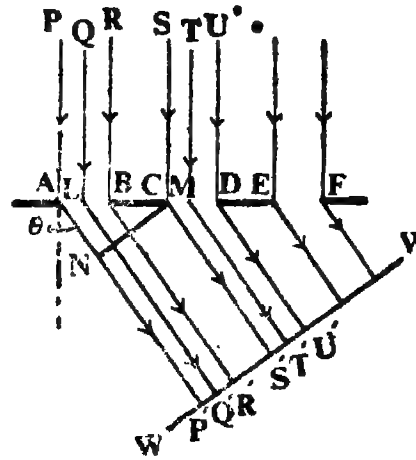

Fig.–75
Theory of a plane transmission grating.

Let us consider two corresponding rays AP′ and CS′. From C draw CN perpendicular to AP′. Then AN obviously represents the path difference between these two rays. Any other pair of rays situated symmetrically in any adjacent clear spaces shall have the same path difference. Now,

$$AN = AC \sin \theta = (a + b) \sin \theta \qquad \ldots \qquad (1)$$

where a is the width of a clear space and b is the width of a ruled line, i. e. $(a + b)$ is the grating element.

The rays will thus reinforce each other if AN is equal to a whole number of wave-lengths, or a bright line will occur in direction θ provided that

$$(a + b) \sin \theta = n\lambda \qquad \ldots \qquad \ldots \qquad (2)$$

where n may have any integral value including zero. This value of n also determines the order of the spectrum observed with the help of the grating.

It will thus be seen from the above equation that there is a definite relation between the wave-length λ of the diffracted light, the order n of the spectrum, the width $(a + b)$ of the grating element, and the angle of diffraction θ. Thus, the formation of diffraction spectra by a grating gives us the means of determining the wave-length of light.

Method—

(i) Before starting the actual measurement, make the following adjustments of the spectrometer—

(a) The axes of the telescope and the collimator must intersect at and be perpendicular to the main axis of the instrument.

The mechanical adjustment embodied in the above condition is generally made by the manufactures of the instrument and hence this adjustment need not be undertaken.

(b) The telescope must be set for receiving parallel rays and it must form a well defined image of an object on the cross-wires of the eye-piece.

First of all, focus the eye-piece on the cross-wires. For this purpose direct the eye-piece towards a white wall a few feet away and alter the distance of the eye-piece from the cross-wires by drawing it in and out till the cross-wires are distinctly seen against the white background. Do not disturb this distance between the two during subsequent adjustments.

For focussing the telescope for parallel light employ *Schuster's method.* (Note—For details see expt.—20.) Illuminate the slit of the collimator with monochromatic light (say, light from a sodium lamp) and without any focussing, place a prism approximately in the minimum deviation position. Turn the prism slightly away from this position, bringing the refracting edge towards the *telescope*, Focus the *telescope* on the image as distinctly as possible, slightly rotating the telescope which might be necessary to keep the image in the field of view. Now rotate the prism slightly towards the other side of the minimum deviation position and focus the collimator until on looking into the telescope the image is again seen as distinctly as possible. Repeat this procedure of alternately focussing the telescope and the collimator until the rotations of the prism do not cause the image to go out of focus. When this condition is achieved the rays entering and leaving the prism are parallel, *i. e.*, the telescope and the collimator are both set for parallel rays. Usually, only a few alternate focussing are necessary.

[**Note**—(i) If the prism is first turned so that its refracting edge moves towards the collimator, then the first focussing must be done by means of the collimator. If any mistake is committed in this process, the image will at once become more and more out of focus, and this condition, therefore, will at once point to the mistake.

(ii) The other usual method of focussing the telescope is to take the instrument to an open window and to focus it on a distant object (such as a distant telegraph post) taking care in this process that there is no parallax between the image and the cross-wires. The instrument is now replaced in its position in the dark room and the telescope is turned towards the collimator, adjusting the latter with the help of rack and opinion arrangement, till the image of the slit is distinctly seen on the cross wire without parallax. The apparatus is now adjusted so that the parallel rays come from the collimator and are received by the telescope, also adjusted for parallel rays.

It should, however, be borne in mind ***that Schuster's method is always to be preferred for this adjustment.***]

(c) The collimator must be adjusted for emitting parallel beam.

[**Note**—First of all *align the collimator with the source* (see exp-30), then set the collimator for parallel rays. This will be achieved when the slit is situated in the focal plane of the collimating lens. The procedure for effecting this adjustment has already been described above.]

(d) The prism table should be adjusted horizontal.

For this purpose, keep the prism in the centre of the prism table (Fig.–76) so that the refracting edge. A coincides with the centre of the table and faces the collimator and two of the reflecting surfaces AB and AC receive light simultaneously. By keeping the prism table fixed, rotate the telescope to receive the light reflected from the face AB. An image of the slit will be seen but it may not necessarily lie centrally in the field of view. Use the levelling screws Q and R in such a way that the image is bisected at the point of intersection of the cross-wires. Next turn the telescope to receive the rays from the other face (AC) and *by operating the third screw P alone* brings the image centrally on the cross-wire as before. Thus, the horizontally of the prism table is established. Remove the prism now.

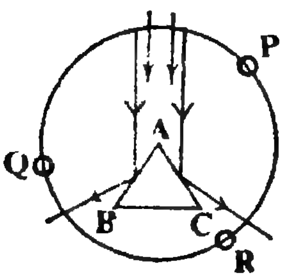

Fig.–76
Adjustment of the prism table.

(ii) The plane of the grating should be adjusted normal to the incident light and should face the telescope.

For this purpose, set the telescope and the collimator in a line so that the direct image of the slit falls on the vertical cross-wire

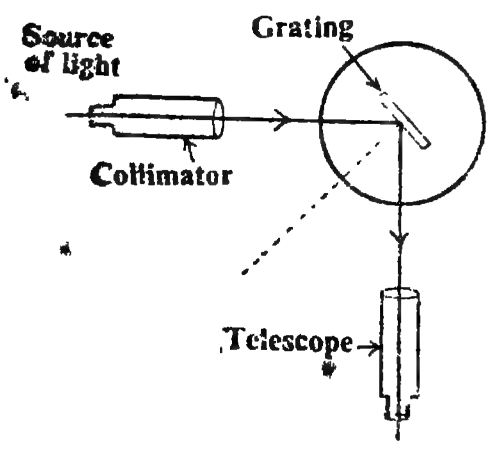

Fig.–77 (a)
Adjustment of the grating.

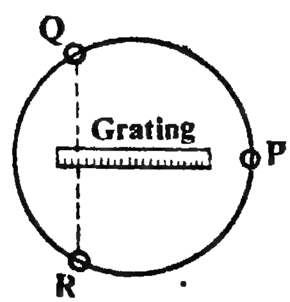

Fig.–77 (b)
Adjustment of the grating.

and is bisected by its point of intersection. Note the readings on both the verniers, and let a be the reading on one of them. Next mount the grating with its clamping device so that its ruled surface lies at the centre of the prism table and also perpendicular (Fig.–77 (a)

to the line joining two of the levelling screws (say, Q and R). Now rotate the telescope till the reading of the verning becomes $(90°+a)$ and clamp it. It is obvious that the axis of the collimator and the telescope are now at right angles to each other. Now turn the grating with the help of the turn-table so that the reflected image falls centrally on the intersection of the cross-wires (Fig.-77 (b.) Adjust the levelling screws Q and R to get the image* centrally in the field of view.

It can be easily understood that in the above position. the grating is inclined at 45° to the incident beam. Take the reading of the verniers. Turn the table from this position through 45° or 135°, as the case may be, so that *the ruled surface of the grating is normal to the incident beam and faces the telescope.*

(iii) **The rulings of the grating should be so adjusted that they are parallel to the main axis of the instrument.**

When the direct image of the slit has been received on the vertical cross-wire, turn the telescope about the axis of the instrument till the diffracted image of the first order is visible on the cross-wires. If necessary, adjust the screw P to get the centre of this diffracted image on the point of intersection of the cross-wires. Now the rulings of the grating are parallel to the main axis of the instrument.

(iv) **The slit should be adjusted parallel to the rulings of the grating.**

For this setting, adjust the telescope on a diffracted image and slowly rotate the slit in its own plane till the image becomes well defined and is brightly illuminated. However, in this process, *do not disturb even slightly the collimating arrangement done previously.*

[Note—Now the adjustments of the instrument are complete for starting the measurements of the angles of diffraction. In these settings, the slit may be kept slightly wider, but thereafter *it should be made narrow as far as permissible.*]

(v) Now illuminate the slit with mercury light. Turn the telescope to the right to bring the first spectral line of the first order spectrum on the cross-wire (Fig.-78), final setting being done by the tangent screw of the telescope. Note down the readings of the two verniers. Similarly, note down the readings for the other spectral lines, mentioning the colour of the line in one column of the table.

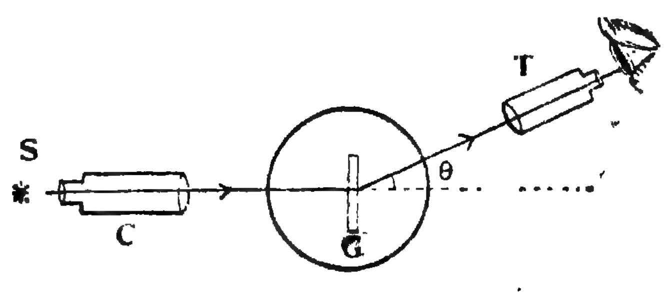

Fig.-78. Telescope adjusted on a diffracted image.

* In this setting the reflected image should be the one obtained by reflection from the *ruled surface*. Care must be taken to select this, for two or more images are sometimes seen.

Next turn the telescope to the left of the direct image and setting as before note down the readings of the two verniers. The difference in the readings of the *same vernier* on the two sides gives twice the angle of diffraction *i. e.*, 2θ. Thus, knowing the angle of diffraction (θ) and the grating element calculate the wave-length of each spectral line.

Repeat the experiment with the second order of the spectrum. Finally, calculate the mean wave-length by taking the average of those obtained from observations on the first and second order spectra.

Observations—

[A] *Readings for setting the grating normal to the collimator axis.*

(i) Reading of the telescope set for direct image, $\theta_1 = \ldots\ldots$

(ii) Telescope turned through $90°$, $\theta_2 = \theta_1 + 90° = \ldots\ldots$

(iii) Reflected image obtained on the cross-wire.
Reading of circular scale now $\theta_3 = \ldots\ldots$

(iv) Prism table turned through $135°$ (or $45°$), $\theta_4 = \theta_3 + 135° = \ldots\ldots$

[B] *Readings for the determination of the angles diffraction.*

Order of spectrum	Colour of spectral line	Position of telescope on the right		Position of telescope on the left		2θ	Mean θ	Remarks
		Vernier	Reading	Vernier	Reading			
I		V_1	...	V_1	...	...	...	(1) Value of one main scale division = ... (2) No. of divisions on the vernier = ... (3) Least count = ...
		V_2	...	V_2	...			
		...		...				
		...		...				
		...		...				
II								Number of lines in the grating = ... per inch

Calculations-

Grating element, $(a + b) \quad \dfrac{2 \cdot 54}{N}$ cm

I Order Spectrum } $\lambda \text{ (violet)} = \dfrac{(a + b) \sin \theta}{n}$

$= \ldots \ldots$ A. U.

(**Note**—Calculate the value of λ for other spectral lines.)

II Order Spectrum } (**Note**—Calculate as above.)

$\therefore$ Mean value of λ (violet) = ...A. U.

etc. etc. etc.

Result—The wave-lengths of spectral lines given out by mercury light are as follows :—

Colour of the spectral line	Obtained from experiment	Standard value*	Error
Violet I	.........A.U.	4047 A. U.	.........A. U.
Violet II		4078 ,,	
Blue		4358 ,,	
Green-Blue		4916 ,,	
Green		5561 ,,	
Yellow I		5770 ,,	
Yellow II		5791 ,,	
Red		6234 ,,	

[**Note**—Plot a graph between λ (taken along the X-axis) and $\sin\theta$ (taken along the Y-axis). For each order of the spectrum you will get a straight line. The slopes of these lines will be found to be in the ratio of their orders.]

Precautions and Sources of Error—

(1) The axes of the telescope and the collimator must intersect at and be perpendicular to the main axis of the spectrometer.

* The above standard values have been taken from the emission are spectrum of mercury. Report the result for wave-lengths of those spectral lines which are observed in the source employed in your experiment.

(2) The telescope must be set for parallel rays and it must form a well-defined image of an object on the cross-wires of the eye-piece.

(3) The collimator must be adjusted for giving out parallel rays.

(4) The prism table should be adjusted horizontal.

(5) The rulings of the grating should be so adjusted that they are parallel to the main axis of the instrument.

(6) The slit should be as narrow as permissible and it should be adjusted parallel to the rulings of the grating.

(7) The faces of the glass of the grating should under no circumstances be touched. The grating should always be handled by the edges only.

(8) The grating should be so adjusted that its ruled surface is normal to the incident light, and it should face the telescope. For this purpose, the table should be rotated through 45^0 or 135° so that *the grating face is towards the telescope*. It should not face the collimator.

[**Note**—Adjustment of the grating surface normal to the collimator axis (*i. e.*, normal to the incident light) should be done with extreme care, *i e.*, the readings for this adjustment should be taken correct to $1'$, the least count of the instrument].

(9) While taking the reading of the telescope, the turn-table should remain clamped and vice-versa.

(10) The eliminate the error due to non-coincidence of the centre of the graduated scale with the main axis of rotation of the spectrometer, both the verniers should be read.

(11) There are certain errors in the grating. An ideal grating is one having rulings which are straight and parallel, equally spaced and have identical form. Corresponding to these requirements, the grating may have the following errors :—

(a) Error due to curvature and non-parallelism of the rulings.

(b) The "amplitude error", *i. e.*, the error due to variation of the ruling.

(c) The "phase error", *i. e.*, the error of spacing.

(12) The above formula, $(a+b)\sin\theta = n\lambda$, is deduced on the assumption that the incident light is normal to the grating face. Hence, if this condition, is not fully achieved in the adjustment, an error shall creep in. Further, a small variation δi in the angle of

incidence causes a correspondingly large error in the angle of diffraction.*

ADDITIONAL EXPERIMENTS

No.—33.

To determine the wave-length of spectral lines emitted by mercury light with a plane transmission grating by observing two deviations.†

Theory—Let the light issuing out of the collimator (Fig.–79) be incident on the grating at an angle i. Let the ray PA of wave-length λ be deviated in the direction AP_1 to the right, and AP_2 to the left, thus making the corresponding angles of deviation δ_1 and δ_2 as shown in the figure. The respective angles of diffraction in the two directions are θ_1 ond θ_2. Thus, $\theta_1 = i + \delta_1$. Now for the ray AP_1 we have

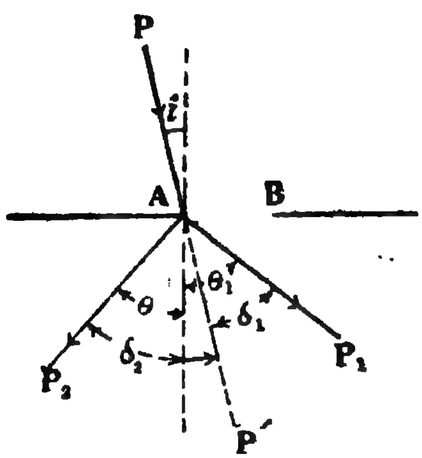

Fig.–79
Two deviations with a grating.

$$(a + b)\ (\sin\theta_1 - \sin i) = n\lambda$$

or $$(a + b)\ [\sin(i + \delta_1) - \sin i] = n\lambda$$

or $$\frac{n\lambda}{a + b} = 2\cos\left(i + \frac{\delta_1}{2}\right) . \sin\frac{\delta_1}{2}$$

$$2\cos\left[\left(i + \frac{\delta_2 - \delta_2}{2}\right) - \frac{\delta_1}{2}\right] . \sin\frac{\phi_1}{2}$$

* While taking readings for the angles of diffraction to the right and to the left, the student may find that the two values are not exactly equal. This discrepancy is due to the fact that the grating face is not exactly normal to the incident rays.

The error due to this slight non-normality of the grating can be fairly well-eliminated by noting this small difference between the two values of the angle of diffraction and turning the table through half the difference when the new readings will be found to be approximately equal.

† From the discussion of the above method (Expt.—32) it is clear that unless the difficulty of setting the grating surface normal to the incident beam is completely overcome, the result will not attain that accuracy which the grating method is capable of yielding. This difficulty can, however, be overcome by observing two deviations for the same ray in the same order of spectrum on the two sides of the zero order.

$$= 2\cos\left(\phi - \frac{\delta_2}{2}\right) . \sin\frac{\delta_1}{2}$$

$$= 2\sin\frac{\delta_1}{2}\cos\frac{\delta_2}{2}\cos\phi - 2\sin\frac{\delta_1}{2}\sin\frac{\delta_2}{2}\sin\phi \qquad \ldots (3)$$

where the angle $\left(i + \frac{\delta_1 - \delta_2}{2}\right)$ has been replaced by ϕ

Similarly, it can be proved that for the ray AP_2

$$\frac{n\lambda}{a+b} = 2\cos\frac{\delta_1}{2}\sin\frac{\delta_2}{2}\cos\phi + 2\sin\frac{\delta_1}{2}\sin\frac{\delta_2}{2}\sin\phi \ldots (4)$$

From equations (1) and (2), it is easy to get

$$(a+b)\cos\phi \cdot \sin\frac{\delta_1+\delta_2}{2} = n\lambda \qquad \ldots (5)$$

and
$$\tan\phi = \frac{\sin\frac{\delta_1-\delta_2}{2}}{2\sin \qquad \sin\frac{\delta_2}{}} \qquad \ldots (6)$$

Thus, from equation (3) it is clear that the wave-length λ can be evaluated by simply measuring the two deviations δ_1 and δ_2. Further from (4) the angle ϕ can be evaluated without a knowledge of the angle of incidence i.

Method—

First make the optical and mechanical adjustments as described in the main experiment above. Then set the grating on the table of the instrument such that its surface makes a small angle to the pencil of rays issuing out of the collimator. Now, adjust the cross-wire of the telescope on the zero order image and take the reading of the two verniers. Then set the cross-wire on the first spectral line of the first order spectrum on the right of the zero order line and note the readings of the verniers. Thus calculate δ_1. Now turn the telescope to the left and similarly determine δ_2 for the same spectral line in the same order of the spectrum. From these two values of δ_1 and δ_2 calculate the value of $\tan\phi$ with the help of equation (4) and hence get the value of $\cos\phi$. Then finally with the help of equation (3) obtain the value of λ.

Repeat the above procedure for the same wave-length for the next order of the spectrum. In the same way, evaluate the wave-lengths of the other spectral lines.

No.—34.

To determine the dispersive power* for the given grating.

Theory.—Let λ_1 and λ_2 be two close wave-lengths (*e. g.*, the D_1 and D_2 lines of sodium light) and θ_1 and θ_2 be their corresponding angles of diffraction for the n^{th} order of the spectrum. Thus, $(\theta_1 - \theta_2)$ measures the angular separation for the wave-length difference $(\lambda_1 - \lambda_2)$. The quantity $\frac{\theta_1 - \theta_2}{\lambda_1 - \lambda_2}$ measures the dispersive power, which, when the wave-lengths are very nearly the same, reduces to $d\theta/d\lambda$. Now, for a grating

$$(a + b) \sin \theta = n \lambda$$

$$(a + b) \cos \theta . d\theta = n . d\lambda$$

or
$$\frac{d\theta}{d\lambda} = \frac{n}{(a+b) \cos\theta} \quad \ldots \quad (7)$$

[**Note**—It is clear from the expression (equation–7) for the dispersive power of a grating that its value varies inversely as the grating element. In other words, we can say that the dispersive power depends on the closeness of the rulings drawn on the grating. It is for this reason that gratings are manufactured with a large number of lines (upto 30,000 lines per inch) on them. Further, from relation (7) it follows that the dispersive power of a given grating is directly proportional to n, *i. e.*, it increases with the order of the spectrum. With increasing n the value of $\cos \theta$ decreases, hence due to this cause also the dispersion increases. Thus, in order to get good separation between spectral lines we must use spectra of high orders.]

Method—First make the usual adjustments of the instrument. Then determine in the usual manner the angle of diffraction θ_1 for the D_1 line of the sodium spectrum. Similarly, determine θ_2 for the D_2 line. Then

$$\text{Dispersive power} = \frac{d\theta}{d\lambda} = \frac{\theta_1 - \theta_2}{\lambda_1 - \lambda_2} = \frac{\theta_1 - \theta_2}{6 \times 10^{-8}}$$

* *Dispersive power of the grating is represented by $d\theta/d\lambda$, and can be defined as the change in the angle of diffraction corresponding to a unit change in wave-length.*

since $d\lambda$ for the two D-lines of sodium is 6 A. U. Repeat the experiment for the second order spectrum and calculate* the dispersive power as above. It will be seen that the dispersive power increases with the order of the spectrum.

No.—35.

To determine the resolving power† of a plane transmission grating.

Theory—The spectral image of a linear slit as obtained by diffraction of light through a grating is, in practice, never a geometrical line, but it has a finite width. The distribution of intensity of light in the image is maximum in the middle of the line and sharply fades away on both sides towards the edges. Now, if the incident beam consists of two wave-lengths, λ and $\lambda + d\lambda$, there will be a partial overlapping of the two corresponding images and the resultant illumination will exhibit a definite dip in the middle. It is this finite fall in the middle which furnishes us with the clue of the presence of two close wave-lengths. If the interval $d\lambda$ is further reduced, there is a greater overlapping of the two images so much so that the eye fails to distinguish between the two wave-lengths and no dip in the intensity distribution curve is perceptible. *Hence for the two wave-lengths to be resolved, the degree of closeness which their images can have, is that the principal maximum of one should lie on the first minimum of the other.* This is, therefore, the criterion* which establishes the limit of resolution. If the maxima lie at this distance or further apart, they can be distinguished separately; if they lie closer than this, they will be seen as one image.

Now, in a plane transmission grating the maxima for the wave-length λ are given by the equation

$$N(a+b)\sin\theta = Nn\lambda$$

where N is the total number of lines in the grating. If $d\phi$ be the half width of the line, then the minima are given by

* Compare the experimentally observed value of the dispersive power with the one calculated from the formula :—

$$\frac{d\theta}{d\lambda} = \frac{n}{(a+b)\cos\theta}$$

† *The resolving power of a grating is represented by the expression $\lambda/d\lambda$, and may be defined as the wave-length at any point in the spectrum divided by the smallest change in wave-length that can be detected at that point.*

* This criterion was first furnished by Lord Rayleigh.

$$N(a+b).\sin(\theta+d\phi) = Nn\lambda+\lambda$$

or $$N(a+b).(\sin\theta+\cos\theta.d\phi) = Nn\lambda+\lambda \quad \ldots \quad (1)$$

since $d\phi$ is small, hence $\sin d\phi = d\phi$, and $\cos d\phi = 1$

If θ and $(\theta + d\theta)$ be the angles of diffraction corresponding to the two wave-lengths, we have

$$(a+b)\sin\theta = n\lambda \quad \ldots \quad (2)$$

and $$(a+b)\sin(\theta+d\theta) = n(\lambda+d\lambda)$$

or $$(a+b)(\sin\theta+\cos\theta.d\theta) = n(\lambda+d\lambda) \quad \ldots \quad (3)$$

Hence from (2) and (3) we get

$$(a+b)\cos\theta.d\theta = n\,d\lambda \quad \ldots \quad (4)$$

Thus, from (1) and (4) we have

$$d\phi = \frac{\lambda}{N(a+b)\cos\theta} \quad \ldots \quad (5)$$

and $$d\theta = \frac{n.d\lambda}{(a+b)\cos\theta} \quad \ldots \quad (6)$$

Now $d\theta$ is the angular separation between the maxima of the two wave-lengths, λ and $\lambda + d\lambda$, and $d\phi$ is the angular separation between the maximum and the minimum of the same wave-length. Thus, according to the criterion laid for the just resolution of the two wave-lengths we have

$$d\phi = d\theta$$

or $$\frac{\lambda}{N(a+b)\cos\theta} = \frac{n.d\lambda}{(a+b)\cos\theta}$$

or $$\frac{\lambda}{d\lambda} = Nn \quad \ldots \quad (7)$$

Therefore, the resolving power of a grating is equal to the product of the number of lines* on the grating and the order of the spectrum observed.

[Note—For a grating with 15,000 lines used in the second order, the resolving power is equal to 30,000. For λ=6000 A. U., this give $d\lambda$=1/6=012 A. U. Thus, any two spectral lines having a separation of 0·2 A. U., or greater than this, will be shown separates (or will be resolved) in this spectrum.

* If only a part of the width of the grating is employed for resolution, the resolving power is proportional to this width.

Method—

Mount the grating on the turn-table of the spectrometer and make its adjustments as described in the main experiment above. L is the sodium lamp, C the collimator, G the grating and T the telescope. In front of G is an adjustable slit which can be operated by the micrometer screw M, which also measures accurately the width of the slit. The operation of the slit thus limits the part of the grating producing the spectrum.

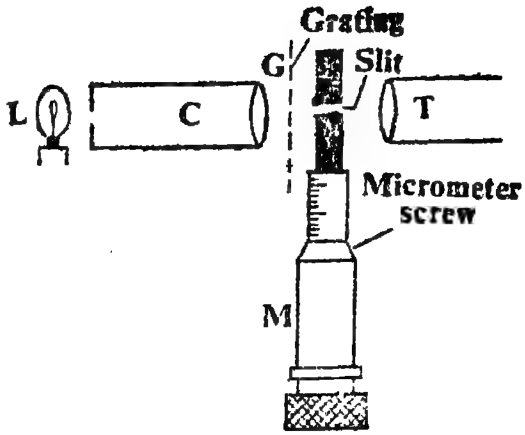

Fig.–80
Resolving power of a grating.

Now, turn the telescope to get the first order images of the two D-lines of sodium spectrum in the field of view. Now begin reducing the width of the slit S and watch carefully the behaviour of the two lines. The two lines will be found to approach each other, till for a particular width of the slit the two lines just cease to be resolved. Take the reading of the micrometer screw in this position. Now close the slit completely and again note the reading of the micrometer screw. Thus, by taking the difference of the two readings calculate the width of the slit which gives the width of that part of the grating from which the diffraction of light is taking place. Since the number of lines per inch of the grating is known, calculate how many lines are included in the measured width of the grating. The product of this calculated number of lines and the order of the spectrum gives the resolving power of the grating.

Repeat the experiment for the same order on the other side of the direct image and thus calculate the mean resolving power for the first order of the spectrum and compare the result with the value $\lambda/d\lambda$ for the sodium lines.

Again repeat the experiment for the second order of the spectrum when it will be observed that the width of the grating, for the resolution of the two lines just to cease, has to be further reduced. Make a calculation for the resolving power for the second order also.

[**Example**—In an experiment with a grating containing 14000 lines per inch, it was observed that in the first order spectrum the two D-lines of sodium just ceased to be resolved when the width of the slit was 0·181 cm.

Thus, the number of lines contained in the width equal to 0·181 cm. of the grating

$$= \frac{\cdot 181 \times 14000}{2 \cdot 54} = 1000$$

The required resolving power $= Nn = 1000 \times 1 = 1000$

Also $$\frac{\lambda}{d\lambda} = \frac{5893}{6} = 982$$

which verifies the experimentally obtained result.

$\therefore$ The resolving power for the whole grating = 14000.]

EXPERMIENT—36

Object—To verify the expression for the resolving power of a telescope.

Apparatus Required—A telescope, a rectangular aperture with width variable with the help of a micrometer screw, a black cardboard with narrow white strips drawn on it, and a micrometer microscope.

Formula Employed—The object of the experiment is to compare the theoretical and practical resolving powers.

$$\text{The theoretical resolving power} = \frac{\lambda}{a}$$

and $$\text{The practical resolving power} = \frac{d}{D}$$

where λ = wave-length of light employed.

a = width of the aperture placed in front of the objective of the telescope for just resolution of the two objects.

d = the separation of the two objects which are to be resolved with the telescope.

D = the distance of the objects from the objective of the telescope.

PRINCIPLE AND THEORY OF THE EXPERIMENT

When a telescope is employed to view a distant point object, its objectives lens behaves like a circular aperture, and consequently the image is formed by diffraction. It is for this reason that the image of a point object is actually never a point but it is a circle, surrounded by very faint concentric dark and bright rings.

To calculate the radius r of the image, let us refer to the accompanying figure, where P is the point where all the rays parallel to the axis coming from a distant point object are focussed. The point which is the centre of the image, will obviously have maximum brightness since all the rays arrive here in the same phase.

The point Q is situated at the edge of the image and will be a point with minimum intensity. This will be so when $AQ - BQ = \lambda$, because under this circumstance the sets of rays

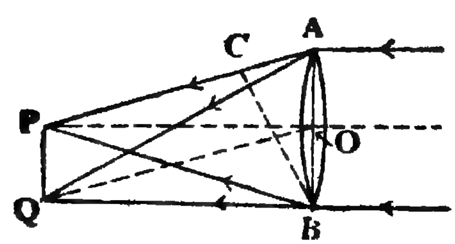

Fig.-81
Diffraction through a lens.

coming from the two-halves, OA and OB, of the lens will differ in phase by $\lambda/2$.

Now, with Q as centre and QB as radius, draw and an arc BC. This will be sensible a straight line, and consequently, the triangle ABC may be regarded as a right-angled triangle. Hence

$$\text{the angle ABC} = \frac{\lambda}{a}$$

where $AC = \lambda$

and $AB = a$ (the aperture of the lens).

But the angle ABC = the angle POQ

Thus $$\frac{\lambda}{a} = \frac{r}{f}$$

where $r = PO$ (the radius of the image)

$f = OP$ (the focal length of the lens)

Thus $$r = f.\frac{\lambda}{a} \qquad (1)^*$$

The intensity distribution of the image is shown in Fig.–82. If, instead of a single point object, we have two distance point objects, their images will now be two circles each of radius r, and the distance x between their centres is given by

$$x = f \tan \theta \qquad \ldots \qquad (2)$$

where θ is the angle subtended by the two objects at the objective of the telescope.

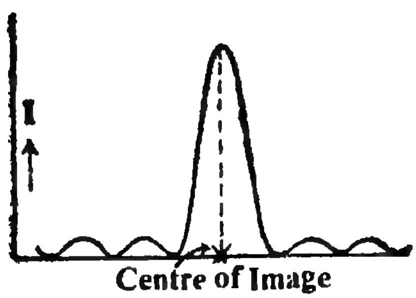

Fig.–82
Intensity distribution of an image.

Now, it is clear that the two images will appear distinctly separate when the edge of one image touches that of the other,

* From this formula it is clear that the radius of the image is inversely proportional to the aperture of the objective lens of the telescope. Thus telescopes with objectives of large aperture tend to produce point images of point objects.

i. e., when $d = 2r$. If the images come closer than this, overlapping of the two shall begin and the two images shall cease to be distinguished as two. The intensity distribution of the two might then be represented as shown in Fig.-83. The two continuous lines represent the intensity distribution of the separate images, while the upper dotted curve shows the two compounded.

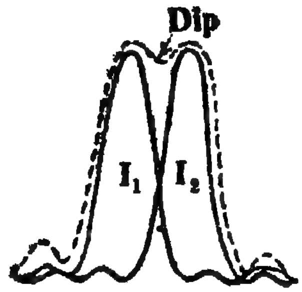

Fig -83
Intensity distribution of two images.

The resultant has two pronounced maxima with an appreciable dip in between. It will thus be possible, with the aid of this falling-off in intensity between two bright regions, to distinguish the two objects, or the two objects *will be resolved.* Lord Rayleigh has shown that the resolution of the images is possible only when d is not less than r, *i. e.*, the centre of one image may, at the most, lie on the edge of the other as shown in Fig.-82. If d is less than r, the dip seen in the figure shall disappear and the curve shall resemble one for a single point object. Hence, under this circumstance, the images shall not be resolved. Thus, in order to secure resolution of the two images we must have $x = r$ or

$$r = f \tan \theta = f\,\theta \qquad \text{(since } \theta \text{ is small)}$$

$$f \qquad (3)$$

This is taken as the limiting case, and the resolving power is measured by this ratio. But from equation—(1)

$$\frac{\lambda}{a}$$

$$\text{Theoretical resolving power} \quad \frac{\lambda}{a} \qquad (4)$$

Now this angle θ is also equal to d/D where d is the distance between the two objects and D is their distance from the objective lens. Hence

$$\text{Practical resolving power} = \frac{d}{D} \qquad (5)$$

The object of this experiment is to compare* the theoretical and practical resolving powers.

Method—

(i) Mount the telescope on a stand and adjust such that its axis lies horizontally and meets normally the cardboard which provides the two sources. This cardboard is mounted on another stand and is placed at a distance of 15—20 ft. from the telescope. Adjust the cardboard in such a way that the white rectangular objects are vertical.

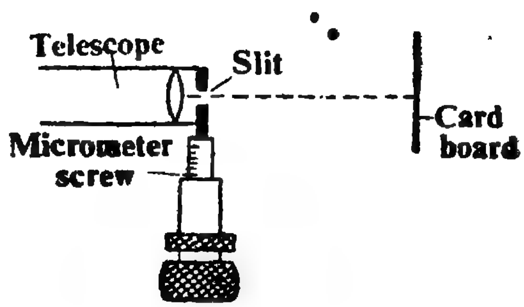

Fig.–84
Resolving power of a telescope.

(ii) Keep the slit wide open and focus the telescope on two white rectangles in the first row. By a horizontal movement of the telescope at right angles to its axis adjust that the two images lie symmetrically with respect to the intersection of the cross-wires in the field of view.

(iii) Now, gradually diminish the width of this aperture till the two images just cease to appear at two. Take the reading of the micrometer screw. Close the slit completely by operating the screw further and take the reading again. The difference of the two readings gives the width 'a' of the slit which is just sufficient to resolve the white lens situated in the first row of the cardboard.

(iv) Measure the distance between the centres of the two consecutive lines in the first row with a travelling microscope. This gives d of the formula.

Then measure with a tape the distance between the cardboard and the objective lens of the telescope. This gives D of the formula.

(vi) Repeat the experiment with objects lying in the second and third rows and thus obtain different values of 'a' and 'd'.

(vii) The only quantity to be known now is λ, the wave-length of light employed. In the present case, this can be taken as 6000×10^{-8} cm. (lying in the yellow region of the solar spectrum), which is the mean wave-length of diffused sun-light.

In order to verify this theory, a telescope is fitted with an adjustable slit (which can be operated by a micrometer screw) placed as close to its objective lens as possible. A serviceable object for this experiment can be conveniently made by taking a black cardboard with white strips of, say, 1 mm. width separated by black strips of 1 mm., 2 mm., 3 mm., etc., width in different rows.

Observations—

[A] (i) Pitch of the micrometer screw =......cm

(ii) No. of divisions on the cap =......

(iii) ∴ Least count of the screw =......cm

[B] Mean value of $\lambda = 6000 \times 10^{-8}$ cm

No. of row	Slit readings			(λ/a) Theoretical resolving power	Distance between the two objects (d)	Distance between the objects and the objective (D)	Practical resolving power (d/D)
	when resolution ceases	when the slit is closed	Width of the slit (a)				

Result—The resolving power of the telescope for the two objects

(a) ......mm. apart = ...

(b) ......mm. apart = ...

(c) ......mm. apart = ...

[**Note**—It will be seen from this study that for resolving those two lines which are very close to each other a larger width of the slit is required. This clearly shows that a large aperture of the objective of a telescope resolves any two objects subtending a very small angle. It is for this reason that objective glasses of astronomical telescopes are made as large as possible. For instance, gigantic telescopes used in the astronomical observations as Mt. Wilson and Mt Pamyara in California are provided with objectives* of 100″ and 200″ size respectively. Moreover, whenever a stop is required to reduce the spherical aberration, it is employed to cut off the central portion rather than the peripheral one, since it leaves the resolving power unimpaired.]

* These objectives are, however, not convergent lenses, but concave mirrors. The theoretical arguments for the resolving power remain unchanged.

Precautions and Sources of Error—

(1) The cardboard as well as the white rectangular objects drawn on it should each be vertical, and the slit employed in front of the objective lens should be parallel to these lines.

(2) The axis of the telescope should be horizontal and should meet the surface of the cardboard normally.

(3) Due care must be taken in measuring the width of the aperture and in measuring the distance between the two linear sources. Avoid back-lash error in these measurements.

(4) The minimum width of the aperture for resolution of the two linear objects should be found *both by opening as well as by closing the slit.*

(5) The distance D has to be measured *from the lens* of the telescope to the cardboard.

(6) *Do not close the slit too tighty*, otherwise it shall get damaged.

A NOTE ON THIS EXPERIMENT

The principle of this method can be employed for the determination of the wave-length of light. For this purpose, the cardboard containing the linear objects is replaced by a net of wires stretched vertically at equal spaces in a frame. These are illuminated by a powerful sodium lamp placed behind the frame. The wires are viewed through the telescope, and the width of the aperture is gradually diminished till resolution disappears. All the quantities in the formula $\lambda/a = d/D$ are known except λ which can, therefore, be calculated out.

11

Polarisation of Light

If we look through a tourmaline crystal cut parallel to its axis, nothing remarkable is noticed except that the light is slightly coloured due to the natural colour of the crystal. If we now place two crystals face to face with their axes parallel, the only observable difference produced is an increased colouration of the emergent light. Rotating both crystals together in a plane parallel to their faces produces no alteration in the field of view. If, on the other hand, one of the crystals is rotated with respect to the other, the light transmitted through them gets fainter and fainter, until the light is completely extinguished when the axes of the crystals are at right angles to each other. If the rotation is further continued, more and more light is transmitted till the original state of affairs is attained.

This experiment leads us to the inevitable conclusion that *the direction of displacement in a light wave is perpendicular to the direction of propagation.* After traversing the first tourmaline crystal, the light waves exhibit an absence of symmetry about the axis of the ray and the displacements in the transmitted waves are confined to a single direction, and the light is said to be *polarised.* The first crystal transmits only those vibrations which make a certain angle with its axis. If the axis of the second crystal is parallel to that of the first the waves transmitted through the first crystal are allowed to pass through the second crystal also. If, however, the axes of the crystals are at right angles to each other, the light transmitted through the first crystal consists of vibrations in a direction at right angles to that in which alone they could be transmitted through the second crystal. It is due to this reason that the two crystals, when placed with their axes perpendicular to each other, do not permit any light to pass.

The plane perpendicular to one in which the vibrations of the light waves are confined is called the plane of polarisation.

The phenomena of interference and diffraction, as discussed in the last two chapters, clearly support the view that light consists of waves. The phenomenon of polarisation, as discussed here, now shows that these waves are transverse. In longitudinal waves, no polarisation of light can take place.

In actual practice, a tourmaline crystal is never employed for the production of polarised light, but is replaced by a Nicol prism* which was first employed in 1826 by a Scotch physicist, Nicol.

A Nicol prism (or simply, a Nicol) is made from a crystal of calcite or Iceland spar,† which is a transparent crystalline form of calcium carbonate. The end faces of a rhombohedron of this crystal having a length nearly three times its width, are ground away until they make an angle of 68° with the long edges. It is then cut diagonally by a plane at right angles to the new end faces and the principal section of the crystal. These new diagonal surfaces so obtained are ground flat, polished and then cemented together with a thin layer of Canada balsam so as to to occupy the same position. The refractive index of Canada balsam is 1·55, while the values of the refractive index of the crystal for the ordinary and extraordinary rays are 1·66 and 1·48 respectively. The ordinary ray strikes the surface of separation of the balsam at an angle greater than the critical angle for the two media (and is passing from a denser to a rarer medium) and is therefore totally reflected aside, while the extraordinary ray travels straight through. Thus, a Nicol prism is the most perfect polariser and furnishes exceedingly easy device to procure plane polarised beam, whose direction of vibration is parallel to the shorter diagonal of the face from which it emerges.

[**Note**—(1) The phenomenon of double refraction as exhibited by many minerals, including calcite, is also shown by tourmaline. When ordinary light falls on a crystal of tourmaline, it is split up into an ordinary and extraordinary ray, and even for very small thicknesses (1 to 2 mm) of the crystal, the ordinary ray is completely absorbed by the crystal. What then emerges out of the crystal is the extraordinary plane-polarised beam.

(2) A modern commercial method for obtaing plane-polarised light is by means of a *Polaroid*. which consists of ultra-microscopic crystals of the organic compound iodo-sulphate of quinine‡,

* For detailed study, read author's book : "A Critical Study of Practical Physics and Viva-Voce".

† The crystal derives this name from the fact that at one time it was found in great quantities in Iceland.

‡ This compound is also known as *Herapathite*, after Herapath, who was the first to carry out investigations upon the crystals of this substance.

which are embedded in nitro-cellulose films in such a way that their axes are parallel to one another. These crystals, like tourmaline, absorb the ordinary ray, and hence easily furnish plane-polarised light.]

A very interesting phenomenon takes place with plane polarised light. When polarised light passes through certain substances, its nature is not changed (i. e., light remains plane polarised), but its plane of polarisation goes on rotating about the direction of the ray as the light travels farther and farther into the medium. This phenomenon is known as *rotatory polarisation.* An experiment based on this phenomenon is described below.

EXPERIMENT—37

Object—To study the variation in the angle of rotation of the plane of polarisation with the concentration of sugar solution, and to calculate the specific rotation using a Laurent's saccharimeter.

Apparatus Required—Laurent's saccharimeter, sodium lamp, a chemical balance, weight box, cane-sugar, funnel beaker, pipette and filter paper.

Description of the Apparatus—The essential parts of the saccharimeter are illustrated in the figure given below. In this figure N_1 is a Nicol prism (known as a "*polariser*") which serves to polarise a beam of light passing through it, while the other Nicol prism N_2 (known as the "*analyser*") analyses the transmitted beam and detects its plane of polarisation. In between the two Nicols is placed the tube T containing the liquid under investigation. The tube is closed on both sides by optically worked glass-plates which are held in position with the help of metallic caps and rubber

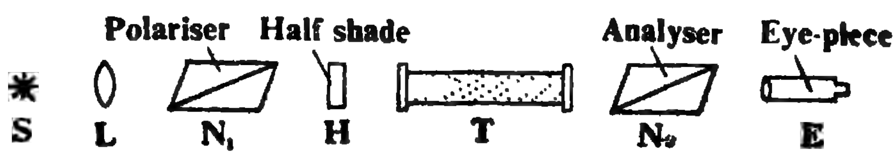

Fig.–85
Schematic arrangement of a half-shade polarimeter.

washers. S is a source of monochromatic radiation (*e. g.*, a sodium lamp) whose light is rendered parallel with the help of lens L. E. is a low power telescope employed to note readings on the graduated circular scale provided with the analyser.

H is the sensitive arrangement known as half shade or laurent plate.

Principle and use of the half-shade—When the Nicol N_1 reduces the light vibrations to a particular direction (namely, parallel to the

short diagonal at the end of the prism), all the light transmitted by it can pass through N_2, if N_2 is oriented exactly in the same way as N_1 (*i. e.*, if its short diagonal lies parallel to that of N_1 and its length lies parallel to that of N_1). In this setting *the Nicols are said to be parallel.* If, however, N_2 is turned from this position through a right angle, no light from N_1 can pass through N_2, since N_2 is now so oriented that the light vibrations falling on it are in a direction perpendicular to its short diagonal, and such vibrations are consequently not transmitted by the analyser. In this setting, *the two Nicols are said to be crossed.*

If an optically active substance is inserted between the Nicols when they are crossed, the plane of vibration of light shall be rotated, and consequently, on account of the change in the direction of vibration some light will pass through the analyser. However, by turning the analyser one way or the other, the light can once more be extinguished, showing thereby that the light is still polarised but its vibrations have changed direction in traversing the active medium. The magnitude of this rotation can be measured by measuring the angle through which the analyser has to be turned. Unfortunately, this measurement cannot be effected with accuracy, since it is found by actual experiment that the analyser can be rotated through an appreciable angle wheh the light is cut out without any apparent return of the light. *This lack of sensitivity in the instrument is overcome by employing the half-shade device.*

The half-shade or the Laurent plate is really a glass-quartz combination in the form of a complete circle. (Fig.–86 a), which covers completely the open end of the polariser. It consists of a semi-circular sheet of quartz (ACB) cut parallel to the optic axis, (*i. e.*, the optic axis is parallel to AB). The complete circle is made up by attaching a semi-circular sheet of glass (ADB) of such thickness that it absorbs the same amount of light as the quartz.

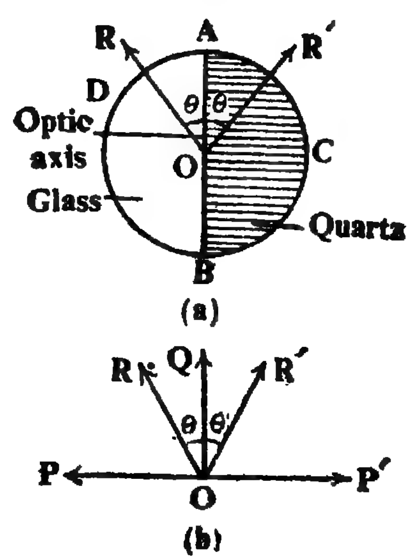

Fig.–86
Principle of a half-shade.

Let us assume that the vibrations of light at O take place in the direction OR (Fig –b) so that its component directions are OP and OQ. When this light falls on quartz, it is separated in two components, OP and OQ, polarised normally to each other, which travel through the quartz price with different velocities. Consequently, when the light

disturbance passes through this plate, there will be a gradual change of phase between these two components on account of the differing velocities of transmission. After traversing a certain distance in the quartz, the disturbance will reach a point in the plate where one of the component displacements is along OQ while the other is along OP′. These will obviously, combine to give the resultant vibration along OR′.

The quartz plate is so cut that when the light disturbance just leaves the plate on the other side, this difference of phase exists between the two components. This difference is obviously half a period, and consequently, the plate is generally called a *half-wave plate*. Of course, the light which transverses the glass side proceeds undisturbed, and its vibrations are still confined along the original direction, *i. e.*, along OR as shown in Fig.–86 (a).

Thus, on emergence from the glass-quartz combination we have two plane-polarised beams, one having vibrations parallel to OR and the other parallel to OR′. If the principal plane of the analyser be set either parallel to AB or normal to it, the components transmitted are the same for both sides and hence *the two sides present equal illumination.* A slight rotation of the analyser results in an unequal illumination of the two-halves which can be readily detected by the eye.

If the analyser is set for equal illumination on both sides, and an optically active substance is then interposed, it will be necessary to rotate the analyser once more to attain the position of equal illumination. The amount of rotation measures the angle of rotation of the plane of polarisation. As this device is very sensitive, this angle can be measured very accurately.

[**Note**—The chief defect of the half-shade device lies in the fact that it is suitable only for the particular wave-length for which the path-difference between the ordinary and the extraordinary beams is half the wave-length. Hence, sometimes other devices, which can be worked on white light, are employed. Two such devices are described and discussed below :—

(1) **Bi-quartz**—This consists of two semi-circular discs of quartz cemented together to form a complete circle. One of the crystals rotates the plane of polarisation of the incident light in a clockwise, and the other in a counter-clockwise direction and the two plates are so cut that the optic axis lies normal to the faces of the plate.

When plane-polarised white light falls on the bi-quartz, each colour is rotated through different angles, hence *rotatory dispersion* takes place in each plate. For the red part of the spectrum the rotation is minimum while for the violet it is maximum. The thickness of each plate is so chosen* as to rotate the plane of polarisation of the yellow light through 90°. The accompanying figure illustrates the process clearly. AB represents the plane of vibration of the incident light, and the planes of vibration of different colours are depicted in each half separately.

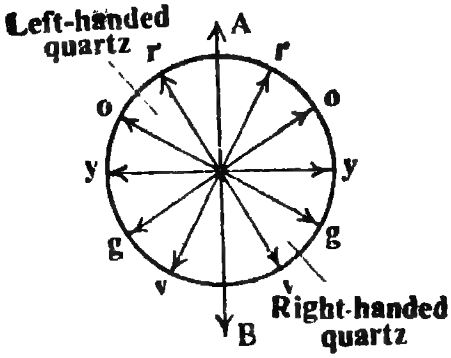

Fig.–87
Principle of a bi-quartz.

Since the rotation of the plane of polarisation for yellow by the two halves is 90°, this colour (y y) falls in a straight line, and hence both the vibrations can be simultaneously extinguished for one single adjustment of the analyser. The remaining colours of the two-halves will be an admixture of red and blue which will present a greyish appearance. This is called the *tint of passage*. If the analyser is slightly displaced from this position, one-half of the field of view will be predominantly blue and the other red or *vice-versa* depending upon the direction of rotation of the analyser. The position of this tint of passage is very sensitive and consequently great accuracy can be achieved in the measurement of the angle of rotation of the plane of polarisation.

(2) **The Lippich Polarising System***—In this system, the field is divided in three parts—the two outer ones similarly illuminated for all positions of the analyser and the central portion, which may be differently illuminated from the neighbouring regions and which has to be matched with the outer parts. of the field. The mode of producing the divided field is illustrated in the accompanying figure. N_1 is the polarising Nicol in front of which are placed two small Nicols L_1 and L_2. The central portion of the light emerging from the combination passes through the Nicol N_1 only, while the other parts pass through N and L.

Fig.–88
Lippich polarising system.

* This thickness is nearly 3·75 mm.

* In this system we have either three-prism arrangement or two-prism arrangement. Herein is described the former device.

In a particular setting of the analyser N_2, the whole field appears uniformly illuminated, while for other positions of N_2 the field is not uniform. Thus, to measure the amount of rotation of any active material placed at O, the analyser is first set into a position corresponding to a uniform illumination of the whole field of view. The substance under investigation is then inserted in position and the analysing Nicol re-adjusted till the uniformity of the field of view is restored. The angle of rotation of N_2 measures the required rotation due to the presence of the optically active substance.

Formula Employed—*The rotation produced per decimetre length of the solution divided by the number of gms. of the active substance per c. c. of the solution is called the specific rotation of the dissolved substance.*

Thus, if x gms. of cane-sugar are dissolved in v. c. c. of the solution, and the length of the tube containing the solution be l decimetres, and θ be the rotation produced, then the specific rotation,† $[\alpha]_t$, of cane-sugar at temperature t is given by—

$$[\alpha]_t = \frac{\theta/l}{x/v} = \frac{\theta \cdot v}{x \cdot l}$$

[**Note**—From the definition of specific rotation as given above it is clear that its units will be *degree* $(dm)^{-1}$ $(gm/c.\,c.)^{-1}$. The value of specific rotation varies with wavelengths, being larger for shorter wave-lengths. Usually, it is expressed for the D-line of sodium.]

Method—

(i) Before starting the actual experiment, examine carefully whether your apparatus employs a half-shade or a bi-quartz device, and whether you require for your experiment a source of monochromatic radiation or of white light.

Now with the sample tube removed *align the axis af the polarimeter with the source of light*. Focus the eye-piece on the circular scale attached to the analyser. Lastly, focus the telescope on the half-shade, *whose dividing line between the two semi-circles should be seen very sharply.*

(ii) Now clean the end-pieces of the polarimeter tube free of dust and grease till the passage of light through them is quite clear. Fill the tube with clear (preferably distilled) water and see that

† The relation for specific rotation of cane-sugar with temperature is given by the formula—

$$[\alpha]_t = 66{\cdot}5^\circ - 0{\cdot}0184\,(t - 20).$$

Thus at 20°C the specific rotation of cane-sugar is 66·5°

there is no air-bubble in the main body of the tube when the end caps have been screwed.*

Illuminate the apparatus with sodium light† and after placing the tube full of water in its position adjust the analyser such that *the two-halves of the half-shade acquire the same intensity.* Note the readings of the main scale and the vernier scale Note also these readings when the analyser is again set for equal intensity after turning it through 180° from this position.

[**Note**—There are actually *four positions* for which intensities of the two-halves match. But only two of them are sensitive positions, *i. e.*, a small rotation of the analyser shows a distinct contrast at once. *Matching has to be done only in these positions.*]

(iii) Prepare sugar solution† of known strength and after removing water from the tube, *rinse it well with sugar solution.* After completely filling the tube with sugar solution replace it in position. Rotate the analyser till the previous setting is restored end take the readings of the main scale and the vernier scale. Rotate the analyser through 180° from this position and after adjusting it for equal brightness of the two-halves of the half-shade take the readings again. After subtracting from these readings the corresponding values obtained in the first setting, (*i. e.*, with pure water) of the analyser get the mean value of the angle of rotation of the plane of polarisation.

(iv) Repeat the experiment with sugar solutions of different concentrations. Plot a graph‡ between the strength of sugar solution and the rotation produced.

(v) Measure the length of the tube with an ordinary scale. *Change it in decimetres.* Finally. calculate the value of specific rotation** with the help of the formula given above.

---

* It is necessary to have rubber washers between the glass ends and the tube *to avoid strain* when screwing up, because a strained end will produce rotation.

† If your instrument employs a bi-quartz as the sensitive device, the source of illumination will be an electric lamp, and in that case the field of view will present different colours, red and blue. In this case rotate the analyser till the two portions of the field of view acquire the same tint, called the *tint of passage.* This condition has to be re-established for subsequent readings with sugar solutions of different concentrations.

‡ The graph shall be a *straight line* proving thereby that the rotation of the plane of polarisation produce by an obtically active substance is directly proportional to the mass of the active substance present in the solution.

** Do not fail to report *the temperature* at which the determination of specific rotation has been conducted.

Observations —

[A] *Preparation of sample solution.*

(i) For the parent solution :—

(a) Mass of the watch-glass = ...gm

(b) ,, ,, ,, + sugar = ...gm

∴ mass of sugar taken = mgm

(c) Dissolved in water and solution made upon volume = v c. c.

∴ concentration of parent solution (C_o) ... m/v gm per c. c.

(ii) For other solutions :—

Concentration of the previous solution	Volume of previous solution	Volume of water added	Concentration obtained
C_o	80 c. c.	20 c. c.	0·8 C_o
0 8 C_o	75 c. c	25 c. c.	0·6 C_o
0·6 C_o	66·7 c. c.	33·3 c. c.	0·4 C_o
0·4 C_o	50 c. c.	50 c. c.	0·2 C_o

[B] *Readings with pure water in the tube.*

S. No.	First position of analyser			Second position of analyser (180° apart)			Remark
	Main scale reading	Vernier reading	Total reading	Main scale reading	Vernier reading	Total reading	
							Least count of the vernier of the analyser =.........

[C] *Readings with sugar solution in the tube.*

S. No.	Strength of sugar solution	First position of analyser			Second position of analyser (180° apart)			Remarks
		Main scale reading	Vernier reading	Total reading	Main scale reading	Vernier reading	Total reading	
1	C_0							(1) Length of the tube = . cm
2	$0{\cdot}8\ C_0$							(2) Temp. of the solution =...°C
3	$0{\cdot}6\ C_0$							
4	$0{\cdot}4\ C_0$							
5	$0{\cdot}2\ C_0$							

Calculations—The value of the angle of solution with different strengths of sugar solution are tabulated below :—

S. No.	Strength of sugar solution	Mean position of analyser	Mean position of analyser (with water)	Angle of rotation

The specific rotation is given by

$$[\alpha]t = \frac{\theta \cdot v}{l \cdot v} = \text{......} \text{ degrees dm}^{-1}\ (\text{gm/c. c.})^{-1}$$

[**Example**—In an experiment, the following observations were taken

(i) Mass of sugar dissolved = 5 gm.

(ii) Volume of the solution = 100 c. c.

(iii) Length of the tube = 22 cm. = 2·2 dm.

(iv) Angle of rotation = 7·35°

(v) Temperature of solution = 22°C

$$\text{Specific rotation* (at 22°C)} = \frac{7{\cdot}35 \times 100}{2{\cdot}2 \times 5}$$

$$= 66{\cdot}8° \text{]}$$

Result—The specific rotation of cane-sugar at......°C corresponding to the D–line of solution is......degree dm^{-1} $(gm/c.c)^{-1}$

[Standard value = ... ; Error – ......%].

Precautions and Sources of Error—

(1) Whenever a solution is changed *rinse the tube at least twice with the new solution* before finally filling it.

(2) Take care that no air-bubble is contained in the polarimeter tube, specially in that part of the tube where light traverses the liquid column.

(3) Filter the solution to make it dust-free. Use a pure variety of sugar for this purpose, otherwise there will be unnecessary dimunition of intensity of light.

(4) While screwing the cap of the tube, do not apply large pressure, otherwise the glass end is likely to be strained and this will produce spurious rotation.

[**Note**—While filling the tube, *keep margin for a small air-bubble*. This will avoid straining of the end plate while screwing. When the tube will be placed horizontally in the apparatus this tiny bubble shall be transferred to the bulged portion provided at the end of the tube. Thus, the air-bubble will not fall in the path of the light transmitted through the solution.]

(5) Set the analyser in correct position with respect to the polariser and set it (the analyser) again in a position 180° apart. During one set of observation, the position of the polariser should not be disturbed.

[**Note**—Before trying to match the intensities of the two halves of the half-shade, distinguish between the two less sensitive and the two more sensitive positions of matching. *Only the latter positions should be used.* If a distinct difference is not detected, it means that the half-shade is not properly set relative to the polariser.]

* Optically active substances are of two types, (1) right-handed or dextro-rotatory, and (2) left-handed or laevo-rotary. Cone-sugar belongs to the former class, hence, its specific rotation should be expressed as +66·8°. When looking towards the source of light, if the optically active substance rotates the plane of polarisation to the right, (*i. e.*, clockwise), it is said to be right-handed ; others which rotate the plane of polarisation to the left (*i. e.*, anti-clockwise) are termed left-handed (see Table–15 at the end of the book.)

(6) Since specific rotation varies with temperature, the latter should be recorded and reported with the result.

(7) While drawing the straight line on the graph, care should be taken that if some points are left out from the line, an equal number lies on either side of the line, *i. e.*, the points should indicate the mean course of the straight line.

EXPERIMENT—38

Object—To determine the polarising angle for the given glass prism surface and to determine therefrom the refractive index of the material of the prism by making use of Brewster's Law.

Apparatus Required—A spectrometer, the given prism, a polaroid with attachment, sodium lamp, and a reading lens.

Formula Employed—Brewster's law states that *the tangent of the angle of polarisation is numerically equal to the index of refraction of the reflecting medium.* Thus

$$\mu = \tan \phi$$

where ϕ is the angle of polarisation.

PRINCIPLE AND THEORY OF THE EXPERIMENT

When light is reflected from surfaces the reflected beam is partially polarised. Brewster in 1811 carried out a series of experiments with regard to the polarisation of light by reflection at the surfaces of the various media. He found that light is almost completely polarised in the plane of incidence when reflected from a transparent medium at a particular angle, known as the *Angle of Polarisation.* According to him, maximum polarisation in the reflected beam occurs when the reflected and refracted rays are mutually perpendicular to each other. Thus, from the accompanying figure we have—

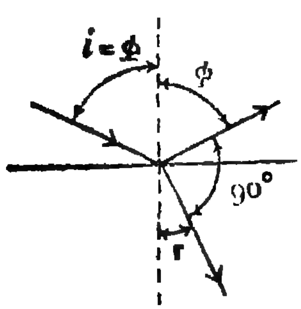

Fig.-89
Brewster's Law.

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin i}{\sin (90° - i)} = \frac{\sin i}{\cos i} = \tan i = \tan \phi$$

since, under this condition, the angle of incidence i is equal to the angle of polarisation ϕ.

Thus, if ϕ be determined, the refractive index* can be easily calculated.

* Since the refractive index of a substance varies with the wave-length of the incident light, it follows that a substance will possess different angles of polarisation with respect to the different components of white light, and polarisation can be complete for only one particular wave-length at a time.

Now the figure given below depicts the principal section ABC of a prism. PO is on incident ray giving rise to the reflected ray OQ. The angle of incidence is equal to the angle of polarisation. Let the angle MOQ be θ.

Now, $\angle AOP = \angle QOC$
But $\angle AOP = \angle MOC$
Hence $\angle QOC = \angle MOC = \theta/2$

Thus, the angle of reflection $= 90^\circ \theta - /2$

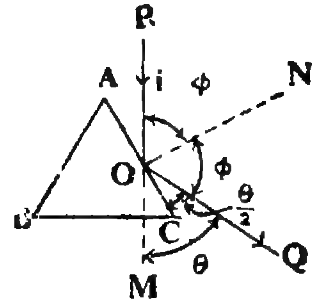

Fig.-90
μ of a prism by
Brewster's Law.

Therefore, the angle of incidence, which is the angle of polarisation, is also equal to $90^\circ - \theta/2$. Hence

$$\phi = (90^\circ - \theta/2)$$

The angle θ can be measured with the help of a spectrometer and hence ϕ can be calculated.

Method—

(i) First make the mechanical and optical adjustments* of the spectrometer. Now place the prism on the turn-table in such a way that the axis of rotation of the table passes through the centre of the circumcircle of the base of the prism. Attach also the polaroid (of a nicol) attachment to the telescope objective.

(ii) Rotate the prism such that the light coming out of the collimator falls on one of its clear faces, and receive the image on the cross-wires. Rotate the polaroid slowly and observe carefully the variation of intensity. Mark specially the position of minimum intensity, which, in general, will not be zero.

(iii) Turn the prism further to increase the angle of incidence and again test for the zero intensity of light by rotating the polaroid. Continue this process till a position is reached when on rotating the polaroid light is completely extinguished.

(iv) Now note the readings of the two verniers. Next remove the prism and set the telescope of the direct image of the slit and again note the readings of the verniers. Calculate the angle between these two positions. This is the angle θ of the above formula. Then calculate the angle of polarisation from the formula, $\phi = 90^\circ - \theta/2$, and hence calculate the refractive index from the formula, $\mu = \tan \phi$.

* These have been fully described and discussed in Exp.–28.

Observations—

Readings for the determination of the angle of polarisation.

S. No.	Vernier	Position of telescope for extinction of the image	Position of direct image	Difference of the two readings of the same vernier	Remark
1	V_1				
	V_2				
⋮					L C. of the spectrometer =... ..
⋮					
⋮					
			Mean θ		

Calculation—

$$\text{Angle of polarisation } \phi = \left(90^\circ - \frac{\theta}{2}\right) = \ldots.$$

$$\therefore \quad \mu = \tan\phi = \ldots ..$$

Result—(i) The angle of polarisation =......

(ii) The refractive index of the material of the prism corresponding to the D-line of sodium =......

Precautions and Sources of Error—

[**Note**—For the precautions to be observed with the use of a spectrometer, refer to Experiment—28. In addition, observe the following precautions.]

(1) In order to allow more reflected light enter into the telescope, keep the slit a bit wide open, but take care in adjusting the cross-wire on the image.

(2) The position for the zero intensity of the reflected light should be accurately found out. This position should be checked by rotating the polaroid both in the clockwise and the anti-clockwise direction.

(3) If the position of zero intensity is not obtained in the experiment, then find out that position where it is very nearly so.

A NOTE ON BREWSTER'S LAW

Jamin, after performing very careful experiments, has come to the conclusion that, in practice, only a few substances of refractive index nearly equal to 1·46, completely polarise light by reflection. As the angle of incidence is slowly increased, the polarised content of the reflected beam first increases, attains a maximum value and then begins to decrease. Lord Rayleigh, after carrying out a series of accurate experiments, has concluded that incomplete polarisation of light when reflected from the surfaces of many substances at the angles given by Brewster's law, is due to unavoidable imperfections of the polish of the surfaces. This appears to be corroborated by Conroy's observation that the optical properties of a glass surface change rapidly during the first few days after the final polishing.

12

Photometry

Illuminating Power—The illuminating power of a source is the quantity of light falling per second on a unit area placed at a unit distance from the source in a direction normal to the rays.

The unit in which illuminating power of a source is measured is the *Candle Power*. One unit candle power is defined as the quantity of light falling per second on a unit area placed normally to the rays and at a unit distance from the *standard candle*. A standard candle* is one made of spermaceti wax, $\frac{7}{8}''$ in diameter, weighing six to a pound and burning at the rate of 120 grains per hour.

Intensity of Illumination—The intensity of illumination at a point is defined as the light falling per second on a unit area of the surface placed at the point under consideration.

The unit of intensity of illumination is the *lux*, which is defined as the intensity at a point on a surface placed normally one metre distant from a source of one candle power.

Another unit which is in common usage is the *foot-candle* which is defined as the intensity at a point on a surface placed normally at a distance of one foot from a source of one candle-power.

Now, intensity of illumination (I) and candle-power (P) are related by the formula—

$$\text{Intensity of Illumination} = \frac{\text{Candle-power}}{(\text{Distance})^2}$$

* As a standard of light, the standard candle defined here is unreliable and therefore unsuitable in practice. Hence, nowadays it has been superseded by more reliable standards, *e. g.*, Harcourt pentane lamp or Hefner amyl-acetate lamp.

$$\text{or} \qquad I = \frac{P}{d^2}$$

where d is the distance of the surface from the source.

Principle of a Photometer*—If two sources with illuminating power P_1 and P_2 be situated respectively at a distance d_1, d_2 from a screen, and if under this circumstance the intensity of illumination due to either source on it be the same, then

$$I_1 = I_2$$

where I_1 and I_2 are respectively the intensities of illumination due to the two sources. Now

$$I_1 = \frac{P_1}{d_1^2} \quad \text{and} \quad I_2 = \frac{P_2}{d_2^2}$$

$$\text{Hence} \qquad \frac{P_1}{d_1^2} = \frac{P_1}{d_2^2} \quad \text{or} \quad \frac{P_1}{P_2} = \frac{d_1^2}{d_2^2}$$

Thus, the candle powers of the two sources are, under this adjustment, directly proportional to the square of their distances from the screen. This is the principle of all photometric measurements.

[**Note**—it is assumed that the student is familiar with simple photometers, such as the Rumford's and Bunsen's. It is difficult to make accurate comparisons of the illuminating powers of sources with these instruments. In the following experiment is described and discussed a very sensitive type of photometer which is known as a Lummer-Brodhun photometer. Even with this, care and practice are needed, but a skilled observer can obtain accurate results.]

EXPERIMENT—39

Object—To compare the illuminating powers of two sources of light using a Lummer-Brodhun photometer.

Apparatus Required—An optical bench, two sources of light, and a Lummer-Brodhun photometer.

Description cf the Apparatus—The Lummer-Brodhun photometer is illustrated in Figure-91. It consists of a box ABCD containing four prisms, P_1, P_2, P_3 and P_4 by means of which light is reflected and transmitted into the long-fours microscope T placed at 45° to the sides of the box, with its object glass in one corner.

LM is a slab of magnesium carbonate whose faces receive the light from the two given sources S_1 and S_2. From the diffuse

For further discussion, read author's book, "A Critical Study of Practical Physics and Viva-Voce".

sides of the slab rays arc scattered and absorbed by the interior of the blackened sides of the box. Only those rays, which cut the sides normally enter into the prisms P_1 and P_2 from the hypotenuse faces of which they are reflected into the right-angled prisms P_3 and P_4 These prisms constitute the principal part of the apparatus.

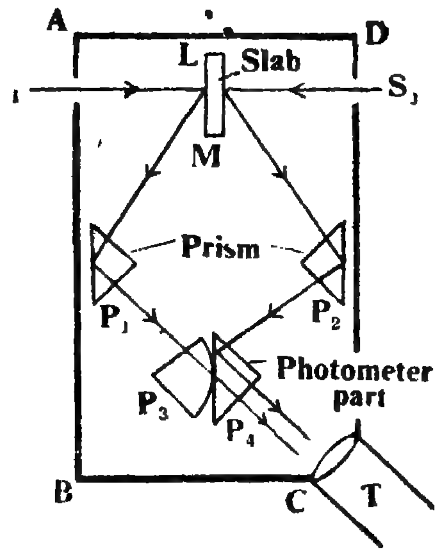

Fig.-91

Sectional figure of a Lummer-Brodhun photometer.

The hypotenuse of the prism P_3 is rounded off, except for a central circular portion which is placed in optical cŏntact with the large face of P_4. The reason for this rays of light may pass from P_3 to P_4 at this junction as if the prism formed one solid medium. Rays of light falling on other parts of the hypotenuse faces are totally reflected.

In this way, rays coming from the prism P_1 pass on through the prism P_4, forming the central bundle of rays in the beam emerging from the right and entering into the microscope (Fig.-92) The rays outside this circle are reflected and are absorbed by the sides of the box. In the same way, rays coming from the prism P_2 are reflected outside the circle while those falling on the circular region are transmitted to the left. Thus, the field of view of the microscope is illuminated by a central circle of rays which originate from the source S_1. The outer portion of the field of view surrounding the central patch is illuminated with light coming from the second source S_2. Generally, these two parts will have different brightness, and on displacing S_1 or S_2, the two parts can be made equally

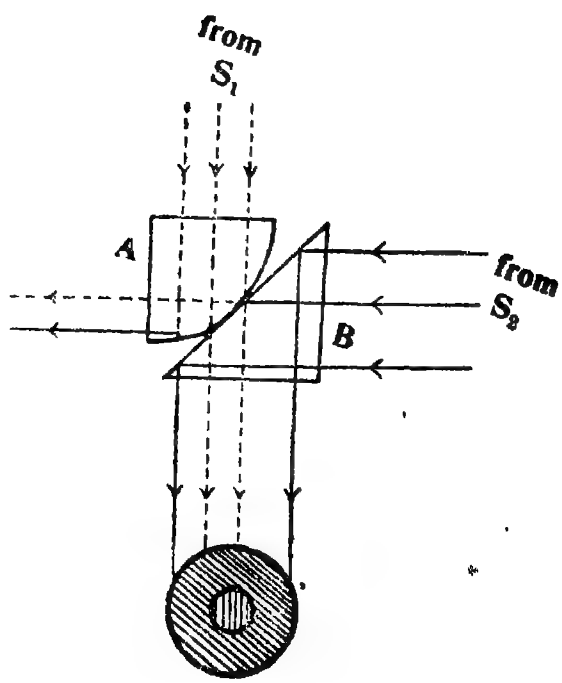

Fig.-92
Field of view in a Lummer-Brodhun photometer.

bright*. The eye can judge this easily, and it can readily appreciate a slight deviation from equality of illumination. It is on this fact that the sensitiveness of this instrument depends. As a matter of fact, this photometer is one of the most accurate instruments for the comparison of the illuminating powers of two sources of light of the same colour.

Formula Employed—If the illuminating powers of two sources of light be P_1 and P_2, and their respective distances from the photometer-head (when they produce equal illumination on this) be d_1 and d_2, we have

$$\frac{P_1}{P_2}=\frac{d_1^2}{d_2^2}$$

Method-

(i) Mount the photometer-head on an upright nearly in the middle of the optical bench such that the line joining the two sources† mounted on either side of the head is normal to the plate LM (see Fig.–91) forms the microscope on contact surface of the prism combination of P_3 and P_4.

(ii) Note the position of the photometer-head and do not disturb its position till one set of observation is completed. Clamp one of the lamps at a convenient position and move the other lamp from the end of the optical bench towards the photometer and watch carefully through the microscope. It will be observed that the two portions in the field of view interchange their brightness as the lamp is moved from its farthest position on the bench to the nearest one. In one position the central portion will be brighter than the surrounding one, while in the other position the surrounding portion shall become brighter than the central one. Now, by moving this lamp in small steps, adjust it in a position till the whole field of view presents uniform illumination. Note the readings of the lamps. Repeat** this setting at least thrice and finally take the mean of these readings.

(iii) Change the position of the first lamp and repeat the experiment as above. In this way, take several readings and calcu-

* It can be easily seen from the figure that the two portions of light received by the microscope traverse the same thickness of glass, any absorption by it will effect the intensity equally and hence will not interfere with the adjustment.

† The lamps should be housed in boxes provided with proper ventilation arrangement and a circular apperture for the light to pass through. The insides of the boxes should be blackened.

** During this repetition, the position of the first lamp should never be changed.

late the ratio of the illuminating powers for each set separately, and thus obtain the mean value of P_1/P_2.

Observations—

Readings for the determination nf d_1 and d_2.

S. No.	Position of the first lamp	Position of the photometer head	Position of the second lamp	Mean position of the second lamp	d_1	
	...cm	.cm	1. ...cm 2. ...cm 3. ...cm	...cm	...cm	...cm

Calculations--

Set I

$$\frac{P_1}{P_2} = \frac{d_1^2}{d_2^2} = \ldots\ldots$$

[**Note**—In this way, calculate the value of P_1/P_2 for each set separately]

$$\therefore \quad \text{Mean} \quad \frac{P_1}{P_2} = \ldots\ldots\ldots\ldots$$

Result—The ratio of the illuminating powers of the two given sources of light*= ....

[Actual value of P_1/P_2 = ...

$\therefore$ Error = ...%]

[**Note**—If a direct-reading instrument (*e.g. a foot-candle meter*) be available in the laboratory, the illuminating powers can be directly ascertained with its help and thus the above result can be

easily verified. The working of a foot-candle meter is simple and depends on photo-electric phenomenon. Just as in a photo-electric cell, light from a lamp falls on the sensitive surface (which may consist of a coating of siiver on a layer of copper oxide, or a coating of gold on a layer of selenium), electrons are liberated as usual and a photo-electric current begins to flow which deflects a needle on a scale graduated directly to read foot-fcandle For this purpope, the lamp is held vertically above the sensitive surface and its distancc is so adjusted that the filament of the lamp is situated at a distanee of one foot above this surface.]

Precautions and Sources of Error—

(1) Set the photometer-head in such a way that the screne provided in the photometer-head lies normally to the line joining the two lamps.

(2) The height of the filaments should be carefully adjusted so that the light falls normally on the screen.

(3) While adjusting the position of the lamp to attain the condition of equality of illumination of the field of view, the readings for the position of the lamp should be taken both when moving it *towards* the photometer as well as when moving it *away* from it. These readings should be repeated.

(4) In this experiment the chief source of error lies in the fact that the distances of the sources of light from the surface of the screen cannot be ascertained with sufficient accuracy. Moreover, the screen provided in the photometer has sufficient thickness so the distances of the lamps should actually be measured upto the nearest surface only.*

(5) If the two surfaces of the screen have unequal reflecting power, an error shall creep in due to this discrepancy.†

ADDITIONAL EXPERIMENT

No.—39 (a)

Object—To study the variation of the illuminating power of an electric glow lamp with the voltage fed at the ends of its filament.

* The latter error can be eliminated by subtracting half the thickness of the screen from the observed values of d_1 and d_2.

† This error can be removed by rotating the photometer-head through 180° with respect to the sources of light so that the surface of the screen interchange their positions to receive the light from the lamps.

For this experiment, a compact apparatus containing an *auto-transformer* and a voltmete should preferably be employed. As is well-known that whenever the voltage in an alternating current circuit is to be adjusted, a step-up or step-down transformer is employed. But when the ratio of transformation is not large, an auto-transformer is employed, which results in efficiency of operation and economy of construction. The entire coil AB (Fig.–93) is the primary and the part AC is the secondary, the potential difference at the ends of which is fed to the electric lamp, its value being given by the voltmeter connected as shown. The potential difference can be varied by shifting the position of C with the help of a knob provided on the body of the apparatus.

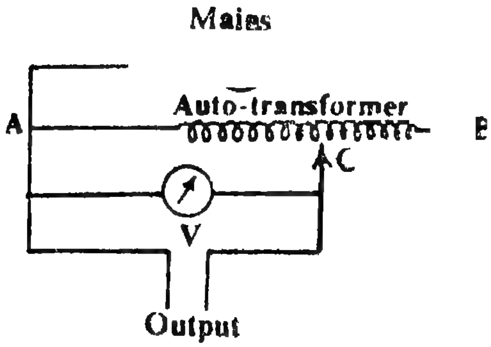

Fig.–93
Connections with an auto-transformer.

Now, take a lamp of known candle-power (or determine its candle-power with a foot-candle meter) and mount it on one eide of the photometer-head. Connect it directly to the mains. Adjust the other lamp under test on the other side and connect it to the output of the auto-transformer set. Match the intensity of illumination of the two portions of the field of view by keeping the distance of the experimental lamp constant and by moving the other lamp (of known candle-power). Note the voltage applied at the ends of the filament and the position of the lamps and the photometer-head. From these readings, calculate the candle-power of the experimental lamp.

Now, change the potential difference at the ends of the filament and attain the equality* of illumination by the displacing the other lamp as before. Calculate the candle-power of the lamp for this altered value of the potential difference. In this manner, go on changing the voltage applied to the lamp in equal steps of, say, 10 volts and go on calculating the candle-power of the lamp in each case.

Plot a graph between the applied voltage and the corresponding candle-power of the lamp. The graph will be a straight line,

As the voltage fed on the lamp are reduced, the colour of light emitted by the lamp undergoes a distinctive change, hence, exact coincidence of luminosity of the two portions becomes a bit tedious. However, with a little practice on the part of the experimenter, the matching of brightness can still be effected with sufficient accuracy.

proving thereby that the candle-power of a lamp is directly proportional to the potential difference applied at the ends of its filament.

EXPERIMENT–40

Object—To compare the illuminating powers of two sources of light and to varify the inverse square law by using a photo-voltaic cell.

Apparatus Required—An optical bench, a photo-voltaic cell, two sources of light, a suspended-coil galvanometer of low resistance, and a commutator.

Description of the Apparatus—When light falls on a metallic surface, electrons are emitted from it. The electrons so emitted are known as *photo-electrons* and this phenomenon is known as *photo-electricity*. Instruments which make use of this phenomenon are known as *photo-electric cells*, consist of an emitter of electrons and a collector which collects these electrons. In such a cell a positive potential is applied to the collector which attracts the electrons and therefore a battery is necessary. *In the case of a photo-voltaic cell no auxiliary battery is needed.* The electrons ejected by light, themselves produce a potential difference between the two plates. This potential difference is capable of producing the current in the external circuit.

The most common type of photo-voltaicc ell is the *seleuium cell*, which is schematically shown in Fig.–94 (a).

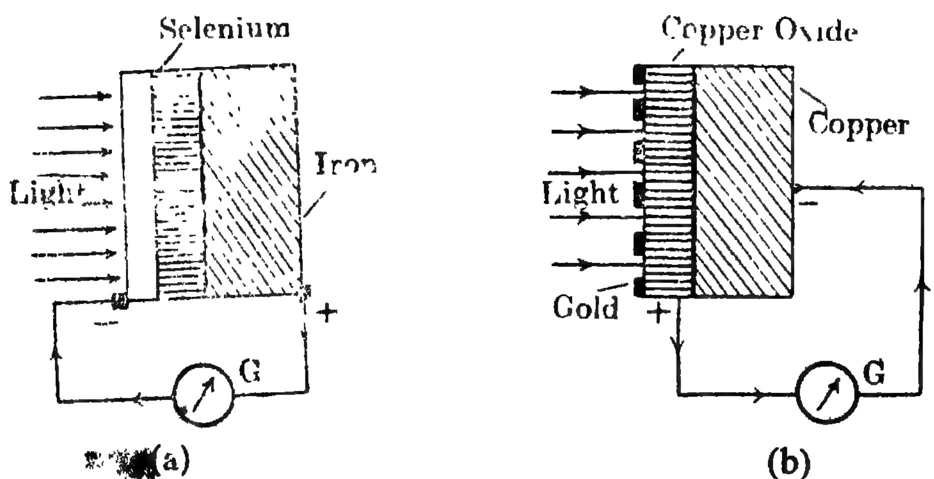

Fig.–94. Photo-voltaic Cells.

It consists of a thin film of selenium coated on an iron plate. Under the influence of the light falling on the cell, it generates an e. m. f. due to difference in electron densities between the iron base and the selenium. The e. m. f. so developed causes a current to flow in the external circuit and the current (or the deflection produced in the galvanometer) is proportional to the intensity

of the light falling on the sensitive surface provided the external circuit has a low resistance.

Fig.–94 (b) represents another type of photo-voltaic cell, which consists of a copper plate and a film of cuprons oxide formed on it. The cuprous oxide is coated with a very thin layer of gold or silver.

Formula Employed—If the two sources of light of illuminating powers P_1 and P_2 produce the same deflection θ in the galvanometer when they placed respectively at distances d_1 and d_2 from the photo-electric cell, then

$$\frac{P_1}{P_2} = \frac{d_1^2}{d_2^2}$$

If a graph is plotted between $1/d^2$ and θ, we get a straight line which proves the validity of the inverse square law.

PRINCIPLE AND THEORY OF THE EXPERIMENT

When the resistance of the external circuit is low, the current produced by the photo-electric cell is proportional to the intensity of illumination. Hence, if P_1 be the illuminating power of one source, and if it be placed at a distance d_1 from the photo-electric cell, the intensity of illumination produced at the sensitive surface of the cell is

$$I = P_1/d_1^2 \quad \ldots \quad \ldots (1)$$

This illumination causes a current to flow through the galvanometer which shows a steady deflection. Now, because the deflection of the galvanometer is proportional to the current flowing through it, this deflection is also proportional to the illumination at the cell. Thus, if θ be the constant deflection, we have

$$I = P_1/d_1^2 = k\,\theta \quad \ldots \quad \ldots (2)$$

where k is the current constant of the galvonometer.

Similarly, if the second source of light placed at a distance d_2 from the photo-electric cell produces the same deflection θ, it means that the intensity of illumination at the cell is the same as before, hence

$$I = P_2\, d_2^2 = k\,\theta \quad \ldots \quad \ldots (3)$$

thus, from equations (2) and (3), we have

$$\frac{P_1}{P_2} = \frac{d_1^2}{d_2^2} \quad \ldots \quad \ldots (4)$$

Thus, if we draw a graph between $1/d^2$ and θ, we shall get a straight line. This proves the validity of the inverse square law in optics.

Methods

(i) Adjust the galvanometer so that its coil is quite free to move in between the pole-pieces of the magnet. Try to get the spot of light on the zero mark of the scale of the galvanometer. Connect the photo-voltaic cell (P. V.) and the galvanometer (G) to the commutator (C) as shown in the figure.

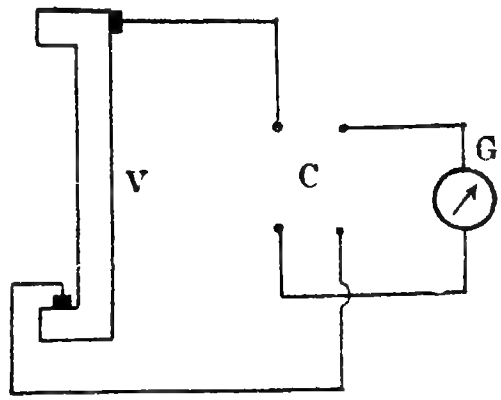

Fig.–95
Connections with a photo-voltaic cell.

(ii) Mount the cell on one of the uprights of the optical bench and note its position on the scale of the bench. This position of the cell should not be disturbed during the course of the experiment. Now place one of the lamps in the lamp-housing upright of the bench and switch on the light. Permit the light to fall on the photo-cell by raising the lid of its window and observe the steady deflection produced in the galvanometer. Obtain a good deflection (say, 5 cm) of the spot of light on the galvanometer scale by adjusting the distance of the lamp from the cell. Reverse the current in the galvanometer with the help of the commutator and observe the deflection again. If the two deflections of the spot of light are equal, it means that the scale is normal to the pencil of light replaced by the mirror of the galvonometer.

(iii) Now, place one of the lamps at a distance d_1 from the photo-cell, and note the deflections of the spot of light both for the direct and reverse currents. Calculate the deflection θ. The difference of the readings of the upright of the lamp and the upright of the photo-cell gives the distance d_1.

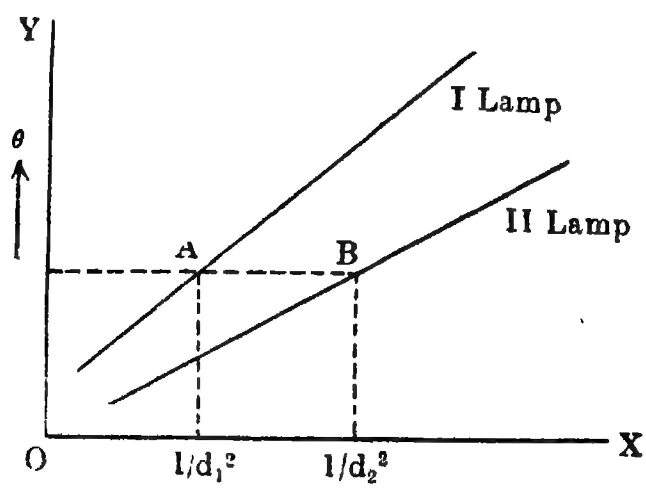

Fig.–96
$1/d^2$—θ graph with a photo-cell.

(iv) Now, change the distance of the lamp in suitable steps (of say, 5 cm) from the photo-cell and record the mean deflection in each case.

(v) Replace this lamp by the other given lamp and repeat the observations with this lamp in the same manner as given above.

(vi) Now, plot graphs between $1/d^2$ (represented along the x-axis) and θ (represented along the y-axis) for both lamps as shown in Fig.-96. From the graphs find the values of $1/d_1^2$ and $1/d_2^2$ for the same values of the deflection θ. With the help of these values, calculate the ratio of the illuminating powers of the two lamps. In this ways get several values of P_1/P_2 from the graph and calculate the mean value of this ratio.

Observations

(i) Position of the photo-voltaic cell = cm

(ii) Initial position of the spot of light on the galvanometer scale = θ cm

[A] *Readings for the determination of* θ *with the first lamp.*

S. No.	Position of the lamp	Distance of the lamp from the cell (d_1)	Deflection of the spot of light		Mean deflection (θ)
			with direct current	with current reversed	
1					
2					
......					
......					

[**Note**—Make a similar table for the other lamp.]

Calculations

[**Note**—With the help of the graph calculate the values of $1/d^2$ for the two lamps for different values of θ and tabulate the values as given below.]

Deflection of the galvanometer (θ)	For the first lamp ($1/d_1^2$)	For the second lamp ($1/d_2^2$)	$P_1/P_2=\frac{1/d_1^2}{1/d_2^2}$
		Mean	

Result—(i) The ratio of the illuminating powers of the two given lamps is ... .

(ii) From the straight line graph it follows that the deflection θ is proportional to $1/d^2$. Now, because the deflection is proportional to the intensity of illumination I, we have I proportional to $1/d^2$. This verifies the inverse square law.

Precautions and Sources of Error

(1) The galvanometer should be properly levelled so that its coil is *quite free* to move in the clearance space in between the pole-pieces. The scale should be adjusted *normal* to the reflected beam of light by noting the equality of the two deflections obtained with the help of the commutator.

(2) Since the photo-electric current is proportional to the illuminating power of the source only when the external circuit has a low resistance, *hence the galvanometer used in this experiment should be a low resistance one.*

(3) The heights of the lamps should be adjusted equal to the height of the photo-cell.

(4) The lid on the window of the photo-cell should be raised only when readings are to be taken with it.

(5) If the galvanometer used in this experiment is not a low resistance one,, then the intensity of illumination on the sensitive surface of the photo-cell will not be proportion to the output current. Thus, the accuracy of the result shall suffer due to this cause.

(6) As the sensitive surface of the photo-cell as also the filament of the bulb cannot be directly reached, the bench error cannot be determined in this experiment. Thus, the values of d_1 and d_2 cannot be accurately determined. Due to this cause, sufficient error can be introduced in the result.

TABLES OF

PHYSICAL CONSTANTS

AND SOME

MATHEMATICAL FUNCTIONS

GENERAL PROPERTIES OF MATTER

Table 1—*Some Useful Formulae*

Area (sq. cm.)	(Volume (c. c.)
Circle : radius r $= \pi r^2$	*Cylinder* : height h, radius r $= \pi r^2 h$
Ellipse : axes 2a and 2b $= \pi ab$	
Curved surface of a *Cylinder* : height h, radius r $= 2\pi rh$	*Cone* : height h, base radius r $= \frac{1}{3}\pi r^2 h$
Curved surface of a *Cone* : height h, base radius r $= \pi r\sqrt{r^2 + h^2}$	*Sphere* : radius r $= \frac{4}{3}\pi r^3$
Curved surface of a *Sphere* : radius r $= 4\pi r^2$	*Ellipsoid* : axes 2a, 2b, and 2c $= \frac{4}{3}\pi abc$

Table 2—*Moment of Inertia* (gm-cm^2)

Body	Axis of Rotation	Formulae
Disc : radius r	Through the centre and perpendicular to its surface	$\frac{1}{2} M r^2$
Ring : radii r_1, r_2	,, ,,	$\frac{1}{2} M (r_2^2 + r_1^2)$
Cylinder : length l, radius r	(i) About its own axis (ii) About an axis through its centre and perpendicular to its axis	(i) $\frac{1}{2} M r^2$ (ii) $M \left[\frac{l^2}{12} + \frac{r^2}{4} \right]$
Cone : base radius r	About its own axis	$\frac{3}{10} M r^2$
Hollow Sphere : outer radius r_1 inner radius r_2	About any diameter	$\frac{2}{5} M \left[\frac{r_1^5 - r_2^5}{r_1^3 - r_2^3} \right]$
Sphere : radius r	About any diameter	$\frac{2}{5} M r^2$

Note—In these formulae M is the mass of the body.

Table 3—*Acceleration due to gravity* (cm/sec²)

Place	g	Place	g
Pole	**982·22**	Delhi	979·15
		Gorakhpur	979·05
Equator	**978·03**	Gwalior	978·97
		Indore	978·60
Agra	979·06	Jaipur	978·52
Ajmer	978·90	Kanpur	979·01
Aligarh	978·08	Lucknow	979·00
Allahabad	978·95	Madras	978·28
Bombay	978·65	Meerut	979·15
Calcutta	978·78	Nagpur	978·54
Dehra Dun	979·07	Varanasi	278·99

g can be calculated at any other place with the help of the formula : $g = 980{\cdot}616 - 2{\cdot}593 \cos 2\lambda + {\cdot}0068 \cos^2 2\lambda - {\cdot}0003 h$, where λ is the latitude of the place and h is the height in metres above mean sea level.

Table 4—*Elastic Constants*

Substance	Young's modulus (Y) (dynes/cm²	Rigidity modulus (n) (dynes/cm²	Poisson's ratio (α)	Breaking stress dynes/cm²
	$\times 10^{11}$	$\times 10^{11}$		$\times 10^{8}$
Aluminium	6·9– 7·2	2·4–2·7	·33–·35	17– 20
Brass	9·0–10·2	3·4–2·3	·39–·40	31– 39
Copper	11·0–12·9	3·4–4·6	·25–·35	28– 46
Glass (crown)	6·0- 7·8	2 6–3·2	·20–·27	3– 9
Glass (flint)	5·0– 6·0	2·0–2·5	·22–·26	...
India rubber	·048–·052	·00016	·46–·49	...
Iron (wrought)	19·0–22·0	7·7–8·3	·27–·29	46– 62
Iron (cast)	10·0–13·0	3·5–5·3	·23–·31	8– 23
Phosphor bronze	11·9–12·1	4·3–4·5	·37–·39	69–108
Quartz fibre	5·2– 5·4	2·9–3·1	·27–·29	95–106
Steel (cast)	19·0–21·0	7·4–7·6	...	...
Steel (mild)	21·0–23·0	8·0–8·9	·25–·31	100–120
Zinc	8·0–11·0	3·9–3·8	·20–·30	11– 15

Table 5—*Density of Common Substances* (gm./c. c.)

Substance	Density	Substance	Density	Substance	Density
Solids		Silver	10·5	Methylated	
		Steel	7·8	spirit	0·83
Aluminium	2·7	Zinc	7·1	Turpentine	
Brass	8·6			Xylol	0·87
Copper	8·89	**Liquids**			
Glass (crown)	2·6			**Gases**	
Glass (flint)	4·0	Alcohol	0·80		
Gold	19·3	Benzene	0 88	Air	·00129
Iron (cast)	7·5	Ether	0 74	Carbon	
Iron		Glycerine	1·26	di-oxide	·00198
(wrought)	7·9	Lubricating		Helium	·000179
Lead	11 34	oil	0·91	Hydrogen	·000090
Platinum	21·45	Mercury	13·60	Steam	
				(100°C)	·00061

Table 6—*Densities of Water and Mercury*

Temp. °C	Water	Mercury	Temp. °C	Water	Mercury	Temp. °C	Water	Mercury
0	·9999	13·5955	40	·9922	13·4973	90	·9653	13·3759
4	1·0000	·5836	50	·9881	·4729	100	·9584	·3518
10	·9997	·5708	60	·9832	·4486	150	·9170	·2330
20	·9982	·5462	70	·9778	·4243	200	·8630	·1150
30	·9957	·5217	80	·9718	·4001	300	·7000	12·8810

Table 7—*Surface Tension of Liquids* (dynes/cm.)

Substance	Surface tension	Substance	Surface tension
Alcohol (ethyl)	22·0	Glycerine	63·5
Alcohol (methyl)	23	Mercury*	465
Benzene	29·2	Olive oil	32
Chloroform	27·2	Soap solution	20—40
Ether	16·5	Turpentine	27·3

* Formula for temperature variation is : $T_t = T_o - 0{\cdot}02t$. Values are liable to vary with the nature of the surface.

Table 8—*Surface Tension of Water* (dynes/cm.)

Temp. °C	Surface Tension	Temp. °C	Surface Tension	Temp. °C	Surface Tension
0	75·0	30	70·6	70	63·8
10	73·5	40	68·9	80	62·0
15	72·8	50	67·3	90	60·2
20	72·1	60	65·6	100	58·2

Table 9—*Viscocity of Liquids* (in poise)

Substance	Viscosity	Substance	Viscosity
Alcohol (ethyl)	·0119	Ether	·00234
Alcohol (methyl)	·00591	Glycerine	8·5
Benzene	·00649	Mercury	·0156
Castor oil	9·86	Olive oil	·98
Chloroform	·00564	Turpentine	·0149

Table 10—*Viscosity of Water* (in poise)

Temp.	Viscosity	Temp.	Viscosity	Temp.	Viscosity
0°C	·01793	30°C	·00800	70	·00406
10	·01311	40	·00657	80	·00356
15	·01142	50	·00550	90	·00316
20	·01006	60	·00469	100	·00284
25	·00893	...	...	...	...

SOUND

Table 11—*Velocity of Sound* (metres/sec)

Substance	Velocity	Substance	Velocity	Substance	Velocity
Solids (20°C)		**Liquids** (20°C)			
Aluminium	5100	Alcohol	1275	Carbon di-oxide	259
Brass	3400	Mercury	1407	Hydrogen	1262
Copper	3560	Turpentine	1326	Nitrogen	338
Glass	5000	Water	1447	Oxygen	316
Iron	5130	**Gases** (0°C)		Sulphur di-oxide	209
Steel	4990	Air	331·1	Water vapour	401

The velocity of sound in gases increases at the following rates : Air ·16, Carbon di-oxide ·47, Nitrogen ·61, and Oxygen ·60 metres per sec. per 1°C rise of temperature.

The velocity of sound in water increases by 3·3 metres per sec. per 1°C rise of temperature.

LIGHT

Table 12—*Refractive Indices* (μ) *and Dispersive Powers* (ω)
(λ = 5893 A. U., Temp. = 15°C)

Substance	μ_D	ω	Substance	μ_D	ω
Isotropic solids			**Liquids**		
Canada balsam ...	1·530	...	Alcohol (ethyl) ...	1·362	·017
Diamond ..	2·417	...	Alcohol (methyl)..	1·329	·016
Ice	1·310	...	Benzene ...	1·501	·033
Quartz (fused) ...	1·458	·015	Chloroform ...	1·446	·020
Sylvine ...	1·490	·023	Ether ...	1·354	·017
			Glycerine ...	1·474	...
Glasses			Turpentine ...	1·470	·021
			Water	1·333	·018
Crown ...	1·500	·015			
Dense crown ...	1·620	·018	**Uniaxial crystals**		
Flint ...	1·560	·020	Calcite ord.	1·658	·020
Dense flint ...	1·620	·027	Calcite ext. ord.	1·486	·013
Extra dense flint ..	1·650	·030	Quartz ord.	1·544	·014
Very dense flint...	1·720	·033	Quartz ext. ord.	1·553	·015

Refractive index of any gas can be taken approximately as unity.

Refractive indices of aqueous solutions are dependent on their concentrations and are generally between 1·33 and 1·38.

Refractive indices of glycerine-water mixtures for different percentages by weight of glycerine :

25% — 1·364 ; 50% — 1·399 ; 75% — 1·436.

Table 13—*Emission Spectra* (in A. U.)
[The visible spectrum colours are indicated—r, o, y, g, b, v.]

Hydrogen	6678 r	6152 o
3970 v	7065 r	6232 o
4102 (δ) v	**Mercury**	**Neon**
4340 (γ) b	4047 v	5765 y
(F) **4861** (β) gb	4078 v	**5853** y
(C) **6563** (α) r	4358 v	5882 o
Helium	4916 bg	6507 r
3889 v	4960 g	7245 r
4026 v	**5461** g	**Sodium**
4471 b	**5770** y	(D_2) **5890** o
(D_3) **5876** y	**5791** y	(D_1) **5896** o

Table 14—*Electro-magnetic Spectrum*

Wireless waves	... ...		5 metres and above
Infra-red ...	3×10^{-2} cm.	to	$7{\cdot}5 \times 10^{-5}$ cm.
Visible red ...	$7{\cdot}5 \times 10^{-5}$ cm.	to	$6{\cdot}5 \times 10^{-5}$ cm.
„ orange ...	$6{\cdot}5 \times 10^{-5}$ cm.	to	$5{\cdot}9 \times 10^{-5}$ cm.
„ yellow ...	$5{\cdot}9 \times 10^{-5}$ cm.	to	$5{\cdot}3 \times 10^{-5}$ cm.
„ green ...	$5{\cdot}3 \times 10^{-5}$ cm.	to	$4{\cdot}9 \times 10^{-5}$ cm.
„ blue ...	$4{\cdot}9 \times 10^{-5}$ cm.	to	$4{\cdot}2 \times 10^{-5}$ cm.
„ violet ...	$4{\cdot}2 \times 10^{-5}$ cm.	to	$3{\cdot}9 \times 10^{-5}$ cm.
Ultra-violet ...	$3{\cdot}9 \times 10^{-5}$ cm.	to	$1{\cdot}8 \times 10^{-5}$ cm.
Soft X-rays ...	$2{\cdot}0 \times 10^{-7}$ cm.	to	$1{\cdot}0 \times 10^{-8}$ cm.
Hard X-rays ...	$1{\cdot}0 \times 10^{-8}$ cm.	to	$1{\cdot}0 \times 10^{-9}$ cm.
Gamma rays ...	$5{\cdot}0 \times 10^{-9}$ cm.	to	$5{\cdot}0 \times 10^{-10}$ cm.
Cosmic rays ...	$5{\cdot}0 \times 10^{-12}$ cm.	to	

Table 15—*Rotatory Powers*

Rotation by quartz for D line = 21·72° per mm. thickness

Specific rotations of solutions and pure liquids

Optically active substance	Solvent	Specific rotation
Cane-sugar	Water	+ 66·5°
Glucose	Water	+ 52°
Fructose	Water	− 91°
Invert sugar	Water	− 19·5°
Tartaric Acid	Water	+ 8·9°
Camphor	Alcohol	+ 41°
Turpentine	Pure	− 37°
Nicotine	Pure	− 162°

The rotation is called *positive* or *right-handed* (*dextro*) if the plane of polarisation appears to be rotated in a clockwise direction when looking through the liquid *towards* the source of light. The contrary rotation is called *negative* or *left-handed* (*laevo*).

LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170						5	9	13	17	21	26	30	34	38
						0212	0253	0294	0334	0374	4	8	12	16	20	24	28	32	36
11	0414	0453	0492	0531	0569						4	8	12	16	20	23	27	31	35
						0607	0645	0682	0719	0755	4	7	11	15	18	22	26	29	33
12	0792	0828	0864	0899	0934						3	7	11	14	18	21	25	28	32
						0969	1004	1038	1072	1106	3	7	10	14	17	20	24	27	31
13	1139	1173	1206	1239	1271						3	6	10	13	16	19	23	26	29
						1303	1335	1367	1399	1430	3	7	10	13	16	19	22	25	29
14	1461	1492	1523	1553	1584						3	6	9	12	15	19	22	25	28
						1614	1644	1673	1703	1732	3	6	9	12	14	17	20	23	26
15	1761	1790	1818	1847	1875						3	6	9	11	14	17	20	23	26
						1903	1931	1959	1987	2014	3	6	8	11	14	17	19	22	25
16	2041	2068	2095	2122	2148						3	6	8	11	14	16	19	22	24
						2175	2201	2227	2253	2279	3	5	8	10	13	16	18	21	23
17	2304	2330	2355	2380	2405						3	5	8	10	13	15	18	20	23
						2430	2455	2480	2504	2529	3	5	8	10	12	15	17	20	22
18	2553	2577	2601	2625	2648						2	5	7	9	12	14	17	19	21
						2672	2695	2718	2742	2765	2	4	7	9	11	14	16	18	21
19	2788	2810	2833	2856	2878						2	4	7	9	11	13	16	18	20
						2900	2923	2945	2967	2989	2	4	6	8	11	13	15	17	19
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8

LOGARITHMS

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50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

ANTILOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
·01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
·02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
·03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
·04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
·05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
·06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
·07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
·08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
·09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
·10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
·11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
·12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
·13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
·14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
·15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
·16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
·17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
·18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
·19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
·20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
·21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
·22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
·23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
·24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
·25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
·26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
·27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
·28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
·29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
·30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
·31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
·32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
·33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
·34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
·35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
·36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
·37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
·38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
·39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
·40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
·41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
·42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
·43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
·44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
·45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
·46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
·47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
·48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
·49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6

ANTILOGARITHMS

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·50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
·51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
·52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
·53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
·54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
·55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
·56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
·57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
·58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
·59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
·60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
·61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
·62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
·63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
·64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
·65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
·66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
·67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
·68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
·69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
·70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
·71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
·72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
·73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
·74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
·75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
·76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
·77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
·78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
·79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
·80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
·81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
·82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
·83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
·84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
·85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
·86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
·87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
·88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
·89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
·90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
·91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
·92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
·93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
·94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
·95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
·96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
·97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
·98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
·99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

NATURAL SINES

Degrees	0′ 0°·0	6′ 0°·1	12′ 0°·2	18 0°·3	24′ 0°·4	30′ 0°·5	36′ 0°·6	42′ 0°·7	48′ 0°·8	54′ 0°·9	Mean Differences 1	2	3	4	5
0	0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	15
1	0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15
2	0349	0367	0384	0402	0419	0437	0454	0472	0489	0507	3	6	9	12	15
3	0524	0542	0559	0577	0594	0612	0629	0647	0664	0682	3	6	9	12	15
4	0699	0717	0734	0752	0769	0787	0805	0822	0840	0857	3	6	9	12	15
5	0875	0892	0910	0928	0945	0963	0981	0998	1016	1033	3	6	9	12	15
6	1051	1069	1086	1104	1122	1139	1157	1175	1192	1210	3	6	9	12	15
7	1228	1246	1263	1281	1299	1317	1334	1352	1370	1388	3	6	9	12	15
8	1405	1423	1441	1459	1477	1495	1512	1530	1548	1566	3	6	9	12	15
9	1584	1602	1620	1638	1655	1673	1691	1709	1727	1745	3	6	9	12	15
10	1763	1781	1799	1817	1835	1853	1871	1890	1908	1926	3	6	9	12	15
11	1944	1962	1980	1998	2016	2035	2053	2071	2089	2107	3	6	9	12	15
12	2126	2144	2162	2180	2199	2217	2235	2254	2272	2290	3	6	9	12	15
13	2309	2327	2345	2364	2382	2401	2419	2438	2456	2475	3	6	9	12	15
14	2493	2512	2530	2549	2568	2586	2605	2623	2642	2661	3	6	9	12	16
15	2679	2698	2717	2736	2754	2773	2792	2811	2830	2849	3	6	9	13	16
16	2867	2886	2905	2924	2943	2962	2981	3000	3019	3038	3	6	9	13	16
17	3057	3076	3096	3115	3134	3153	3172	3191	3211	3230	3	6	10	13	16
18	3249	3269	3288	3307	3327	3346	3365	3385	3404	3424	3	6	10	13	16
19	3443	3463	3482	3502	3522	3541	3561	3581	3600	3620	3	7	10	13	16
20	3640	3659	3679	3699	3719	3739	3759	3779	3799	3819	3	7	10	13	17
21	3839	3859	3879	3899	3919	3939	3959	3979	4000	4020	3	7	10	13	17
22	4040	4061	4081	4101	4122	4142	4163	4183	4204	4224	3	7	10	14	17
23	4245	4265	4286	4307	4327	4348	4369	4390	4411	4431	3	7	10	14	17
24	4452	4473	4494	4515	4536	4557	4578	4599	4621	4642	4	7	11	14	18
25	4663	4684	4706	4727	4748	4770	4791	4813	4834	4856	4	7	11	14	18
26	4877	4899	4921	4942	4964	4986	5008	5029	5051	5073	4	7	11	15	18
27	5095	5117	5139	5161	5184	5206	5228	5250	5272	5295	4	7	11	15	18
28	5317	5340	5362	5384	5407	5430	5452	5475	5498	5520	4	8	11	15	19
29	5543	5566	5589	5612	5635	5658	5681	5704	5727	5750	4	8	12	15	19
30	5774	5797	5820	5844	5867	5890	5914	5938	5961	5985	4	8	12	16	20
31	6009	6032	6056	6080	6104	6128	6152	6176	6200	6224	4	8	12	16	20
32	6249	6273	6297	6322	6346	6371	6395	6420	6445	6469	4	8	12	16	20
33	6494	6519	6544	6569	6594	6619	6644	6669	6694	6720	4	8	13	17	21
34	6745	6771	6796	6822	6847	6873	6899	6924	6950	6976	4	9	13	17	21
35	7002	7028	7054	7080	7107	7133	7159	7186	7212	7239	4	9	13	18	22
36	7265	7292	7319	7346	7373	7400	7427	7454	7481	7508	5	9	14	18	23
37	7536	7563	7590	7618	7646	7673	7701	7729	7757	7785	5	9	14	18	23
38	7813	7841	7869	7898	7926	7954	7983	8012	8040	8069	5	9	14	19	24
39	8098	8127	8156	8185	8214	8243	8273	8302	8332	8361	5	10	15	20	24
40	8391	8421	8451	8481	8511	8541	8571	8601	8632	8662	5	10	15	20	25
41	8693	8724	8754	8785	8816	8847	8878	8910	8941	8972	5	10	16	21	26
42	9004	9036	9067	9099	9131	9163	9195	9228	9260	9293	5	11	16	21	27
43	9325	9358	9391	9424	9457	9490	9523	9556	9590	9623	6	11	17	22	28
44	9657	9691	9725	9759	9793	9827	9861	9896	9930	9965	6	11	17	23	29

NATURAL SINES

Degrees	0′ 0°·0	6′ 0°·1	12′ 0°·2	18′ 0°·3	24′ 0°·4	30′ 0°·5	36′ 0°·6	42′ 0°·7	48′ 0°·8	54′ 0°·9	Mean Differences 1	2	3	4	5
45	7071	7083	7096	7108	7120	7133	7145	7157	7169	7181	2	4	6	8	10
46	7193	7206	7218	7230	7242	7254	7266	7278	7290	7302	2	4	6	8	10
47	7314	7325	7337	7349	7361	7373	7385	7396	7408	7420	2	4	6	8	10
48	7431	7443	7455	7466	7478	7490	7501	7513	7524	7536	2	4	6	8	10
49	7547	7558	7570	7581	7593	7604	7615	7627	7638	7649	2	4	6	8	9
50	7660	7672	7683	7694	7705	7716	7727	7738	7749	7760	2	4	6	7	9
51	7771	7782	7793	7804	7815	7826	7837	7848	7859	7869	2	4	5	7	9
52	7880	7891	7902	7912	7923	7934	7944	7955	7965	7976	2	4	5	7	9
53	7986	7997	8007	8018	8028	8039	8049	8059	8070	8080	2	3	5	7	9
54	8090	8100	8111	8121	8131	8141	8151	8161	8171	8181	2	3	5	7	8
55	8192	8202	8211	8221	8231	8241	8251	8261	8271	8281	2	3	5	7	8
56	8290	8300	8310	8320	8329	8339	8348	8358	8368	8377	2	3	5	6	8
57	8387	8396	8406	8415	8425	8434	8443	8453	8462	8471	2	3	5	6	8
58	8480	8490	8499	8508	8517	8526	8536	8545	8554	8563	2	3	5	6	8
59	8572	8581	8590	8599	8607	8616	8625	8634	8643	8652	1	3	4	6	7
60	8660	8669	8678	8686	8695	8704	8712	8721	8729	8738	1	3	4	6	7
61	8746	8755	8763	8771	8780	8788	8796	8805	8813	8821	1	3	4	6	7
62	8829	8838	8846	8854	8862	8870	8878	8886	8894	8902	1	3	4	5	7
63	8910	8918	8926	8934	8942	8949	8957	8965	8973	8980	1	3	4	5	6
64	8988	8996	9003	9011	9018	9026	9033	9041	9048	9056	1	3	4	5	6
65	9063	9070	9078	9085	9092	9100	9107	9114	9121	9128	1	2	4	5	6
66	9135	9143	9150	9157	9164	9171	9178	9184	9191	9198	1	2	3	5	6
67	9205	9212	9219	9225	9232	9239	9245	9252	9259	9265	1	2	3	4	6
68	9272	9278	9285	9291	9298	9304	9311	9317	9323	9330	1	2	3	4	5
69	9336	9342	9348	9354	9361	9367	9373	9379	9385	9391	1	2	3	4	5
70	9397	9403	9409	9415	9421	9426	9432	9438	9444	9449	1	2	3	4	5
71	9455	9461	9466	9472	9478	9483	9489	9494	9500	9505	1	2	3	4	5
72	9511	9516	9521	9527	9532	9537	9542	9548	9553	9558	1	2	3	3	4
73	9563	9568	9573	9578	9583	9588	9593	9598	9603	9608	1	2	2	3	4
74	9613	9617	9622	9627	9632	9636	9641	9646	9650	9655	1	2	2	3	4
75	9659	9664	9668	9673	9677	9681	9686	9690	9694	9699	1	1	2	3	4
76	9703	9707	9711	9715	9720	9724	9728	9732	9736	9740	1	1	2	3	3
77	9744	9748	9751	9755	9759	9763	9767	9770	9774	9778	1	1	2	3	3
78	9781	9785	9789	9792	9796	9799	9803	9806	9810	9813	1	1	2	2	3
79	9816	9820	9823	9826	9829	9833	9836	9839	9842	9845	1	1	2	2	3
80	9848	9851	9854	9857	9860	9863	9866	9869	9871	9874	0	1	1	2	2
81	9877	9880	9882	9885	9888	9890	9893	9895	9898	9900	0	1	1	2	2
82	9903	9905	9907	9910	9912	9914	9917	9919	9921	9923	0	1	1	2	2
83	9925	9928	9930	9932	9934	9936	9938	9940	9942	9943	0	1	1	1	2
84	9945	9947	9949	9951	9952	9954	9956	9957	9959	9960	0	1	1	1	2
85	9962	9963	9965	9966	9968	9969	9971	9972	9973	9974	0	0	1	1	1
86	9976	9977	9978	9979	9980	9981	9982	9983	9984	9985	0	0	1	1	1
87	9986	9987	9988	9989	9990	9990	9991	9992	9993	9993	0	0	0	1	1
88	9994	9995	9995	9996	9996	9997	9997	9997	9998	9998	0	0	0	0	0
89	9998	9999	9999	9999	9999	1·000	1·000	1·000	1·000	1·000	0	0	0	0	0
90	1·000														

NATURAL COSINES

Degrees.	0′ 0°·0	6′ 0°·1	12′ 0°·2	18′ 0°·3	24′ 0°·4	30′ 0°·5	36′ 0°·6	42′ 0°·7	48′ 0°·8	54′ 0°·9	Mean Difference. 1	2	3	4	5
0	1·000	1·000	1·000	1·000	1·000	1·000	·9999	9999	9999	9999	0	0	0	0	0
1	9998	9998	9998	9997	9997	9997	9996	9996	9995	9995	0	0	0	0	0
2	9994	9993	9993	9992	9991	9990	9990	9989	9988	9987	0	0	0	1	1
3	9986	9985	9984	9983	9982	9981	9980	9979	9978	9977	0	0	1	1	1
4	9976	9974	9973	9972	9971	9969	9968	9966	9965	9963	0	0	1	1	1
5	9962	9960	9959	9957	9956	9954	9952	9951	9949	9947	0	1	1	1	2
6	9945	9943	9942	9940	9938	9936	9934	9932	9930	9928	0	1	1	1	2
7	9925	9923	9921	9919	9917	9914	9912	9910	9907	9905	0	1	1	2	2
8	9903	9900	9898	9895	9893	9890	9888	9885	9882	9880	0	1	1	2	2
9	9877	9874	9871	9869	9866	9863	9860	9857	9854	9851	0	1	1	2	2
10	9848	9845	9842	9839	9836	9833	9829	9826	9823	9820	1	1	2	2	3
11	9816	9813	9810	9806	9803	9799	9796	9792	9789	9785	1	1	2	2	3
12	9781	9778	9774	9770	9767	9763	9759	9755	9751	9748	1	1	2	3	3
13	9744	9740	9736	9732	9728	9724	9720	9715	9711	9707	1	1	2	3	3
14	9703	9699	9694	9690	9686	9681	9677	9673	9668	9664	1	1	2	3	4
15	9659	9655	9650	9646	9641	9636	9632	9627	9622	9617	1	2	2	3	4
16	9613	9608	9603	9598	9593	9588	9583	9578	9573	9568	1	2	2	3	4
17	9563	9558	9553	9548	9542	9537	9532	9527	9521	9516	1	2	3	3	4
18	9511	9505	9500	9494	9489	9483	9478	9472	9466	9461	1	2	3	4	5
19	9455	9449	9444	9438	9432	9426	9421	9415	9409	9403	1	2	3	4	5
20	9397	9391	9385	9379	9373	9367	9361	9354	9348	9342	1	2	3	4	5
21	9336	9330	9323	9317	9311	9304	9298	9291	9285	9278	1	2	3	4	5
22	9272	9265	9259	9252	9245	9239	9232	9225	9219	9212	1	2	3	4	6
23	9205	9198	9191	9184	9178	9171	9164	9157	9150	9143	1	2	3	5	6
24	9135	9128	9121	9114	9107	9100	9092	9085	9078	9070	1	2	4	5	6
25	9063	9056	9048	9041	9033	9026	9018	9011	9003	8996	1	3	4	5	6
26	8988	8980	8973	8965	8957	8949	8942	8934	8926	8918	1	3	4	5	6
27	8910	8902	8894	8886	8878	8870	8862	8854	8846	8838	1	3	4	5	7
28	8829	8821	8813	8805	8796	8788	8780	8771	8763	8755	1	3	4	6	7
29	·8746	8738	8729	8721	8712	8704	8695	8686	8678	8669	1	3	4	6	7
30	8660	8652	8643	8634	8625	8616	8607	8599	8590	8581	1	3	4	6	7
31	8572	8563	8554	8545	8536	8526	8517	8508	8499	8490	2	3	5	6	8
32	8480	8471	8462	8453	8443	8434	8425	8415	8406	8396	2	3	5	6	8
33	8387	8377	8368	8358	8348	8339	8329	8320	8310	8300	2	3	5	6	8
34	8290	8281	8271	8261	8251	8241	8231	8221	8211	8202	2	3	5	7	8
35	8192	8181	8171	8161	8151	8141	8131	8121	8111	8100	2	3	5	7	8
36	8090	8080	8070	8059	8049	8039	8028	8018	8007	7997	2	3	5	7	9
37	7986	7976	7965	7955	7944	7934	7923	7912	7902	7891	2	4	5	7	9
38	7880	7869	7859	7848	7837	7826	7815	7804	7793	7782	2	4	5	7	9
39	7771	7760	7749	7738	7727	7716	7705	7694	7683	7672	2	4	6	7	9
40	7660	7649	7638	7627	7615	7604	7593	7581	7570	7559	2	4	6	8	9
41	7547	7536	7524	7513	7501	7490	7478	7466	7455	7443	2	4	6	8	10
42	7431	7420	7408	7396	7385	7373	7361	7349	7337	7325	2	4	6	8	10
43	7314	7302	7290	7278	7266	7254	7242	7230	7218	7206	2	4	6	8	10
44	·7193	7181	7169	7157	7145	7133	7120	7108	7096	7083	2	4	6	8	10

NATURAL COSINES

Degrees.	0′ 0°·0	6′ 0°·1	12′ 0°·2	18′ 0°·3	24′ 0°·4	30′ 0°·5	36′ 0°·6	42′ 0°·7	48′ 0°·8	54′ 0°·9	Mean Differences. 1	2	3	4	5
45	·7071	7059	7046	7034	7022	7009	6997	6984	6972	6959	2	4	6	8	10
46	·6947	6934	6921	6909	6896	6884	6871	6858	6845	6833	2	4	6	8	11
47	6820	6807	6794	6782	6769	6756	6743	6730	6717	6704	2	4	6	9	11
48	6691	6678	6665	6652	6639	6626	6613	6600	6587	6574	2	4	7	9	11
49	6561	6547	6534	6521	6508	6494	6481	6468	6455	6441	2	4	7	9	11
50	6428	6414	6401	6388	6374	6361	6347	6334	6320	6307	2	4	7	9	11
51	6293	6280	6266	6252	6239	6225	6211	6198	6184	6170	2	5	7	9	11
52	·6157	6143	6129	6115	6101	6088	6074	6060	6046	6032	2	5	7	9	12
53	6018	6004	5990	5976	5962	5948	5934	5920	5906	5892	2	5	7	9	12
54	5878	5864	5850	5835	5821	5807	5793	5779	5764	5750	2	5	7	9	12
55	·5736	5721	5707	5693	5678	5664	5650	5635	5621	5606	2	5	7	10	12
56	·5592	5577	5563	5548	5534	5519	5505	5490	5476	5461	2	5	7	10	12
57	·5446	5432	5417	5402	5388	5373	5358	5344	5329	5314	2	5	7	10	12
58	·5299	5284	5270	5255	5240	5225	5210	5195	5180	5165	2	5	7	10	12
59	·5150	5135	5120	5105	5090	5075	5060	5045	5030	5015	3	5	8	10	13
60	·5000	4985	4970	4955	4939	4924	4909	4894	4879	4863	3	5	8	10	13
61	·4848	4833	4818	4802	4787	4772	4756	4741	4726	4710	3	5	8	10	13
62	·4695	4679	4664	4648	4633	4617	4602	4586	4571	4555	3	5	8	10	13
63	·4540	4524	4509	4493	4478	4462	4446	4431	4415	4399	3	5	8	10	13
64	4384	4368	4352	4337	4321	4305	4289	4274	4258	4242	3	5	8	11	13
65	·4226	4210	4195	4179	4163	4147	4131	4115	4099	4083	3	5	8	11	13
66	·4067	4051	4035	4019	4003	3987	3971	3955	3939	3923	3	5	8	11	14
67	·3907	3891	3875	3859	3843	3827	3811	3795	3778	3762	3	5	8	11	14
68	·3746	3730	3714	3697	3681	3665	3649	3633	3616	3600	3	5	8	11	14
69	·3584	3567	3551	3535	3518	3502	3486	3469	3453	3437	3	5	8	11	14
70	·3420	3404	3387	3371	3355	3338	3322	3305	3289	3272	3	5	8	11	14
71	·3256	3239	3223	3206	3190	3173	3156	3140	3123	3107	3	6	8	11	14
72	·3090	3074	3057	3040	3024	3007	2990	2974	2957	2940	3	6	8	11	14
73	·2924	2907	2890	2874	2857	2840	2823	2807	2790	2773	3	6	8	11	14
74	·2756	2740	2723	2706	2689	2672	2656	2639	2622	2605	3	6	8	11	14
75	·2588	2571	2554	2538	2521	2504	2487	2470	2453	2436	3	6	8	11	14
76	·2419	2402	2385	2368	2351	2334	2317	2300	2284	2267	3	6	8	11	14
77	·2250	2233	2215	2198	2181	2164	2147	2130	2113	2096	3	6	9	11	14
78	·2079	2062	2045	2028	2011	1994	1977	1959	1942	1925	3	6	9	11	14
79	·1908	1891	1874	1857	1840	1822	1805	1788	1771	1754	3	6	9	11	14
80	·1736	1719	1702	1685	1668	1650	1633	1616	1599	1582	3	6	9	12	14
81	·1564	1547	1530	1513	1495	1478	1461	1444	1426	1409	3	6	9	12	14
82	·1392	1374	1357	1340	1323	1305	1288	1271	1253	1236	3	6	9	12	14
83	·1219	1201	1184	1167	1149	1132	1115	1097	1080	1063	3	6	9	12	14
84	·1045	1028	1011	0993	0976	0958	0941	0924	0906	0889	3	6	9	12	14
85	·0872	0854	0837	0819	0802	0785	0767	0750	0732	0715	3	6	9	12	15
86	·0698	0680	0663	0645	0628	0610	0593	0576	0558	0541	3	6	9	12	15
87	·0523	0506	0488	0471	0454	0436	0419	0401	0384	0366	3	6	9	12	15
88	·0349	0332	0314	0297	0279	0262	0244	0227	0209	0192	3	6	9	12	15
89	·0175	0157	0140	0122	0105	0087	0070	0052	0035	0017	3	6	9	12	15
90	·0000														

NATURAL TANGENTS

Degrees.	0′ 0°·0	6′ 0°·1	12′ 0°·2	18′ 0°·3	24′ 0°·4	30′ 0°·5	36′ 0°·6	42′ 0°·7	48′ 0°·8	54′ 0°·9	Mean Differences. 1′	2	3	4	5
0	·0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	15
1	0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15
2	0349	0366	0384	0401	0419	0436	0454	0471	0488	0506	3	6	9	12	15
3	0523	0541	0558	0576	0593	0610	0628	0645	0663	0680	3	6	9	12	15
4	0698	0715	0732	0750	0767	0785	0802	0819	0837	0854	3	6	9	12	15
5	0872	0889	0906	0924	0941	0958	0976	0993	1011	1028	3	6	9	12	14
6	1045	1063	1080	1097	1115	1132	1149	1167	1184	1201	3	6	9	12	14
7	1219	1236	1253	1271	1288	1305	1323	1340	1357	1374	3	6	9	12	14
8	1392	1409	1426	1444	1461	1478	1495	1513	1530	1547	3	6	9	12	14
9	1564	1582	1599	1616	1633	1650	1668	1685	1702	1719	3	6	9	12	14
10	1736	1754	1771	1788	1805	1822	1840	1857	1874	1891	3	6	9	12	14
11	1908	1925	1942	1959	1977	1994	2011	2028	2045	2062	3	6	9	11	14
12	2079	2096	2113	2130	2147	2164	2181	2198	2215	2232	3	6	9	11	14
13	2250	2267	2284	2300	2317	2334	2351	2368	2385	2402	3	6	8	11	14
14	2419	2436	2453	2470	2487	2504	2521	2538	2554	2571	3	6	8	11	14
15	2588	2605	2622	2639	2656	2672	2689	2706	2723	2740	3	6	8	11	14
16	2756	2773	2790	2807	2823	2840	2857	2874	2890	2907	3	6	8	11	14
17	2924	2940	2957	2974	2990	3007	3024	3040	3057	3074	3	6	8	11	14
18	3090	3107	3123	3140	3156	3173	3190	3206	3223	3239	3	6	8	11	14
19	3256	3272	3289	3305	3322	3338	3355	3371	3387	3404	3	5	8	11	14
20	3420	3437	3453	3469	3486	3502	3518	3535	3551	3567	3	5	8	11	14
21	3584	3600	3616	3633	3649	3665	3681	3697	3714	3730	3	5	8	11	14
22	3746	3762	3778	3795	3811	3827	3843	3859	3875	3891	3	5	8	11	14
23	3907	3923	3939	3955	3971	3987	4003	4019	4035	4051	3	5	8	11	14
24	·4067	4083	4099	4115	4131	4147	4163	4179	4195	4210	3	5	8	11	13
25	4226	4242	4258	4274	4289	4305	4321	4337	4352	4368	3	5	8	11	13
26	4384	4399	4415	4431	4446	4462	4478	4493	4509	4524	3	5	8	10	13
27	4540	4555	4571	4586	4602	4617	4633	4648	4664	4679	3	5	8	10	13
28	4695	4710	4726	4741	4756	4772	4787	4802	4818	4833	3	5	8	10	13
29	·4848	4863	4879	4894	4909	4924	4939	4955	4970	4985	3	5	8	10	13
30	·5000	5015	5030	5045	5060	5075	5090	5105	5120	5135	3	5	8	10	13
31	5150	5165	5180	5195	5210	5225	5240	5255	5270	5284	2	5	7	10	12
32	·5299	5314	5329	5344	5358	5373	5388	5402	5417	5432	2	5	7	10	12
33	·5446	5461	5476	5490	5505	5519	5534	5548	5563	5577	2	5	7	10	12
34	·5592	5606	5621	5635	5650	5664	5678	5693	5707	5721	2	5	7	10	12
35	5736	5750	5764	5779	5793	5807	5821	5835	5850	5864	2	5	7	10	12
36	5878	5892	5906	5920	5934	5948	5962	5976	5990	6004	2	5	7	9	12
37	·6018	6032	6046	6060	6074	6088	6101	6115	6129	6143	2	5	7	9	12
38	·6157	6170	6184	6198	6211	6225	6239	6252	6266	6280	2	5	7	9	11
39	·6293	6307	6320	6334	6347	6361	6374	6388	6401	6414	2	4	7	9	11
40	6428	6441	6455	6468	6481	6494	6508	6521	6534	6547	2	4	7	9	11
41	6561	6574	6587	6600	6613	6626	6639	6652	6665	6678	2	4	7	9	11
42	6691	6704	6717	6730	6743	6756	6769	6782	6794	6807	2	4	6	9	11
43	6820	6833	6845	6858	6871	6884	6896	6909	6921	6934	2	4	6	8	11
44	6947	6959	6972	6984	6997	7009	7022	7034	7046	7059	2	4	6	8	10

NATURAL TANGENTS

Degrees	0′ 0°·0	6′ 0°·1	12′ 0°·2	18′ 0°·3	24′ 0°·4	30′ 0°·5	36′ 0°·6	42′ 0°·7	48′ 0°·8	54′ 0°·9	Mean Differences 1	2	3	4	5
45	1 0000	0035	0070	0105	0141	0176	0212	0247	0283	0319	6	12	18	24	30
46	1 0355	0392	0428	0464	0501	0538	0575	0612	0649	0686	6	12	18	25	31
47	1 0724	0761	0799	0837	0875	0913	0951	0990	1028	1067	6	13	19	25	32
48	1 1106	1145	1184	1224	1263	1303	1343	1383	1423	1463	7	13	20	27	33
49	1 1504	1544	1585	1626	1667	1708	1750	1792	1833	1875	7	14	21	28	34
50	1 1918	1960	2002	2045	2088	2131	2174	2218	2261	2305	7	14	22	29	36
51	1 2349	2393	2437	2482	2527	2572	2617	2662	2708	2753	8	15	23	30	38
52	1 2799	2846	2892	2938	2985	3032	3079	3127	3175	3222	8	16	24	31	39
53	1 3270	3319	3367	3416	3465	3514	3564	3613	3663	3713	8	16	25	33	41
54	1 3764	3814	3865	3916	3968	4019	4071	4124	4176	4229	9	17	26	34	43
55	1 4281	4335	4388	4442	4496	4550	4605	4659	4715	4770	9	18	27	36	45
56	1 4826	4882	4938	4994	5051	5108	5166	5224	5282	5340	10	19	29	38	48
57	1 5399	5458	5517	5577	5637	5697	5757	5818	5880	5941	10	20	30	40	50
58	1 6003	6066	6128	6191	6255	6319	6383	6447	6512	6577	11	21	32	43	53
59	1 6643	6709	6775	6842	6909	6977	7045	7113	7182	7251	11	23	34	45	56
60	1 7321	7391	7461	7532	7603	7675	7747	7820	7893	7966	12	24	36	48	60
61	1 8040	8115	8190	8265	8341	8418	8495	8572	8650	8728	13	26	38	51	64
62	1 8807	8887	8967	9047	9128	9210	9292	9375	9458	9542	14	27	41	55	68
63	1 9626	9711	9797	9883	9970	2 0057	2 0145	2 0233	2 0323	2 0413	15	29	44	58	73
64	2 0503	0594	0686	0778	0872	0965	1060	1155	1251	1348	16	31	47	63	78
65	2 1445	1543	1642	1742	1842	1943	2045	2148	2251	2355	17	34	51	68	85
66	2 2460	2566	2673	2781	2889	2998	3109	3220	3332	3445	18	37	55	73	92
67	2 3559	3673	3789	3906	4023	4142	4262	4383	4504	4627	20	40	60	79	99
68	2 4751	4876	5002	5129	5257	5386	5517	5649	5782	5916	22	43	65	87	108
69	2 6051	6187	6325	6464	6605	6746	6889	7034	7179	7326	24	47	71	95	119
70	2·7475	7625	7776	7929	8083	8239	8397	8556	8716	8878	26	52	78	104	131
71	2 9042	9208	9375	9544	9714	9887	3 0061	3 0237	3 0415	3 0595	29	58	87	116	145
72	3 0777	0961	1146	1334	1524	1716	1910	2106	2305	2506	32	64	96	129	161
73	3 2709	2914	3122	3332	3544	3759	3977	4197	4420	4646	36	72	108	144	180
74	3 4874	5105	5339	5576	5816	6059	6305	6554	6806	7062	41	81	122	163	204
75	3 7321	7583	7848	8118	8391	8667	8947	9232	9520	9812	46	93	139	186	232
76	4 0108	0408	0713	1022	1335	1653	1976	2303	2635	2972	53	107	160	213	267
77	4 3315	3662	4015	4374	4737	5107	5483	5864	6252	6646					
78	4 7046	7453	7867	8288	8716	9152	9594	5 0045	5 0504	5 0970	Mean differences cease to be sufficiently accurate				
79	5 1446	1929	2422	2924	3435	3955	4486	5026	5578	6140					
80	5 6713	7297	7894	8502	9124	9758	6·0405	6 1066	6 1742	6 2432					
81	6 3138	3859	4596	5350	6122	6912	7720	8548	9395	7 0264					
82	7 1154	2066	3002	3962	4947	5958	6996	8062	9158	8 0285					
83	8 1443	2636	3863	5126	6427	7769	9152	9 0579	9 2052	9 3572					
84	9 5144	9 677	9 845	10 02	10 20	10 39	10 58	10 78	10·99	11 20					
85	11 43	11 66	11 91	12 16	12 43	12 71	13 00	13 30	13 62	13 95					
86	14 30	14 67	15·06	15 46	15 89	16 35	16 83	17 34	17 89	18 46					
87	19 08	19 74	20·45	21 20	22 02	22 90	23 86	24 90	26 03	27 27					
88	28 64	30 14	31 82	33 69	35 80	38 19	40 92	44·07	47 74	52 08					
89	57·29	63 66	71 62	81·85	95 49	114 6	143 2	191 0	286 5	573 0					
90	∞														

POWERS ROOTS & RECIPROCALS

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$
1	1	1	1	1	1
2	4	8	1·414	1·260	·5000
3	9	27	1·732	1·442	·3333
4	16	64	2	1·587	·2500
5	25	125	2·236	1·710	·2000
6	36	216	2·449	1·817	1667
7	49	343	2·646	1·913	·1429
8	64	512	2·828	2·000	·1250
9	81	729	3·000	2·080	·1111
10	100	1000	3·162	2·154	·1000
11	121	1331	3·317	2·224	·09091
12	144	1728	3·464	2·289	·08333
13	169	2197	3·606	2·351	·07692
14	196	2744	3·742	2·410	·07143
15	225	3375	3·873	2·466	·06667
16	256	4096	4·000	2·520	·06250
17	289	4913	4·123	2·571	05882
18	324	5832	4·243	2·621	05556
19	361	6859	4·359	2·668	·05263
20	400	8000	4·472	2·714	·0500
21	441	9261	4·583	2·759	04762
22	484	10648	4·690	2·802	·04545
23	529	12167	4·796	2·844	·04348
24	576	13824	4·899	2·884	·04167
25	625	15625	5·000	2·924	·0400
26	676	17576	5·099	2·962	·03846
27	729	19683	5·196	3·000	·03704
28	784	21952	5·292	3·037	·03571
29	841	24389	5·385	3·072	·03448
30	900	27000	5·477	3·107	·03333
31	961	29791	5·568	3·141	·03226
32	1024	32768	5·657	3·175	·03125
33	1089	35937	5·745	3·208	·03030
34	1156	39304	5·831	3·240	·02941
35	1225	42875	5·916	3·271	·02857
36	1296	46656	6·000	3·302	·02778
37	1369	50653	6·083	3·332	·02703
38	1444	54872	6·164	3·362	·02632
39	1521	59319	6·245	3·391	·02564
40	1600	64000	6·325	3·420	·0250
41	1681	68921	6·403	3·448	·02439
42	1764	74088	6·481	3·476	·02381
43	1849	79507	6·557	3·503	·02326
44	1936	85184	6·633	3·530	·02273
45	2025	91125	6·708	3·557	·02222
46	2116	97336	6·782	3·583	·02174
47	2209	103823	6·856	3·609	·02128
48	2304	110592	6·928	3·634	·02083
49	2401	117649	7·000	3·659	·02041
50	2500	125000	7·071	3·684	·020

POWERS ROOTS & RECIPROCALS

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$
51	2601	132651	7·141	3·708	·01961
52	2704	140608	7·211	3·733	·01923
53	2809	148877	7·280	3·756	·01887
54	2916	157464	7·348	3·780	·01852
55	3025	166375	7·416	3·803	·01818
56	3136	175616	7·483	3·826	·01786
57	3249	185193	7·550	3·849	·01754
58	3364	195112	7·616	3·871	·01724
59	3481	205379	7·681	3·893	·01695
60	3600	216000	7·746	3·915	·01667
61	3721	226981	7·810	3·936	·01639
62	3844	238328	7·874	3·958	·01613
63	3969	250047	7·937	3·979	·01587
64	4096	262144	8·000	4·000	·01562
65	4225	274625	8·062	4·021	·01538
66	4356	287496	8·124	4·041	·01515
67	4489	300763	8·185	4·062	·01493
68	4624	314432	8·246	4·082	·01471
69	4761	328509	8·307	4·102	·01449
70	4900	343000	8·367	4·121	·01429
71	5041	357911	8·426	4·141	·01408
72	5184	373248	8·485	4·160	·01389
73	5329	389017	8·544	4·179	·01370
74	5476	405224	8·602	4·198	·01351
75	5625	421875	8·660	4·217	·01333
76	5776	438976	8·718	4·236	·01316
77	5929	456533	8·775	4·254	·01299
78	6084	474552	8·832	4·273	·01282
79	6241	493039	8·888	4·291	·01266
80	6400	512000	8·944	4·309	·01250
81	6561	531441	9·000	4·327	·01235
82	6724	551368	9·055	4·344	·01220
83	6889	571787	9·110	4·362	·01205
84	7056	592704	9·165	4·380	·01190
85	7225	614125	9·220	4·397	·01176
86	7396	636056	9·274	4·414	·01163
87	7569	658503	9·327	4·431	·01149
88	7744	681472	9·381	4·448	·01136
89	7921	704969	9·434	4·465	·01124
90	8100	729000	9·487	4·481	·01111
91	8281	753571	9·539	4·498	·01099
92	8464	778688	9·592	4·514	·01087
93	8649	804357	9·644	4·531	·01075
94	8836	830584	9·695	4·547	·01064
95	9025	857375	9·747	4·563	·01053
96	9216	884736	9·798	4·579	·01042
97	9409	912673	9·849	4·595	·01031
98	9604	941192	9·899	4·610	·01020
99	9801	970299	9·950	4·626	·01010
100	10000	1000000	10·000	4·642	·0100

SQUARE ROOTS FROM 1 TO 10

	0	1	2	3	4	5	6	7	8	9	Mean Differences. 1	2	3	4	5	6	7	8	9
1·0	1·000	1·005	1·010	1·015	1·020	1·025	1·030	1·034	1·039	1·044	0	1	1	2	2	3	3	4	4
1·1	1·049	1·054	1·058	1·063	1·068	1·072	1·077	1·082	1·086	1·091	0	1	1	2	2	3	3	4	4
1·2	1·095	1·100	1·105	1·109	1·114	1·118	1·122	1·127	1·131	1·136	0	1	1	2	2	3	3	4	4
1·3	1·140	1·145	1·149	1·153	1·158	1·162	1·166	1·170	1·175	1·179	0	1	1	2	2	3	3	3	4
1·4	1·183	1·187	1·192	1·196	1·200	1·204	1·208	1·212	1·217	1·221	0	1	1	2	2	2	3	3	4
1·5	1·225	1·229	1·233	1·237	1·241	1·245	1·249	1·253	1·257	1·261	0	1	1	2	2	2	3	3	4
1·6	1·265	1·269	1·273	1·277	1·281	1·285	1·288	1·292	1·296	1·300	0	1	1	2	2	2	3	3	3
1·7	1·304	1·308	1·311	1·315	1·319	1·323	1·327	1·330	1·334	1·338	0	1	1	2	2	2	3	3	3
1·8	1·342	1·345	1·349	1·353	1·356	1·360	1·364	1·367	1·371	1·375	0	1	1	1	2	2	3	3	3
1·9	1·378	1·382	1·386	1·389	1·393	1·396	1·400	1·404	1·407	1·411	0	1	1	1	2	2	3	3	3
2·0	1·414	1·418	1·421	1·425	1·428	1·432	1·435	1·439	1·442	1·446	0	1	1	1	2	2	2	3	3
2·1	1·449	1·453	1·456	1·459	1·463	1·466	1·470	1·473	1·476	1·480	0	1	1	1	2	2	2	3	3
2·2	1·483	1·487	1·490	1·493	1·497	1·500	1·503	1·507	1·510	1·513	0	1	1	1	2	2	2	3	3
2·3	1·517	1·520	1·523	1·526	1·530	1·533	1·536	1·539	1·543	1·546	0	1	1	1	2	2	2	3	3
2·4	1·549	1·552	1·556	1·559	1·562	1·565	1·568	1·572	1·575	1·578	0	1	1	1	2	2	2	3	3
2·5	1·581	1·584	1·587	1·591	1·594	1·597	1·600	1·603	1·606	1·609	0	1	1	1	2	2	2	3	3
2·6	1·612	1·616	1·619	1·622	1·625	1·628	1·631	1·634	1·637	1·640	0	1	1	1	2	2	2	2	3
2·7	1·643	1·646	1·649	1·652	1·655	1·658	1·661	1·664	1·667	1·670	0	1	1	1	2	2	2	2	3
2·8	1·673	1·676	1·679	1·682	1·685	1·688	1·691	1·694	1·697	1·700	0	1	1	1	1	2	2	2	3
2·9	1·703	1·706	1·709	1·712	1·715	1·718	1·720	1·723	1·726	1·729	0	1	1	1	1	2	2	2	3
3·0	1·732	1·735	1·738	1·741	1·744	1·746	1·749	1·752	1·755	1·758	0	1	1	1	1	2	2	2	3
3·1	1·761	1·764	1·766	1·769	1·772	1·775	1·778	1·780	1·783	1·786	0	1	1	1	1	2	2	2	3
3·2	1·789	1·792	1·794	1·797	1·800	1·803	1·806	1·808	1·811	1·814	0	1	1	1	1	2	2	2	2
3·3	1·817	1·819	1·822	1·825	1·828	1·830	1·833	1·836	1·838	1·841	0	1	1	1	1	2	2	2	2
3·4	1·844	1·847	1·849	1·852	1·855	1·857	1·860	1·863	1·865	1·868	0	1	1	1	1	2	2	2	2
3·5	1·871	1·873	1·876	1·879	1·881	1·884	1·887	1·889	1·892	1·895	0	1	1	1	1	2	2	2	2
3·6	1·897	1·900	1·903	1·905	1·908	1·910	1·913	1·916	1·918	1·921	0	1	1	1	1	2	2	2	2
3·7	1·924	1·926	1·929	1·931	1·934	1·936	1·939	1·942	1·944	1·947	0	1	1	1	1	2	2	2	2
3·8	1·949	1·952	1·954	1·957	1·960	1·962	1·965	1·967	1·970	1·972	0	1	1	1	1	2	2	2	2
3·9	1·975	1·977	1·980	1·982	1·985	1·987	1·990	1·992	1·995	1·997	0	1	1	1	1	2	2	2	2
4·0	2·000	2·002	2·005	2·007	2·010	2·012	2·015	2·017	2·020	2·022	0	0	1	1	1	1	2	2	2
4·1	2·025	2·027	2·030	2·032	2·035	2·037	2·040	2·042	2·045	2·047	0	0	1	1	1	1	2	2	2
4·2	2·049	2·052	2·054	2·057	2·059	2·062	2·064	2·066	2·069	2·071	0	0	1	1	1	1	2	2	2
4·3	2·074	2·076	2·078	2·081	2·083	2·086	2·088	2·090	2·093	2·095	0	0	1	1	1	1	2	2	2
4·4	2·098	2·100	2·102	2·105	2·107	2·110	2·112	2·114	2·117	2·119	0	0	1	1	1	1	2	2	2
4·5	2·121	2·124	2·126	2·128	2·131	2·133	2·135	2·138	2·140	2·142	0	0	1	1	1	1	2	2	2
4·6	2·145	2·147	2·149	2·152	2·154	2·156	2·159	2·161	2·163	2·166	0	0	1	1	1	1	2	2	2
4·7	2·168	2·170	2·173	2·175	2·177	2·179	2·182	2·184	2·186	2·189	0	0	1	1	1	1	2	2	2
4·8	2·191	2·193	2·195	2·198	2·200	2·202	2·205	2·207	2·209	2·211	0	0	1	1	1	1	2	2	2
4·9	2·214	2·216	2·218	2·220	2·223	2·225	2·227	2·229	2·232	2·234	0	0	1	1	1	1	2	2	2
5·0	2·236	2·238	2·241	2·243	2·245	2·247	2·249	2·252	2·254	2·256	0	0	1	1	1	1	2	2	2
5·1	2·258	2·261	2·263	2·265	2·267	2·269	2·272	2·274	2·276	2·278	0	0	1	1	1	1	2	2	2
5·2	2·280	2·283	2·285	2·287	2·289	2·291	2·293	2·296	2·298	2·300	0	0	1	1	1	1	2	2	2
5·3	2·302	2·304	2·307	2·309	2·311	2·313	2·315	2·317	2·319	2·322	0	0	1	1	1	1	2	2	2
5·4	2·324	2·326	2·328	2·330	2·332	2·335	2·337	2·339	2·341	2·343	0	0	1	1	1	1	1	2	2

SQUARE ROOTS FROM 1 TO 10

	0	1	2	3	4	5	6	7	8	9	Mean Differences 1	2	3	4	5	6	7	8	9
5 5	2 345	2 347	2 340	2 352	2 354	2 356	2·358	2 360	2 362	2 364	0	0	1	1	1	1	1	2	2
5 6	2 366	2 369	2 371	2 373	2 375	2 377	2 379	2 381	2 383	2 385	0	0	1	1	1	1	1	2	2
5 7	2 387	2 390	2 392	2 394	2·396	2 398	2 400	2 402	2 404	2 406	0	0	1	1	1	1	1	2	2
5·8	2 408	2 410	2 412	2 415	2 417	2 419	2 421	2 423	2 425	2 427	0	0	1	1	1	1	1	2	2
5 9	2 429	2 431	2 433	2 435	2 437	2 439	2 441	2 443	2 445	2 447	0	0	1	1	1	1	1	2	2
6 0	2 449	2 452	2 454	2 456	2 458	2 460	2 462	2 464	2 466	2 468	0	0	1	1	1	1	1	2	2
6 1	2 470	2 472	2 474	2 476	2 478	2 480	2 482	2 484	2 486	2 488	0	0	1	1	1	1	1	2	2
6 2	2 4900	2 492	2 494	2 496	2 498	2 500	2 502	2 504	2 506	2 508	0	0	1	1	1	1	1	2	2
6 3	2 510	2 512	2 514	2 516	2 518	2·520	2 522	2 524	2 526	2 528	0	0	1	1	1	1	1	2	2
6 4	2 530	2 532	2 534	2 536	2 538	2 540	2 542	2 544	2 546	2 548	0	0	1	1	1	1	1	2	2
6 5	2·550	2 551	2 553	2 555	2 557	2·559	2 561	2 563	2 565	2 567	0	0	1	1	1	1	1	2	2
6 6	2 569	2 571	2 573	2 575	2 577	2 579	2 581	2 583	2 585	2 587	0	0	1	1	1	1	1	2	2
6 7	2 588	2 590	2 592	2 594	2 596	2 598	2 600	2 602	2 604	2 606	0	0	1	1	1	1	1	2	2
6 8	2 608	2 610	2 612	2 613	2 615	2 617	2 619	2 621	2 623	2 625	0	0	1	1	1	1	1	2	2
6 9	2 627	2 629	2 631	2 632	2 634	2 636	2 638	2 640	2 642	2 644	0	0	1	1	1	1	1	2	2
7 0	2 646	2 648	2 650	2 651	2 653	2 655	2 657	2 659	2 661	2 663	0	0	1	1	1	1	1	2	2
7 1	2 605	2 666	2 668	2 670	2·672	2 674	2 676	2 678	2 680	2 681	0	0	1	1	1	1	1	1	2
7 2	2 683	2 685	2 687	2 689	2 691	2 693	2 694	2 696	2 698	2 700	0	0	1	1	1	1	1	1	2
7 3	2 702	2 704	2 700	2 707	2 709	2 711	2 713	2 715	2 717	2 718	0	0	1	1	1	1	1	1	2
7 4	2 720	2 722	2 724	2 726	2 728	2 729	2 731	2 733	2 735	2 737	0	0	1	1	1	1	1	1	2
7 5	2 739	2 740	2 742	2 744	2 746	2 748	2 750	2 751	2 753	2 755	0	0	1	1	1	1	1	1	2
7 6	2 757	2 759	2 760	2 762	2 764	2 766	2 768	2 769	2 771	2 773	0	0	1	1	1	1	1	1	2
7 7	2 775	2 777	2 778	2 780	2 782	2 784	2 786	2 787	2 789	2 791	0	0	1	1	1	1	1	1	2
7 8	2 793	2 795	2 796	2 798	2 800	2 802	2 804	2 805	2 807	2 809	0	0	1	1	1	1	1	1	2
7 9	2 811	2 812	2 814	2 816	2 818	2 820	2 821	2 823	2 825	2 827	0	0	1	1	1	1	1	1	2
8 0	2 828	2 830	2 832	2 834	2 835	2 837	2 839	2 841	2 843	2 844	0	0	1	1	1	1	1	1	2
8 1	2 846	2 848	2 850	2 851	2 853	2·855	2 857	2 858	2 860	2 862	0	0	1	1	1	1	1	1	2
8 2	2 864	2 865	2 867	2 869	2 871	2 872	2 874	2 876	2·877	2 879	0	0	1	1	1	1	1	1	2
8 3	2 881	2 883	2 884	2 886	2 888	2 890	2 891	2 893	2 895	2 897	0	0	1	1	1	1	1	1	2
8 4	2 898	2 900	2 902	2 903	2 905	2 907	2 909	2 910	2 912	2 914	0	0	1	1	1	1	1	1	2
8 5	2 915	2 917	2 919	2 921	2 922	2 924	2 926	2 927	2 929	2 931	0	0	1	1	1	1	1	1	2
8 6	2 933	2 934	2 936	2 938	2 939	2 941	2 943	2 944	2 946	2 948	0	0	1	1	1	1	1	1	2
8 7	2 950	2 951	2 953	2 955	2 956	2 958	2 960	2 961	2 963	2 965	0	0	1	1	1	1	1	1	2
8·8	2 966	2 968	2 970	2 972	2 973	2·975	2 977	2 978	2 980	2 982	0	0	1	1	1	1	1	1	2
8 9	2 983	2 985	2 987	2 988	2 990	2 992	2 993	2 995	2 997	2 998	0	0	1	1	1	1	1	1	2
9 0	3 000	3 002	3 003	3 005	3 007	3 008	3 010	3 012	3 013	3 015	0	0	0	1	1	1	1	1	1
9 1	3 017	3 018	3 020	3 022	3 023	3 025	3·027	3 028	3 030	3 032	0	0	0	1	1	1	1	1	1
9 2	3 033	3 035	3 036	3 038	3 040	3 041	3 043	3 045	3 046	3 048	0	0	0	1	1	1	1	1	1
9·3	3 050	3 051	3 053	3 055	3 056	3 058	3·059	3·061	3 063	3 064	0	0	0	1	1	1	1	1	1
9·4	3 066	3 068	3 069	3 071	3 072	3 074	3 076	3 077	3 079	3 081	0	0	0	1	1	1	1	1	1
9 5	3 082	3 084	3 085	3 087	3 089	3 090	3 092	3 094	3 095	3 097	0	0	0	1	1	1	1	1	1
9·6	3 098	3 100	3 102	3 103	3 105	3 106	3 108	3 110	3 111	3 113	0	0	0	1	1	1	1	1	1
9 7	3 114	3 116	3 118	3 119	3 121	3 122	3 124	3 126	3·127	3 129	0	0	0	1	1	1	1	1	1
9·8	3 130	3·132	3 134	3 135	3 137	3 138	3 140	3·142	3 143	3 145	0	0	0	1	1	1	1	1	1
9 9	3 146	3·148	3 150	3 151	3·153	3 154	3 156	3 158	3 159	3 161	0	0	0	1	1	1	1	1	1

SQUARE ROOTS FROM 10 TO 100

	0	1	2	3	4	5	6	7	8	9	Mean Differences 1	2	3	4	5	6	7	8	9
10	3.162	3.178	3.194	3.209	3.225	3.240	3.256	3.271	3.286	3.302	2	3	5	6	8	9	11	12	14
11	3.317	3.332	3.347	3.362	3.376	3.391	3.406	3.421	3.435	3.450	1	3	4	6	7	9	10	12	13
12	3.464	3.479	3.493	3.507	3.521	3.536	3.550	3.564	3.578	3.592	1	3	4	6	7	8	10	11	13
13	3.606	3.619	3.633	3.647	3.661	3.674	3.688	3.701	3.715	3.728	1	3	4	5	7	8	10	11	12
14	3.742	3.755	3.768	3.782	3.795	3.808	3.821	3.834	3.847	3.860	1	3	4	5	7	8	9	11	12
15	3.873	3.886	3.899	3.912	3.924	3.937	3.950	3.962	3.975	3.987	1	3	4	5	6	8	9	10	11
16	4.000	4.012	4.025	4.037	4.050	4.062	4.074	4.087	4.099	4.111	1	2	4	5	6	7	9	10	11
17	4.123	4.135	4.147	4.159	4.171	4.183	4.195	4.207	4.219	4.231	1	2	4	5	6	7	8	10	11
18	4.243	4.254	4.266	4.278	4.290	4.301	4.313	4.324	4.336	4.347	1	2	3	5	6	7	8	9	10
19	4.359	4.370	4.382	4.393	4.405	4.416	4.427	4.438	4.450	4.461	1	2	3	5	6	7	8	9	10
20	4.472	4.483	4.494	4.506	4.517	4.528	4.539	4.550	4.561	4.572	1	2	3	4	6	7	8	9	10
21	4.583	4.593	4.604	4.615	4.626	4.637	4.648	4.658	4.669	4.680	1	2	3	4	5	6	8	9	10
22	4.690	4.701	4.712	4.722	4.733	4.743	4.754	4.764	4.775	4.785	1	2	3	4	5	6	7	8	9
23	4.796	4.806	4.817	4.827	4.837	4.848	4.858	4.868	4.879	4.889	1	2	3	4	5	6	7	8	9
24	4.899	4.909	4.919	4.930	4.940	4.950	4.960	4.970	4.980	4.990	1	2	3	4	5	6	7	8	9
25	5.000	5.010	5.020	5.030	5.040	5.050	5.060	5.070	5.079	5.089	1	2	3	4	5	6	7	8	9
26	5.099	5.109	5.119	5.128	5.138	5.148	5.158	5.167	5.177	5.187	1	2	3	4	5	6	7	8	9
27	5.196	5.206	5.215	5.225	5.235	5.244	5.254	5.263	5.273	5.282	1	2	3	4	5	6	7	8	9
28	5.292	5.301	5.310	5.320	5.329	5.339	5.348	5.357	5.367	5.376	1	2	3	4	5	6	7	7	8
29	5.385	5.394	5.404	5.413	5.422	5.431	5.441	5.450	5.459	5.468	1	2	3	4	5	5	6	7	8
30	5.477	5.486	5.495	5.505	5.514	5.523	5.532	5.541	5.550	5.559	1	2	3	4	4	5	6	7	8
31	5.568	5.577	5.586	5.595	5.604	5.612	5.621	5.630	5.639	5.648	1	2	3	3	4	5	6	7	8
32	5.657	5.666	5.675	5.683	5.692	5.701	5.710	5.718	5.727	5.736	1	2	3	3	4	5	6	7	8
33	5.745	5.753	5.762	5.771	5.779	5.788	5.797	5.805	5.814	5.822	1	2	3	3	4	5	6	7	8
34	5.831	5.840	5.848	5.857	5.865	5.874	5.882	5.891	5.899	5.908	1	2	3	3	4	5	6	7	8
35	5.916	5.925	5.933	5.941	5.950	5.958	5.967	5.975	5.983	5.992	1	2	2	3	4	5	6	7	8
36	6.000	6.008	6.017	6.025	6.033	6.042	6.050	6.058	6.066	6.075	1	2	2	3	4	5	6	7	7
37	6.083	6.091	6.099	6.107	6.116	6.124	6.132	6.140	6.148	6.156	1	2	2	3	4	5	6	7	7
38	6.164	6.173	6.181	6.189	6.197	6.205	6.213	6.221	6.229	6.237	1	2	2	3	4	5	6	6	7
39	6.245	6.253	6.261	6.269	6.277	6.285	6.293	6.301	6.309	6.317	1	2	2	3	4	5	6	6	7
40	6.325	6.332	6.340	6.348	6.356	6.364	6.372	6.380	6.387	6.395	1	2	2	3	4	5	6	6	7
41	6.403	6.411	6.419	6.427	6.434	6.442	6.450	6.458	6.465	6.473	1	2	2	3	4	5	5	6	7
42	6.481	6.488	6.496	6.504	6.512	6.519	6.527	6.535	6.542	6.550	1	2	2	3	4	5	5	6	7
43	6.557	6.565	6.573	6.580	6.588	6.595	6.603	6.611	6.618	6.626	1	2	2	3	4	5	5	6	7
44	6.633	6.641	6.648	6.656	6.663	6.671	6.678	6.686	6.693	6.701	1	2	2	3	4	5	5	6	7
45	6.708	6.716	6.723	6.731	6.738	6.745	6.753	6.760	6.768	6.775	1	1	2	3	4	4	5	6	7
46	6.782	6.790	6.797	6.804	6.812	6.819	6.826	6.834	6.841	6.848	1	1	2	3	4	4	5	6	7
47	6.856	6.863	6.870	6.877	6.885	6.892	6.899	6.907	6.914	6.921	1	1	2	3	4	4	5	6	7
48	6.928	6.935	6.943	6.950	6.957	6.964	6.971	6.979	6.986	6.993	1	1	2	3	4	4	5	6	6
49	7.000	7.007	7.014	7.021	7.029	7.036	7.043	7.050	7.057	7.064	1	1	2	3	4	4	5	6	6
50	7.071	7.078	7.085	7.092	7.099	7.106	7.113	7.120	7.127	7.134	1	1	2	3	4	4	5	6	6
51	7.141	7.148	7.155	7.162	7.169	7.176	7.183	7.190	7.197	7.204	1	1	2	3	4	4	5	6	6
52	7.211	7.218	7.225	7.232	7.239	7.246	7.253	7.259	7.266	7.273	1	1	2	3	3	4	5	6	6
53	7.280	7.287	7.294	7.301	7.308	7.314	7.321	7.328	7.335	7.342	1	1	2	3	3	4	5	5	6
54	7.348	7.355	7.362	7.369	7.376	7.382	7.389	7.396	7.403	7.409	1	1	2	3	3	4	5	5	6

SQUARE ROOTS FROM 10 TO 100

	0.	1	2	3	4	5	6	7	8	9	Mean Differences 1	2	3	4	5	6	7	8	9
55	7·416	7·423	7·430	7·436	7·443	7·450	7·457	7·463	7·470	7·477	1	1	2	3	3	4	5	5	6
56	7·483	7·490	7·497	7·503	7·510	7·517	7·523	7·530	7·537	7·543	1	1	2	3	3	4	5	5	6
57	7·550	7·556	7·563	7·570	7·576	7·583	7·589	7·596	7·603	7·609	1	1	2	3	3	4	5	5	6
58	7·616	7·622	7·629	7·635	7·642	7·649	7·655	7·662	7·668	7·675	1	1	2	3	3	4	5	5	6
59	7·681	7·688	7·694	7·701	7·707	7·714	7·720	7·727	7·733	7·740	1	1	2	3	3	4	4	5	6
60	7·746	7·752	7·759	7·765	7·772	7·778	7·785	7·791	7·797	7·804	1	1	2	3	3	4	4	5	6
61	7·810	7·817	7·823	7·829	7·836	7·842	7·849	7·855	7·861	7·868	1	1	2	3	3	4	4	5	6
62	7·874	7·880	7·887	7·893	7·899	7·906	7·912	7·918	7·925	7·931	1	1	2	3	3	4	4	5	6
63	7·937	7·944	7·950	7·956	7·962	7·969	7·975	7·981	7·987	7·994	1	1	2	3	3	4	4	5	6
64	8·000	8·006	8·012	8·019	8·025	8·031	8·037	8·044	8·050	8·056	1	1	2	2	3	4	4	5	6
65	8·062	8·068	8·075	8·081	8·087	8·093	8·099	8·106	8·112	8·118	1	1	2	2	3	4	4	5	6
66	8·124	8·130	8·136	8·142	8·149	8·155	8·161	8·167	8·173	8·179	1	1	2	2	3	4	4	5	5
67	8·185	8·191	8·198	8·204	8·210	8·216	8·222	8·228	8·234	8·240	1	1	2	2	3	4	4	5	5
68	8·246	8·252	8·258	8·264	8·270	8·276	8·283	8·289	8·295	8·301	1	1	2	2	3	4	4	5	5
69	8·307	8·313	8·319	8·325	8·331	8·337	8·343	8·349	8·355	8·361	1	1	2	2	3	4	4	5	5
70	8·367	8·373	8·379	8·385	8·390	8·396	8·402	8·408	8·414	8·420	1	1	2	2	3	4	4	5	5
71	8·426	8·432	8·438	8·444	8·450	8·456	8·462	8·468	8·473	8·479	1	1	2	2	3	4	4	5	5
72	8·485	8·491	8·497	8·503	8·509	8·515	8·521	8·526	8·532	8·538	1	1	2	2	3	3	4	5	5
73	8·544	8·550	8·556	8·562	8·567	8·573	8·579	8·585	8·591	8·597	1	1	2	2	3	3	4	5	5
74	8·602	8·608	8·614	8·620	8·626	8·631	8·637	8·643	8·649	8·654	1	1	2	2	3	3	4	5	5
75	8·660	8·666	8·672	8·678	8·683	8·689	8·695	8·701	8·706	8·712	1	1	2	2	3	3	4	5	5
76	8·718	8·724	8·729	8·735	8·742	8·746	8·752	8·758	8·764	8·769	1	1	2	2	3	3	4	5	5
77	8·775	8·781	8·786	8·792	8·798	8·803	8·809	8·815	8·820	8·826	1	1	2	2	3	3	4	4	5
78	8·832	8·837	8·843	8·849	8·854	8·860	8·866	8·871	8·877	8·883	1	1	2	2	3	3	4	4	5
79	8·888	8·894	8·899	8·905	8·911	8·916	8·922	8·927	8·933	8·939	1	1	2	2	3	3	4	4	5
80	8·944	8·950	8·955	8·961	8·967	8·972	8·978	8·983	8·989	8·994	1	1	2	2	3	3	4	4	5
81	9·000	9·006	9·011	9·017	9·022	9·028	9·033	9·039	9·044	9·050	1	1	2	2	3	3	4	4	5
82	9·055	9·061	9·066	9·072	9·077	9·083	9·088	9·094	9·099	9·105	1	1	2	2	3	3	4	4	5
83	9·110	9·116	9·121	9·127	9·132	9·138	9·143	9·149	9·154	9·160	1	1	2	2	3	3	4	4	5
84	9·165	9·171	9·176	9·182	9·187	9·192	9·198	9·203	9·209	9·214	1	1	2	2	3	3	4	4	5
85	9·220	9·225	9·230	9·236	9·241	9·247	9·252	9·257	9·263	9·268	1	1	2	2	3	3	4	4	5
86	9·274	9·279	9·284	9·290	9·295	9·301	9·306	9·311	9·317	9·322	1	1	2	2	3	3	4	4	5
87	9·327	9·333	9·338	9·343	9·349	9·354	9·359	9·365	9·370	9·375	1	1	2	2	3	3	4	4	5
88	9·381	9·386	9·391	9·397	9·402	9·407	9·413	9·418	9·423	9·429	1	1	2	2	3	3	4	4	5
89	9·434	9·439	9·445	9·450	9·455	9·460	9·466	9·471	9·476	9·482	1	1	2	2	3	3	4	4	5
90	9·487	9·492	9·497	9·503	9·508	9·513	9·518	9·524	9·529	9·534	1	1	2	2	3	3	4	4	5
91	9·539	9·545	9·550	9·555	9·560	9·566	9·571	9·576	9·581	9·586	1	1	2	2	3	3	4	4	5
92	9·592	9·597	9·602	9·607	9·612	9·618	9·623	9·628	9·633	9·638	1	1	2	2	3	3	4	4	5
93	9·644	9·649	9·654	9·659	9·664	9·670	9·675	9·680	9·685	9·690	1	1	2	2	3	3	4	4	5
94	9·695	9·701	9·706	9·711	9·716	9·721	9·726	9·731	9·737	9·742	1	1	2	2	3	3	4	4	5
95	9·747	9·752	9·757	9·762	9·767	9·772	9·778	9·783	9·788	9·793	1	1	2	2	3	3	4	4	5
96	9·798	9·803	9·808	9·813	9·818	9·823	9·829	9·834	9·839	9·844	1	1	2	2	3	3	4	4	5
97	9·849	9·854	9·859	9·864	9·869	9·874	9·879	9·884	9·889	9·894	1	1	1	2	3	3	4	4	5
98	9·899	9·905	9·910	9·915	9·920	9·925	9·930	9·935	9·940	9·945	0	1	1	2	2	3	3	4	4
99	9·950	9·955	9·960	9·965	9·970	9·975	9·980	9·985	9·990	9·995	0	1	1	2	2	3	3	4	4

HEAT

THERMAL EXPANSION OF LIQUIDS

EXPERIMENT—1

Object. To determine the coefficient of apparent expansion* of a liquid (glycerine) with a weight thermometer.

Apparatus required. A weight thermometer, glycerine, heating arrangement, a thermometer, a beaker, chemical balance, and a weight box.

Description of the Apparatus. A weight thermometer is generally of the shape depicted in the accompanying figure. It consists of a thin-walled cylindrical glass bulb A drawn out at the upper end into a capillary stem BCD bent twice at right angles. A convenient size is obtained by making A nearly three inches long, and half an inch wide. The apparatus measures the expansion of a liquid in A relative to glass. In order to deduce the real expansion of the liquid, that of the material of the weight thermometer should be known. The instrument is called a weight thermometer since with its help temperatures can be evaluated by measuring the changes in the mass of the liquid required to fill the whole thermometer at different temperatures.

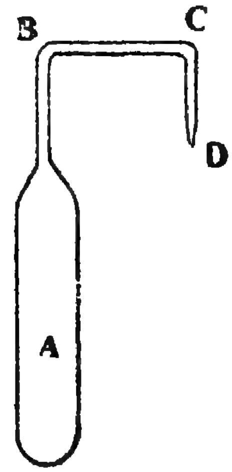

Fig. 1
Weight
Thermometer

Formula Employed. The coefficient of apparent expansion (C_a) of a liquid is calculated with the help of the formula—

$$C_a = \frac{m}{M.t}$$

* The Coefficient of Apparent Expansion of a liquid is defined as the apparent increase in volume per unit volume at 0°C for one degree rise in temperature. Its unit is "per °C".

where m = Mass of the liquid expelled when the weight thermometer is placed in a hot bath.

M = Mass of the liquid *remaining* in the weight thermometer.

t = Rise in temperature of the liquid.

PRINCIPLE AND THEORY OF THE EXPERIMENT

Let us assume that a weight thermometer is completely filled with a liquid at 0°C. Then it is placed in a bath at t°C. when, on account of expansion, some of the liquid overflows. Let the mass of this expelled liquid be m, and let the mass of the liquid remaining in the weight thermometer be M. Hence, the mass of the liquid required to fill the weight thermometer at 0°C is $(m + M)$. If the density of the liquid at 0°C be d_0, then

$$\text{Volume of the weight thermometer} = \frac{m + M}{d_0} \quad \ldots \quad (1)$$

But a mass M of the liquid, whose volume at 0°C is M/d_0, fills completely the weight thermometer at t°C when the volume is $(M + m)/d_0$. Since we are finding the apparent expansion of the liquid the increase in the volume of the glass is neglected. Hence, the apparent expansion of the liquid between 0°C and t°C

$$= \frac{M + m}{d_0} - \frac{M}{d_0} = \frac{m}{d_0} \quad \ldots \quad (2)$$

Thus, by definition, the coefficient of apparent expansion of the liquid is given by

$$C_a = \frac{m/d_0}{M/d_0 \times t} = \frac{m}{M.t} \quad \ldots \quad (3)$$

[**Note**—Instead of filling the weight thermometer at 0°C, as indicated above, it may be filled at the room temperature (t_1). If very great accuracy is not required, C_a can be calculated from the formula—

$$C_a = \frac{m}{M(t_2 - t_1)} \quad \ldots \quad (4)$$

where t_2 is the temperature to which the weight thermometer is subsequently heated.]

Method

(i) Weigh a clean dry weight thermometer in a chemical balance. After this dip the mouth of the weight thermometer in the experimental liquid contained in a crucible, and warm* gently

* It is advisable to heat the bulb with hot air first by holding the burner flame a bit below it.

the bulb, waving the burner flame uniformly underneath it. This process forces a few air bubbles to escape out through the liquid.

(ii) Now remove the burner. Some liquid* will be drawn in. Continue this process of alternate heating and cooling till a sufficient quantity of the liquid collects into the bulb. Now heat carefully till the liquid in the bulb begins to boil and the air inside the weight thermometer is completely expelled by the vapours of the liquid. Let the weight thermometer cool.** The vapours shall condense and the whole thermometer shall be completely filled by the liquid†. Let it attain the room temperature with the nozzle of the weight thermometer still dipping in the liquid.

(iii) When the weight thermometer has acquired the room temperature, wipe out cautiously with a piece of filter paper any liquid sticking to the end of the stem. Weigh the weight thermometer and thus determine the mass of the liquid contained in it at the room temperature.

(iv) Now transfer the weight thermometer in a beaker containing water, which may then be heated to its boiling point‡. Receive the expelled liquid in a vessel. When the expulsion of the liquid is over for some time, take out the weight thermometer, wipe out the water sticking to it and allow it to cool to the room temperature. Again weigh it in the chemical balance. This gives the mass (M) of the liquid remaining in the bulb. From the two weighings calculate the mass (m) of the liquid expelled out.

(v) Note the room temperature (t_1) as well as the temperature (t_2) of the bath. Then calculate the value of the coefficient of

---

* In order to avoid the cracking of the warm glass due to the incoming cool liquid, the latter should be warmed previously. However, if the weight thermometer is made of pyrex glass (coefficient of expansion $= 3 \times 10^{-6}$) or of fused quartz (coefficient of expansion $= 0.4 \times 10^{-6}$), this difficulty of cracking shall not arise. But the weight thermometers of these materials, specially the latter one, are costly and may not be available in the laboratory.

** To cool the weight thermometer quickly, it may be dipped in a beaker of cold water, but, during this process, the nozzle of the weight thermometer *should be kept dipped* in the experimental liquid.

† There should be no air bubble in the weight thermometer. If any is detected, it should be cautiously removed by gently warming the weight thermometer with its nozzle dipped under the liquid.

‡ If water, instead of glycerine, is employed as the experimental liquid, the bath should perferably be maintained (with a thermostat) at (say) 60°C. This will minimise the evaporation of water filling the weight thermometer. In this case, a large-sized weight thermometer should be used, so that the mass of the over-flowing water is appreciable.

apparent expansion for the liquid with the help of the formula (4) given above.

Observations

S. No.	Determinations	Magnitude
1.	Mass of the empty wt. thermometer	......gm.
2.	,, ,, ,, + liquid at t_1°C	......gm.
3.	,, ,, ,, + liquid at t_2°C	......gm.
4.	Room temperature (t_1)	......°C
5.	Temperature of boiling water (t_2)	......°C

Calculations

(1) Mass of the liquid expelled, (m) .gm.

(2) Mass of the liquid remaining, (M) .gm.

(3) Rise in temperature, (t_2-t_1) .°C.

$$\therefore C_a = \frac{m}{M(t_2-t_1)}$$

$$= \text{...per °C.}$$

Result. The coefficient of apparent expansion of glycerine between the range of temperature from......°C to......°C=......per°C.

[Standard value = ... per °C ; Error = ...%]

Precautions and Sources of Error

(1) The filling of the weight thermometer with a liquid requires some skill and patience too. Hence, it should be done slowly and cautiously. Moreover, the liquid should be warmed up previously, so that when it first enters the weight thermometer, it may not crack the latter due to a sudden change in temperature.

(2) The weight thermometer should not be weighed while it is not. It should be weighed only when it is cooled to the room temperature. Weighings should be done after wiping off all liquid sticking to the outside surface of the weight thermometer.

(3) In these determinations, the mass of the expelled liquid is a small quantity, it should, therefore, be determined accurately. Hence all weighings should be done in a chemical balance.

(4) The most important precaution in this experiment is that there should be no air-bubble, however small,* anywhere in the weight thermometer. If any bubble is found obstinately sticking to the neck of the weight thermometer, it should be removed by boiling the liquid, until half of it is evaporated and the process of filling should be repeated.

(5) A part of the stem of the weight thermometer projects outside the bath. Hence, it does not acquire the temperature of the bath, which introduces *some error* in the result.

ADDITIONAL EXPERIMENT

Expt.—1 (a)

Object To determine the coefficient of real expansion of a liquid (glycerine) with a weight thermometer.

We know that the two coefficients of expansion of a liquid are connected by the relation :

$$C_r = C_a + C_g$$

where C_r is the coefficient of real expansion of the liquid, and C_g is the coefficient of cubical expansion of the material (glass) of which the weight thermometer is made.

Now the coefficient of volume expansion of glass (C_g) is equal to three times its linear expansion, which can be known from the Table of Physical Constants. However, it must be admitted that this procedure is not free from objection. When the weight thermometer is blown in this form its thermal properties undergo a material change, hence it is not permissible to assume that its cubical expansion is three times of that value which is given in the table for ordinary glass.

An alternative method is to use an experimental liquid (say, mercury) whose coefficient of real expansion (= 0 000182) is accurately known. Determine the value of C_a for mercury as explained in the main experiment above. Then, calculate the value of C_g from the relation, $C_g = C_r - C_a$. Now this value of C_g can be used in subsequent determinations. This method, however, is not very suitable for a laboratory practice.

Another procedure to determine the value of C_r is to use a weight thermometer made of fused silica, whose coefficient of

* The coefficient of expansion of air is 367×10^{-5} while that of glycerine is 53×10^{-5} (and that of water is 15×10^{-5}). Thus, if there is even a very tiny air-bubble in the weight thermometer, its size will become appreciable at the higher temperature, hence it will drive out extra liquid along with it, thus giving rise to spurious expansion of the liquid. This will then constitute a serious source of error.

expansion ($C_g = 0{\cdot}4 \times 10^{-6}$ nearly) is vanishingly small. Its expansion for ordinary differences of temperature shall be practically zero, hence the coefficient of expansion of the liquid determined with its help shall be the real one.

EXPERIMENT—2

Object. To determine the coefficient of real expansion of a liquid (water) by hydrostatic method.

Apparatus Required. A chemical balance with weight box, a sinker in the form of a glass bulb loaded with lead shots, a thermometer, beaker, stirrer, and a long thin copper wire.

Description of the Apparatus. The apparatus consists of a chemical balance, to the left pan of which a thin (diameter less than one-tenth of a mm.) copper wire can be attached. A hole is provided in the base on the balance-case through which the wire can pass. Another hole is made in the top of the shelf on which the balance-case rests so that the wire passes freely through the two holes. The sinker is suspended from the lower end of the wire, and is immersed in a beaker of water which can be heated to any desired temperature. The beaker also contains a thermometer (T) and a stirrer (S).

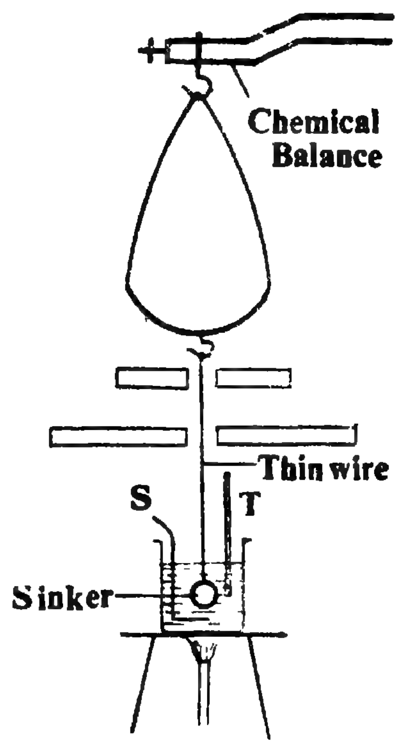

Fig. 2
Hydrostatic Balance

Formula Employed. If the loss in weight of the sinker at t_1°C be W_1, and at t_2°C be W_2, then

$$\frac{W_2}{W_1} = \frac{1 + C_g (t_2 - t_1)}{1 + C_r (t_2 - t_1)}$$

where C_g is the coefficient of cubical expansion of the sinker (glass) and C_r is the required coefficient of expansion of the liquid (water in this case). From the above we get, after simplification, the required formula—

$$C_r = \frac{W_1 - W_2}{W_2 (t_2 - t_1)} + \frac{W_1}{W_2} C_g$$

PRINCIPLE AND THEORY OF THE EXPERIMENT

Let the sinker be weighed in air, and then in water at t_1°C. Let the loss in weight at this temperature be W_1. Then according to the Principle of Archimedes

Weight of the water displaced at $t_1°C = W_1$

Hence. Volume of the water displaced at $t_1°C = W_1/d_1$

where d_1 is the density of water at $t_1°C$.

Now let the water be heated to $t_2°C$ and the loss in weight of the solid be W_2. Then

Volume of the water displaced at $t_2°C = W_2/d_2$

where d_2 is the density of water at $t_2°C$.

Obviously, the volumes of the water displaced at these two temperatures should be equal to the volumes of the sinker at these temperatures. Hence

Volume of the sinker at $t_1°C = W_1/d_1$

and Volume of the sinker at $t_2°C = W_2/d_2$

But, by definition, the volumes of the sinker at the two temperatures are related by the following formula—

$$\frac{W_2}{d_2} = \frac{W_1}{d_1}[1 + C_g(t_2 - t_1)]$$

or

$$\frac{W_2}{W_1} = \frac{d_2}{d_1}[1 + C_g(t_2 - t_1)] \quad \ldots \quad (1)$$

where C_g is the coefficient of cubical expansion of the material (glass) of the sinker.

But*

$$\frac{d_2}{d_1} = \frac{1}{1 + C_r(t_2 - t_1)} \quad \ldots \quad (1)$$

where C_r is the coefficient (real) of cubical expansion of water.

This can be easily proved as follows :—

Let the volume of a certain mass of the liquid at $t_1°C$ be V_1 and its density be d_1, and let these quantities for the same mass of the liquid at $t_2°C$ be respectively V_2 and d_2. Then the mass of the liquid

$$V_1 d_1 = V_2 d_2$$

or

$$\frac{d_2}{d_1} = \frac{V_1}{V_2} = \frac{V_0(1 + C_r t_1)}{V_0(1 + C_r t_2)}$$

where V_0 is the volume of the same liquid at 0°C. From the above expression, we have

$$\frac{d_2}{d_1} = \frac{1}{(1 + C_r t_2)(1 - C_r t_1)} = \frac{1}{1 + C_r(t_2 - t_1)}$$

expanding with the help of the Binomial Theorem and retaining only the first power of C_r which is a small quantity.

Substituting the value of d_2/d_1 from equation (2) in (1), we have

$$\frac{W_2}{W_1} = \frac{1 + C_g\ (t_2 - t_1)}{1 + C_r\ (t_2 - t_1)}$$

from which
$$C_r = \frac{W_1 - W_2}{W_2\,(t_2 - t_1)} + \frac{W_1}{W_2} . C_g \quad \ldots \quad (3)$$

This equation is utilised to determine C_r by determining the losses in weight of a sinker at two known temperatures.

Method

(i) Tie a very thin piece of copper wire* to the left-hand pan of the chemical balance as shown in the figure, and let it pass freely through the two holes below. The sinker attached to the lower end of the wire can be completely immersed when desired, in water contained in a beaker, which can be heated.

(ii) Find the mass of the sinker in air. Heat the water to a high temperature, say 80°C, and immerse the sinker completely in it. Stir the water now and then to ensure uniformity of temperature. As soon as the whirls, produced by vigorous stirring, have subsided the balance is effected and the temperature taken by a thermometer immersed in water with its bulb as near to the sinker as possible.

(iii) In order to make a balance when the liquid is cooling, the weights in the other pan are to be *continually diminished*. Adjust the weights on the pan before an observation so that the sinker appears a little too heavy. After a short interval the pointer of the balance will cross the zero position. At this instant the temperature of the water should be observed. The sum of the weights placed in the pan gives the mass of the solid at this temperature.

---

* (1) The wire should be thin, its diameter being not greater than 0·1 mm, so that surface tension may not cause any appreciable effect upon the wire where it enters the water. In practice a thin copper wire may be employed, though it is always preferable to use a short length of platinum wire, specially treated to diminish surface tension effect for immersion in the water.

(2) Since during some part of the experiment the water will be at a temperature much above that of the immediate surroundings of the balance, it is essential to use a wire about 40 cms., long, so that convection currents may not disturb the equilibrium of the balance. In that case a specially prepared wire should be employed. Nearly 30 cms. of the wire may be of copper and the lower end (nearly 10 cms. long) may be of specially treated platinum as pointed above.

(iv) In this way, record a series of weighings of the sinker for different temperatures. Then calculate C_g, for each separate temperature interval, from the formula given above :

Observations

Weight of the sinker in air	Temperature of water (t_2)	Correspond-ing weight of the sinker in water	Loss in weight at these tempe-ratures	Remarks
				(1) C_g (given) = ... per °C. (2) Room temp. (t_1) = ... per °C.

Calculations

Set I. (For temp. difference = ...°C)

$$C_r = \frac{W_1 - W_2}{W_2 (t_2 - t_1)} + \frac{W_1}{W} C_g$$

$$= \text{......per °C.}$$

[**Note**—Make similar calculations for other temperature differences*.]

Result—The coefficients of real expansion of water are tabulated† below :—

Temperature interval	Coefficient of real expansion

* The coefficient of thermal expansion is not a constant quantity but varies with temperature. Hence report the result for different temperature ranges which should be small.

† For standard values, see Table–4 given at the end of the book.

Precautions and Sources of Error

(1) The wire chosen for suspending the sinker should have a diameter of not more than 0·1 mm., so that capillary action, where the wire cuts the water surface, may not create difficulty in finding the correct weight of the sinker. For this purpose, a properly treated platinum wire should be employed.

(2) The wire should be at least 40 cms. long. By using such a long wire, convection currents produced by the liquid at the higher temperature shall not disturb the equilibrium of the balance.

(3) The liquid should be well stirred to secure uniformity of temperature throughout its entire mass. The beam of the balance should be raised for checking the equilibrium only when the whirls, produced by stirring, subside.

(4) The bulb of the thermometer should be as close to the sinker as possible and its reading should be observed as soon as the pointer of the balance crosses the zero position.

(5) To procure a sufficiently measurable loss in weight, and thus to increase the accuracy of the result, a suitable sinker should be used in the experiment. A convenient form of the sinker is a glass bulb containing lead-shots and properly sealed.

(6) In this experiment, there can be some error due to the value of C_g, as taken from the Table of Constants. The glass bulb may have a different coefficient of expansion*.

* This error can be eliminated by actually determining C_g for the experimental sinker taking a liquid of known expansion and making use of the formula given above. This value of C_g can be employed in subsequent determinations.

CALORIMETRY

EXPERIMENT—3

Object. To determine the specific heat of a liquid (glycerine) by the method of cooling.

Apparatus Required. Two small and exactly identical calorimeters, a double-walled vessel with lid, three thermometers, stop-watch, physical balance, weight box, glycerine, and heating arrangement.

Description of the Apparatus. The double-walled vessel consists of two rectangular chambers in the annular space of which cold water is filled in. This ensures the constancy of the temperature of the surroundings. The cover of the chamber carries two holes closed with bored rubber stoppers. The two thermometers needed to record the temperatures of the two liquids pass through the stoppers. The lower parts of the stems of the thermometer carry two more corks which fit tightly into the calorimeters which consequently remain suspended. The two small calorimeters are exactly alike and are generally made of aluminium. The outside of the calorimeters is lamp-blacked so that when hot liquids are filled in them, heat is lost to the surroundings primarily through radiation. The calorimeters are generally provided with a mark on the inner surface, upto which level the liquid is filled in. This ensures equal volumes (*not masses*) of the liquids taken in the two calorimeters. A thermometer is also kept in the annular space between the two walls of the chamber to record the temperature of the surrounding water.

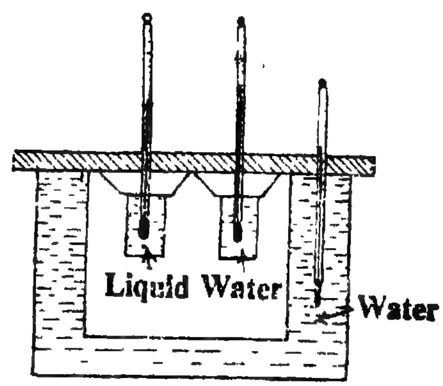

Fig. 3
Specific heat by the method of cooling

Formula Employed. The specific heat (S) of the experimental liquid is given by the formula—

$$\frac{MS + W}{t_1} = \frac{m + W}{t_2}$$

where M = Mass of the experimental liquid
W = Water equivalent* of the calorimeter (= mass × specific heat of calr.)
m = Mass of an *equal volume* of water
t_1 = Time required by the liquid to fall in temp., say, from θ_1 to θ_2.
t_2 = Time required by water to cool through the same range of temperature.

PRINCIPLE AND THEORY OF THE EXPERIMENT

When a hot body is allowed to lose its heat by radiation, the loss of heat depends on the area of the body, the nature of its surface, and the excess of temperature of the body over its surroundings. If at any instant the excess of temperature be θ, the loss of heat dQ in time dt is given by

$$dQ = A_1 f(\theta)\, dt \qquad \ldots \qquad (1)$$

where A_1 is a constant depending on the area of the body and the emissive power of the surface, $f(\theta)$ is some function of the excess of temperature.

If this loss of heat dQ produces a lowering in temperature of the body by $-d\theta$, we have

$$dQ = m_1 S_1\, d\theta \qquad \ldots \qquad (2)$$

where m_1 is the mass and s_1 is the specific heat of the body. Equating (1) and (2) we have

$$A_1 f(\theta)\, dt = -m_1 s_1\, d\theta$$

or

$$\int_0^{t_1} dt = -\frac{m_1 s_1}{A}\int_{\theta_1}^{\theta_2} \frac{d\theta}{f(\theta)}$$

or

$$t_1 = \frac{m_1 s_1}{A_1}\int_{\theta_2}^{\theta_1} \frac{d\theta}{f(\theta)} \qquad \ldots \qquad (3)$$

If another body be also allowed to cool from θ_1 to θ_2 and the time taken be t_2 we have

$$t_2 = \frac{m_2 s_2}{A_2}\int_{\theta_2}^{\theta_1} \frac{d\theta}{f(\theta)} \qquad \ldots \qquad (4)$$

If $A_1 = A_2$ which implies that the surfaces of the two bodies radiat-

* The water equivalent of the two calorimeters should be nearly the same.

ing heat, as well as their powers of emission are equal*, we have from equations (3) and (4) the following relation :

$$\frac{m_1 s_1}{t_1} = \frac{m_2 s_2}{t_2} \quad \ldots \quad (5)$$

The above principle is employed in the determination of the specific heat of a liquid. If two exactly identical calorimeters (thus having equal thermal capacity W) be taken, and equal volumes of water and another experimental liquid be taken in them and if the masses of the two liquids to m and M respectively, we have† from (5)

$$\frac{MS + W}{t_1} = \frac{m + W}{t_2} \quad \ldots \quad (6)$$

Where S is the specific heat of the experimental liquid.

Method

(i) First of all, find the masses of the two calorimeters when empty, then coat the outside of the calorimeters uniformly with lamp-black by holding and turning them in a sooty flame. Heat separately the given liquid and water to the same temperature (between 60° and 70°C). Pour the experimental liquid in one of the calorimeters, and water in the other one, both up to the marks provided inside them. This ensures that the volumes of the two liquids are equal, as demanded by theory. Susperd the calorimeters with the help of the two thermometers as shown in the figure. Fill the annular space between the walls of the enclosure with cold water‡, so that the inner chamber forms a constant temperature enclosure.

(ii) Commence taking readings†† of the two thermometers at intervals of a minute noting time with the help of a stop-watch. Keep the liquids gently stirred.

* To ensure the equality $A_1 = A_2$, the two calorimeters are blackened from outside (thereby their emissivities are equal), and the two liquids in the calorimeters are taken in equal volume (thereby heat radiating surfaces are made equal).

† In this expression t_1 and t_2 are the respective times taken by the experimental liquid and by water to cool down through the *same* range of temperature.

‡ No water is to be poured in the inner chamber in which the calorimeters are to be suspended.

†† Readings should be commenced when the two thermometers indicate approximately the same temperature. A convenient plan to note the temperatures as well as the corresponding time in the stop-watch is to take the reading of the thermometer immersed in the experimental liquid when the second's hand of the stop-watch is at 60, and note the reading of the other thermometer when the second's hand is at 30.

(iii) Continue taking readings of the two thermometers at regular intervals of one minute. Continue the readings till the temperature is a few degrees above the room temperature.* Also note the temperature of the water between the walls a few times.

(iv) At the end of these observations, remove the calorimeters and weigh them to determine the masses of the two liquids. Now plot two cooling curves (see fig–4) on a sheet of graph paper taking time as abscissa and temperature as ordinate. From the curves determine the times (t_1 and t_2) required by the two liquid to cool through same range of temperature (from θ_1 to θ_2), where θ_1 may be taken equal to say, 40°C, and θ_2 equal to 35°C.

(v) By taking the specific heat of the material of the two calorimeters as known, calculate the specific heat of the experimental liquid with the help of formula given above.

Observations—*Readings for the determination of θ and t*

S No.	Time	Temp. of glycerine	Temp of water	Remarks
				(1) Mass† of the first calorimeter = ...gm. (2) ,, ,, + glycerine = ...gm. (3) Mass of the second calorimeter = ...gm. (4) ,, ,, + water = ...gm. (5) Temp. of cold water in the annular space of enclosure : (i) ......°C ; (ii) ......°C. (iii) ......°C ; (iv) ......°C.

* Glycerine cools more rapidly and therefore it will reach the last temperature of observation first. When this is so, the readings of the thermometer dipped in this should be discontinued, but the readings of the other thermometer (dipped in water) should be continued. It should further be noted that it is the time to cool through equal intervals of temperature which has to be observed, and note the temperature change in equal intervals of time.

† By actual weighings it will be seen that the difference, if any, in the masses of the two calorimeters, is negligibly small.

Calculations

(i) Mass of water ... = ...gm.
(ii) Mass of glycerine ... = ...gm.

From the graph*

(iii) Time taken by water to cool from ......°C to ......°C = ...sec.
(iv) Time taken by glycerine to cool through the same range = ... sec.

Now
$$\frac{MS + W}{t_1} = \frac{m + W}{t_2}$$

$\therefore$ S = ... cals per gm. per °C.

Result. The specific heat of glycerine= .. cals. per gm. per °C
[Standard value = ...cals. per gm. per °C
Error= ...%]

Precautions and Sources of Error

(1) The two calorimeters should be properly suspended in the enclosure so that they may not lose their support and fall down. None of the two calorimeters should touch the inside chamber.

(2) The calorimeters should be made of a material whose conductivity is high. Their walls should be thin. Then only the temperature of the liquid as recorded by its thermometer shall be equal to that of the radiating surface of the calorimeter.

(3) The space between the walls of the vessel should be filled with cold water so that it can be safely assumed that the temperature of the surroundings is constant. It should also be recorded a number of times during the course of the experiment to see whether this constancy is maintained or not.

(4) The readings of the two calorimeters should not be commenced till the two thermometers record approximately the same temperature. The initial temperature should be considerably below the boiling point of the two liquids.

(5) According to theory, the formula for the specific heat of the liquid is valid only when the surfaces emitting radiation are equal in area and their radiating powers are also equal. To procure this

(a) The outer surfaces of the two calorimeters should be uniformly lamp-blacked. Surfaces should not be spoiled subsequently during handling etc.

* See the graph given in Fig. 4.

(h) The volumes of the two liquids should be equal.

(6) The determination of the specific heat of a liquid by this method depends on the assumption that the rate of loss of heat is the same for water as well as for the liquid. This is justifiable only when the loss of heat takes place under exactly identical conditions. If during the course of this experiment, this condition is not completely fulfilled, the result shall be adversely effected. Moreover, mercury thermometers are not susceptible of great accuracy, specially when observations on rapidly varying temperatures are to be conducted.

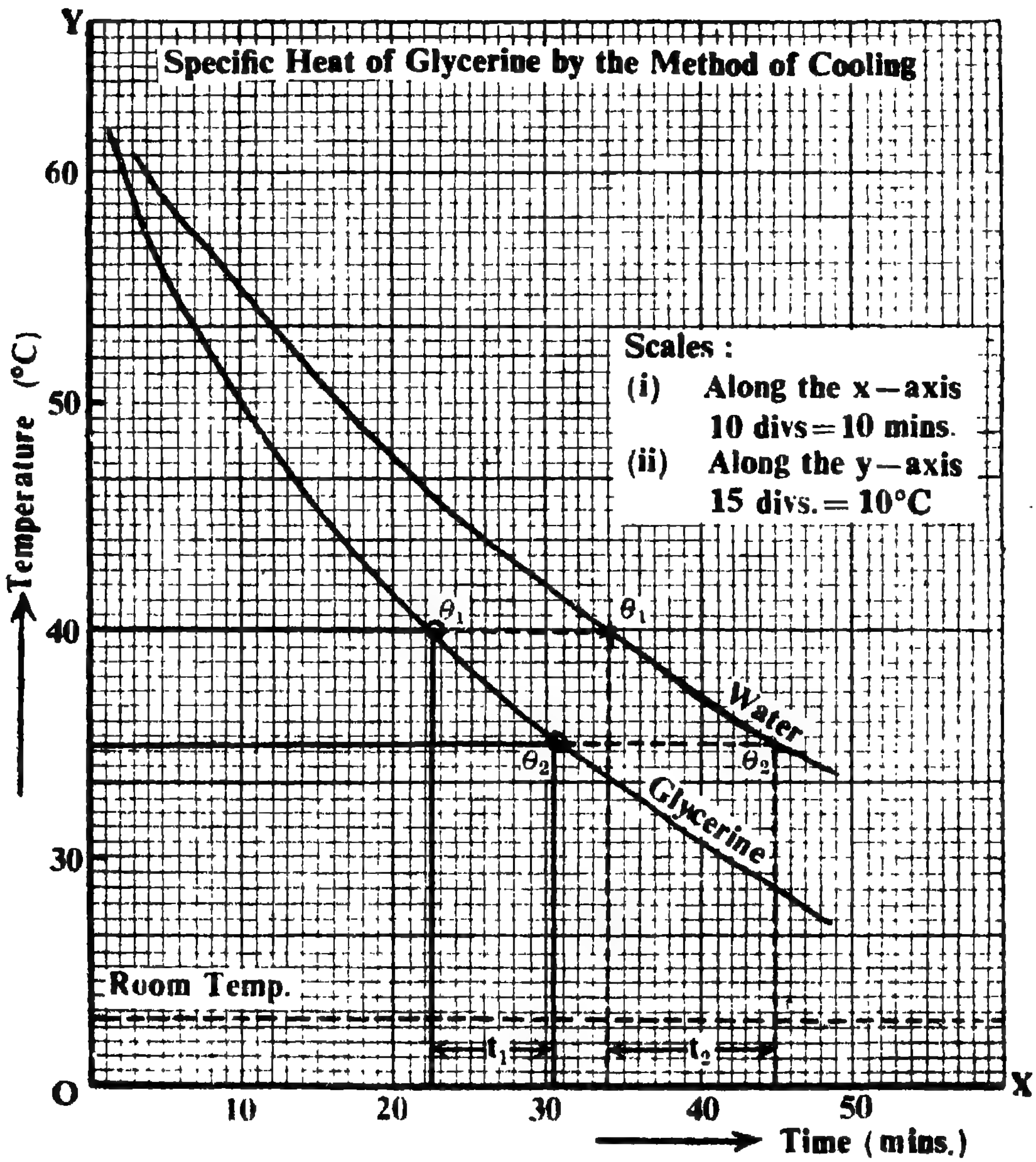

Fig. 4

Cooling curves for water and glycerine

EXPERIMENT—4

Object. To determine the specific heat of copper by using a copper block calorimeter.

Apparatus Required. Copper block calorimeter, a sensitive thermometer (reading upto one-tenth of a degree), a heating coil, battery, rheostat, ammeter, voltmeter and a stop-watch.

Description of the Apparatus. The copper block calorimeter (fig.–5) consists essentially of a block of copper over which is wound evenly a heating coil which is electrically insulated from the calorimeter by a thin paraffined paper. Heat is supplied electrically with the help of a battery, a suitable current being adjusted with the help of a rheostat. A hole is drilled in the middle of the block to receive a thermometer for recording its temperature. In order to minimise losses of heat by conduction and convection, the calorimeter is surrounded by a non-conducting medium.

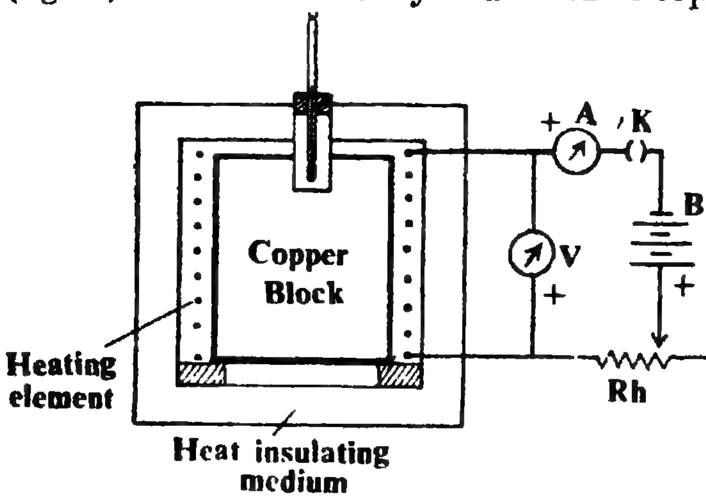

Fig. 5
Copper block calorimeter
(with electric circuit)

Formula Employed. The specific heat (s) is calculated with the help of the following formula—

$$s = \frac{VIt \cdot 10^7}{M\theta \cdot J}$$

or putting J $4{\cdot}2 \times 10^7$ we have

$$s = 0{\cdot}24 . \frac{VIt}{M\theta}$$

where V = The potential difference across the heating coil.

I = The current flowing in the coil.

t = Time during which the current is passed.

M = Mass of the copper block.

θ = Rise in temperature.

PRINCIPLE AND THEORY OF THE EXPERIMENT

Electrical energy is supplied to the copper block whose temperature consequently rises. Thus by knowing the amount of expenditure of electrical energy and the heat gained by the calorimeter, the specific heat of the material of the calorimeter can easily be calculated.

Thus, if a current of I amperes be passed through the heating coil for t seconds, and if V volts be the difference of potential across the coil, the amount of work done (W), according to Joule's laws, is given by

$$W = VIt \,.\, 10^7 \text{ ergs.}$$

Hence, from the first law of thermo-dynamics the amount of heat (H) generated is given by the formula—

$$H = \frac{W}{J} = \frac{V.I.t.\,10^7}{J} \text{ calories}$$

where J is the mechanical equivalent of heat.

If M be the mass, s the specific heat of the material of the calorimeter, and if θ be the rise in temperature, the heat gained by the calorimeter is given by

$$Q = Ms\theta \text{ calories.}$$

If we neglect all losses of heat by radiation etc., we have $H = Q$. Hence

$$Ms\theta = \frac{VIt \,.\, 10^7}{J}$$

or $$s = \frac{VIt \,.\, 10^7}{M\theta \,.\, J} \text{ cals. per gm. per °C.}$$

Thus knowing the value of J and measuring other quantities occurring on the right, the value of s can be calculated.

Method

(i) Take out the copper block calorimeter and weigh it. Now make the electrical connections as shown in the figure, and adjust the values of the current and the voltage before fitting in the calorimeter.

(ii) Now replace the calorimeter. Insert a sensitive thermometer in the hole provided in the block for this purpose.

(iii) Switch on the current and start *immediately* the stop-watch. At regular intervals of time (say, half a minute) go on recording the temperature of the calorimeter. When there has been appreciable rise of temperature (usually about 5 to 6°C), switch off the current and note the time for which the current has been

passed. Record also the readings of the ammeter and the voltmeter.*

(iv) Allow the calorimeter to cool and record the fall in temperature of the calorimeter at regular intervals of time.

(v) From these observations obtain graphically† the corrected rise in temperature of the calorimeter, and calculate the value of the specific heat of copper with the help of the formula given above.

Observations

[A] (i) Mass of the copper calorimeter = ...gms.

(ii) P. D. across the heating coil = ...volts.

(iii) Current flowing through the coil = ...amps.

(iv) Time for which current is passed = ...secs.

[B] *Readings for applying radiation correction.*

S. No.	Readings when calr. is being heated			Readings when heating is discontinued		
	Time	Temp.	Point on graph	Time	Temp.	Point on graph

Calculations

Corrected rise of temperature
(obtained from the group) = ...°C

$$s = \frac{VIt.\ 10^7}{M\theta.\ 4.2 \times 10^7}$$

= .... cals. per gm. per °C

* The readings of these instruments should be kept constant throughout the experiment by adjusting the rheostat. Moreover, special care should be taken to record the time and temperature accurately at the requisite intervals.

† This has been explained fully at the end of the present experiment in this connection, see fig.–6 and 7.

Result. The specific heat of copper between the range of temperatures from ......°C to ......°C = ...cals. per gm. per °C.

[Standard value = ... ; Error = ...%]

Precautions and Sources of Error

(1) The readings of the ammeter and the voltmeter should be adjusted to suitable values prior to putting the copper block calorimeter in its heating coil arrangement. These values should be maintained constant throughout the heating process.

(2) A sensitive thermometer should be employed to record temperatures. It should preferably read upto one-tenth of a degree.

(3) Special care should be devoted in recording temperatures at regular intervals of time. Attempt should be made to note the readings of time and temperature simultaneously.

(4) There is always a certain amount of lag in the record of the temperature with a thermometer, specially when the temperature is rapidly changing. Thus, there will be a difference in the observed value and the actual value of the temperature of the calorimeter when the temperature is rising. No correction has been applied in the present case, hence the result is not free from error on this account.

A NOTE ON THE RADIATION CORRECTION CALORIMETRIC MEASUREMENTS

In an experiment for the determination of the specific heat of a solid by the method of mixtures, let m be the mass of the solid, s its specific heat, and T its initial temperature, and if W be the water equivalent of the calorimeter and it contents, t_1 initial temperature, and let t_2 be its final temperature, then

$$ms (T - t_2) = W (t_2 - t_1)$$

It this equation, it has been assumed that there has been no loss of heat. In practice there is a loss of heat from the calorimeter, which, for accurate work, should be added to the right-hand side of the equation since all the heat given out by the hot solid to the calorimeter is not retained by the latter, and some is invariably lost to the surroundings. Thus, a correction for the error arising from this cause is essential. We may make the correction by the consideration that the final temperature t_2 would have been $t_2 + \Delta t$, had there been no loss of heat by the calorimeter to its surroundings. Hence, the above equation should take the following corrected form :—

$$ms (T - t_2) = W [(t_2 + \Delta t) - t_1]$$

For this purpose, the temperature of the calorimeter is noted immediately before dropping the hot body, and then the temperature is noted after regular intervals of half a minute until the maximum temperature is reached, the observations being continued beyond this point. The readings are then plotted on a graph. This graph is depicted in fig -6 where θ represents temperature and t time. The experimental curve is ABC, the portion AB representing the heating process and BC representing the cooling process.

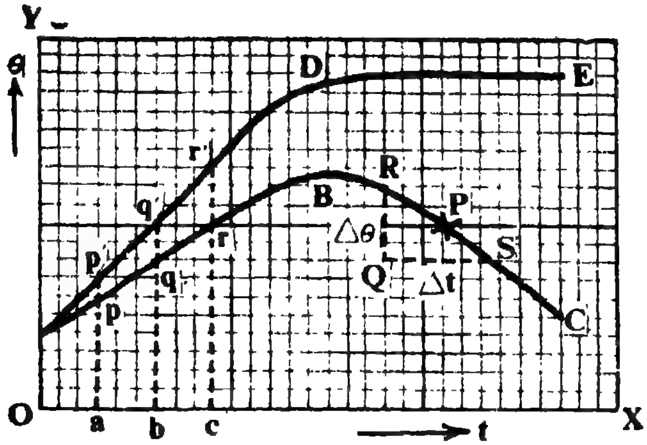

Fig 6

Had there been no losses, the curve should have been similar to one represented at ADE, the final temperature remaining constant at the level DE. The purpose of the experiment is actually to make the correction of the ordinates and draw the curve ADE.

For this purpose we need to know the rate of cooling at any temperature. Take the point P at the mean temperature over the range BC. Take two points R and S near P and complete the right-angled triangle RQS. Let RQ = $\Delta\theta$ and QS = Δt, then obviously the rate of cooling at B is given by the value of $\Delta\theta/\Delta t$ which can thus be determined.

At the room temperature the rate of cooling vanishes, since if the calorimeter be at the same temperature as that of its surroundings, then there would be no exchange of heat between the two. Thus, at the room temperature $d\theta/dt = 0$.

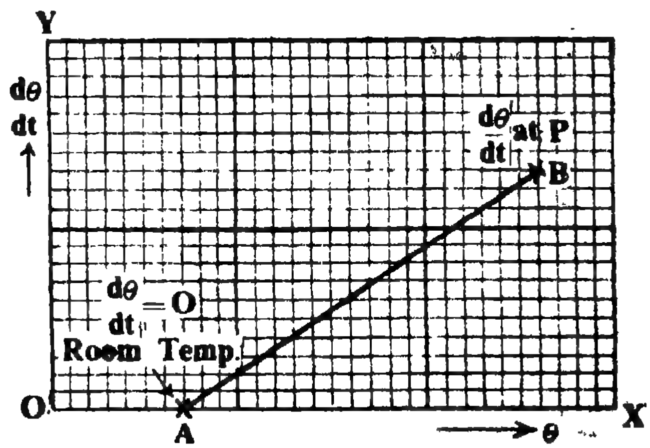

Fig. 7

Assuming the validity of Newton's law of cooling, a graph can be drawn between rate of cooling $(d\theta/dt)$ and temperature (θ), as shewn in fig.-7. The point A on this graph corresponds to the room temperature, and B corresponds to the point P of fig.-6. The

two points are sufficient to determine the course of the curve which should be a straight line.

Now divide the curve ABC into a number of portions which are nearly straight, e. g., Ap, pq, etc. Let the temperatures corresponding to the points A and p be θ_1 and θ_2 respectively. Hence, the mean temperature during the time interval Oa is $\theta_1 + \theta_2/2$. Now from the graph for $d\theta/dt$ (fig.-7) note the rate of cooling corresponding to this temperature. Multiply this by the time Oa, and add the result to the ordinate ap, thus getting ap$'$. Let this correction (= pp$'$) be $\delta\theta_1$.

In the same way calculate the amount $\delta\theta_2$ lost during the interval ab. Add the sum $(\delta\theta_1 + \delta\theta_2)$ to bq and so obtain bq$'$. Continue this process until the maximum ordinates along DE are obtained. It will be revealed on drawing the curve ADE that the portion DE becomes parallel to the x—axis. It means that if there were no loss of heat from the calorimeter, its maximum temperature should have remained constant with time. Hence, the corrected temperature $(t_2 + \Delta t)$ is the ordinate corresponding to this straight portion DE.

Another method, called the *Adiabatic Method*, is to eliminate the loss of heat by continuously adjusting the temperature of the bath, which surrounds the calorimeter, so that it is always equal to the temperature of the calorimeter itself. It is obvious from the manner of carrying out the experiment that heat losses shall be completely eliminated since at any instant the temperature of the calorimeter is equal to that of its immediate surroundings, and therefore there is no temperature gradient existing between the two and hence no exchange of heat. But this method is beyond the scope of ordinary laborating practice.

A simple, but *approximate* method which is often used in the daily laboratory work is to add to the observed maximum temperature half* the cooling produced in a time equai to the duration of the experiment. This calculation is based on the assumption that the average excess of temperature of the calorimeter over its surroundings may be taken equal to half the final excess, hence the cooling is also half the cooling at the final temperature.

EXPERIMENT—5

Object. To determine the latent heat of steam by Joly's steam calorimeter.

* In an experiment with the copper block calorimeter, when heating was continued for 5 mts. the maximum temperature attained was 27°C, and when it was allowed to cool for 5 mts., the temperature reached was 25·4°C. Hence, the required correction is equal to $\frac{1}{2}$ (27 − 25·4)°C = 0·8°C.

Apparatus Required. A double-walled jacket, a chemical balance with weight box, a piece of fine copper wire, a small copper piece, a thermometer, a boiler, a nichrome wire, a battery, and a rheostat.

Description of the Apparatus. A double-walled metal jacket J, the annular space of which is filled with some insulting material, surrounds a small copper pan P suspended by means of a fine copper wire attached to one arm of a chemical balance. The chamber has an inlet tube for the steam to come in from a boiler, and an exit tube for the steam to pass out. A thermometer (Th) inserted through a hole in the jacket gives the temperature of the inside. The jacket is closed at its upper end by a light metallic lid with a hole through which the wire passes without touching the lid. A thin coil* C in the form of a spiral surrounds the wire without touching it.

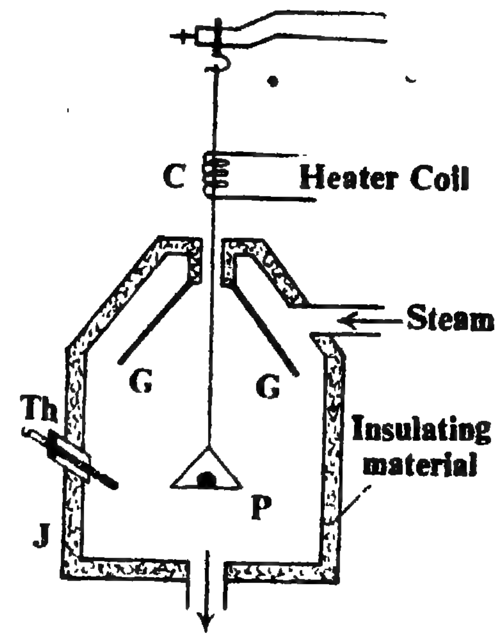

Fig. 8
Joly's steam calorimeter

There are protecting copper shields G, G which do not allow any water drop to fall on the copper pan.

Formula Employed. In this experiment heat is given by the steam in condensation and is taken up by the copper pan and the copper piece.

Heat given out by steam $= ML$ cals.

Heat taken up by the solid $= m_1 s (\theta_2 - \theta_1)$ cals.

Heat taken up by the pan $= m_2 s (\theta_2 - \theta_1)$ cals.

Thus $$ML = (m_1 + m_2)\, S\, (\theta_2 - \theta_1)$$

where M = Mass of the steam condensed on the pan and the body (copper piece); L = Required latent heat of steam; m_1 = Mass of the body (copper piece); m_2 = Mass of the copper pan; S = Specific heat of copper; and $\theta_2 - \theta_1$ = Rise in temperature.

When steam is allowed to enter the chamber, some of it escapes through the hole in the lid andcondenses there forming a thin film of water, and then surface tension makes accurate weighing difficult. To eleminate this error the coil is made to glow (by passing electric current through it) so that the heat developed is just sufficient to prevent condensation of steam at the hole.

PRINCIPLE AND THEORY OF THE EXPERIMENT

Let a body of mass m_1 (and specific heat s_1) be placed on the copper pan of mass m_2 (and specific heat s_2) and let their initial temperature be θ_1. Let steam be passed in the steam jacket, whose temperature begins to rise till at θ_2 it remains steady. Steam is condensed on the pan and the body, which are in dynamic equilibrium with the former, there is a gain in the weight of the pan and the equilibrium of the pan is disturbed. Let the mass of steam condensed be M. and θ_2 be the steady final temperature of the steam jacket. Then

the heat given out by steam $= ML$ cals.

the heat taken by the body $= m_1 s_1 (\theta_2 - \theta_1)$ cals.

the heat taken by the pan $= m_2 s_2 (\theta_2 - \theta_1)$ cals.

Hence $$ML = (m_1 s_1 + m_2 s_2)(\theta_2 - \theta_1)$$

Since in this case $s_1 = s_2$ (say), we have

$$ML = (s_1 + m_2)\, s\, (\theta_2 - \theta_1)$$

Every quantity on the right hand side is known, hence the value of L can be calculated out. †

Method

(i) Suspend the clean and dry copper pan from the left arm of a chemical balance by means of a fine copper wire in the steam jacket in such a way that it hangs in the midple of the jacket and the wire passes freely through the hole in the lid covering the mouth of the jacket.

(ii) Prepare a small helical coil of some resistance (say, nichrome) wire, and pass the suspension wire axially through it, and connect the ends of the coil to a battery, so that the coil may be heated to red glow when desired. Adjust the coil a little above the lid of the jacket.

(iii) Insert a thermometer in the hole provided for the purpose, and find the mass of the pan. Now put the copper piece on this pan and again find the mass of the two together. This gives the mass of the solid body. Note the temperature of the jacket.

(iv) Connect the coil to the battery so that it gets red-hot and connect the delivery tube of the steam generator to the jacket and let a steady current of steam be passed through it. Steam condenses on the solid and the pan, and weights are added on the other pan to maintain equilibrium. When the pan ceases to increase in weight* note the readings and record the final steady temperature. Thus calculate the mass of the steam condensed.

---

† Alternatively, if L be given the specific heat of the material of the solid can be determined by the same process.

* The weight becomes practically constant within six to eight minutes, though a very slow increase of about 4 mgms. per hour may be observed due to radiation.

(v) Calculate the latent heat of steam from the formula given above.

Observations

S. No.	Determinations	Magnitude	Remarks
1.	Mass of the copper pan	...gm.	(1) Mass of the solid = ...gm.
2.	,, ,, ,, & solid body	...gm.	(2) Mass of the steam condensed = ...gm.
3.	Initial temp. of the jacket	...°C	
4.	Mass of the copper pan and solid body & steam condensed	...gm.	(3) Specific heat of copper (given) = ...
5.	Final temp. of the jacket	...°C	

Calculations

(1) Heat given out by the steam $= ML$
$= \ldots\ldots = \ldots\ldots$ cals.

(2) Heat taken by the substance $= m_1 s\,(\theta_2 - \theta_1)$
$= \ldots\ldots = \ldots\ldots$ cals.

(3) Heat taken by the copper pan $= m_2 s\,(\theta_2 - \theta_1)$
$= \ldots\ldots = \ldots\ldots$ cals.

Hence $$ML = m_1 s\,(\theta_2 - \theta_1) + m_2 s\,(\theta_2 - \theta_1)$$
... etc. ... etc.

Result. The latent heat of steam as determined with the help of Joly's steam calorimeter = ...cals. per gm.

[Standard value = ...cals/gm ; Error = ...%]

Precautions and Sources of Error

(1) The suspension wire should be so suspended that it touches neither the lid of the steam jacket nor the glowing helical coil.

(2) The suspension wire should be long so that the steam jacket is situated fairly distant from the pan of the balance, otherwise the pan may be disturbed due to the presence of convection currents.

(3) The suspension wire should be thin, since no account has been taken of the steam condensed on the wire enclosed inside the jacket.

(4) The temperatures of the enclosure should be recorded when they are absolutely steady, specially the temperature of the steam. An interval of certainly not less than fifteen to twenty minutes is required to allow the solid to acquire this temperature.

(5) M does not actually represent the correct weight of the steam condensed since the first weighing is done in air at θ_1 °C and the second in steam at θ_3 °C. Hence, for accurate work all the weighings should be reduced to those in vaccum and then the increase in mass should be calculated.

ADIABATIC TRANSFORMATION

EXPERIMENT—6

Object. To determine the value of γ, the ratio between the two specific heats of a gas (air), by Clement and Desorme's method.

Apparatus Required. A large flask (capacity about 5 litres), a liquid manometer, and a compression pump.

Description of the Apparatus. Clement and Desorme's apparatus consists of a large flask F of nearly 5 litre capacity. This is kept inside a wooden box packed with some non-conducting material (*e.g.*, cotton-wool, asbestos, etc.) to minimise the possibility of exchanges of heat with the surroundings. The flask is fitted with an airtight metallic neck in which pass three tubes. One of the side tubes is connected to a compression pump through a stop-cock T_1 and the other is connected to a liquid manometer M, the levels of the liquid being read with a scale S. The central tube of the flask is comparatively of a wider bore which can be put in communication with the outside atmosphere by operating the stop-cock T_2.

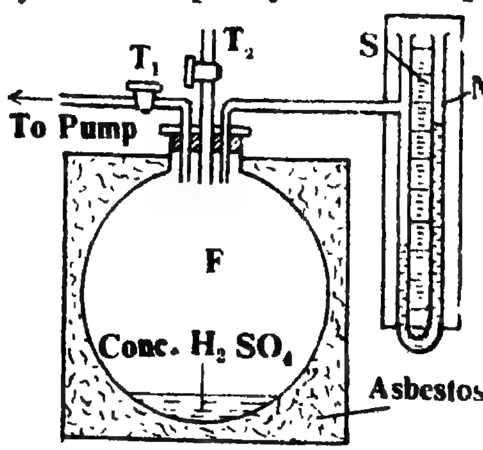

Fig. 9
Clement and Desorme's apparatus

Formula Employed*. The value of γ is determined with the help of the following formula :—

$$\gamma = \frac{h_1}{h_1 - h_2}$$

* This formula is applicable only when the values of h are small (say, a few centimetres of mercury). If the values of h are not small, the exact formula involving logarithms should be employed, as discussed in the Theory of the Experiment.

where h_1 = Initial reading of the manometer when air is compressed into the flask and steady condition is attained.

h_2 = Final reading of the manometer when air has suffered adiabatic expansion and the flask has again attained steady condition.

PRINCIPLE AND THEORY OF THE EXPERIMENT

Let some air be compressed in the flask till the manometer registers an excess of pressure over the atmospheric pressure. When the heat generated during compression is radiated away and the levels of the manometer liquid remain stationary, let the reading of the pressure difference as recorded by the manometer be h_1. If the atmospheric pressure be P, the pressure of the enclosed air P_1 is given by

$$P_1 = P + h_1 \rho g \quad \ldots \quad (1)$$

where ρ is the density of the manometer liquid.

Now let the enclosed air be put in communication with the outside atmosphere by opening, for a very short interval of time, the stop-cock T_2. The air inside expands adiabatically till its pressure becomes equal to that of the atmosphere when the stop-cock is closed. At this instant, the volume of the air remaining in the flask is equal to V, the volume of the flask. Obviously, this air remaining in the flask after adiabatic expansion must have occupied less volume, say V_1, before the expansion when its pressure was P_1 Thus

$$P_1 V_1^{\gamma} = P V^{\gamma}$$

or

$$\frac{P_1}{P} = \left(\frac{V}{V_1}\right)^{\gamma}$$

or

$$\log P_1 - \log P = \gamma [\log V - V_1] \quad \ldots \quad (2)$$

Owing to the sudden expansion of the compressed air there is a fall in the temperature of the enclosed air. As the flask is allowed to stand, it acquires heat from the surroundings, consequently the pressure inside the flask increases as is indicated by a gradual rise in the difference of the two levels of the manometer. After sometime, the liquid levels in the manometer attain a steady value, indicating a pressure difference h_2 cms. of the liquid. Thus, the pressure P_2 of the enclosed gas is given by

$$P_2 = P + h_2 \rho g \quad \ldots \quad (3)$$

Now since this air has the same temperature as it had when its volume was V_1 at pressure P_1, we have

$$P_1 V_1 = P_2 V$$

or

$$\log P_1 + \log V_1 = \log P_2 + \log V$$

or

$$\log P_1 - \log P_2 = \log V - \log V_1 \quad \ldots \quad (4)$$

From equations (2) and (4) we have

$$\log P_1 - \log P = \gamma [\log P_1 - \log P_2]$$

Hence
$$\gamma = \frac{\log P_1 - \log P}{\log P_1 - \log P_2} \quad \ldots \quad (5)$$

Equation (5) can be utilised for the evaluation of γ, the values of P_1 and P_2 being obtained from (1) and (3). However, if h_1 and h_2 are small, equation (5) can be reduced further to a very simple form. Thus

$$\gamma = \frac{\log (P + h_1\rho g) - \log P}{\log (P + h_1\rho g) - \log (P + h_2\rho g)}$$

$$= \frac{\log \left(1 + \frac{h_1\rho g}{P}\right)}{\log \left(1 + \frac{h_1\rho g}{P}\right) - \log \left(1 + \frac{h_2\rho g}{P}\right)}$$

Now expanding the logarithmic series on the right and retaining only the first powers of h_1 and h_2 we have

$$\gamma = \frac{h_1}{h_1 - h_2} \quad \ldots \quad (6)$$

Method

(i) After closing the stop-cock T_2 and opening the stop-cock T_1 compress, with a few strokes of the compression pump*, some air in the flask. Now close the stop-cock T_1 also.

(ii) Wait for sometime so that the heat produced by compression is lost to the surroundings and the gas inside acquires a steady temperature equal to the room temperature. The steady state will be indicated when the levels of the manometer liquid are stationary.† Note down the levels of the liquid column in the two limbs and take their difference which gives h_1.

(iii) Now open the stock-cock T_2 for a very short time. (Actually, turn the stop-cock just once from one side to the other). During this process, the compressed air inside the flask is put in communication with the outside atmosphere. The compressed air

* While compressing air into the flask keep an eye on the levels of the liquid in the manometer. In careless pumping, the liquid may not be expelled out. If the shorter formula embodied in equation (6) is to be utilised later, then the reading h_1 of the manometer should not be too much.

† Do not be impatient for performing the next operation. When the levels in the manometer remain steady for at least five minutes, then only pass on the next step.

expands adiabatically and due to sudden expansion the temperature of the inside air falls down. Wait for sometime till the temperature inside becomes equal to that of the outside. This will be indicated when the two levels of the liquid column become stationary. When the levels are perfectly constant, read them and calculate the difference of pressure which gives h_2. Calculate the value of γ from equation (6) given above.

(iv) Repeat the above process a number of times and calculate the mean value of γ. Record also the room temperature.

Observations*

Room Temperature = ...°C.

S. No.	Readings of the manometer when air is compressed			Readings of the manometer when air expands adiabatically			γ
	Left limb	Right limb	Pressure difference (h_1)	Left limb	Right limb	Pressure difference (h_2)	
						Mean	

Calculations

Set I $\gamma = \dfrac{h_1}{h_1 - h_2}$

[**Note.** Calculate similarly for other sets.]

$\because$ Mean value of γ = ...

Result. The value of γ for air at...... °C = ...

[Standard value at 17°C = 1·40 ; Error = ...%]

Precautions and Sources of Error

(1) In this experiment, it is essential that the enclosed air is perfectly dry. For this purpose, some hygroscopic substance, like concentrated sulphuric acid, should be kept in the flask.

(2) The apparatus should be perfectly airtight.

* If equation (5) is employed for the calculation of γ, the following readings should also be recorded—
(i) Barometric height = ... cms. of mercury
(ii) Density of the manometer liquid = ... gm/c.c.
(iii) Density of mercury = ... gm/c.c.

(3) The liquid employed in the manometer should be of low density and low vapour pressure, so that the pressure difference as recorded by the manometer is sufficiently large to admit of accurate measurement.*

(4) For the applicability of the approximate formula (equation–6), the values of h_1 and h_2 should be small†.

(5) When the air is being compressed into the flask, an eye should be kept on the levels of the liquid in the manometer. In careless pumping, the liquid may not be expelled out of the manometer.

(6) The readings for h_1, and h_2 should be recorded only when the temperature of the enclosed gas becomes steady. This will be so when the levels of the liquid in the manometer do not alter their positions.

(7) In this experiment, there is a source of serious error. In the theory of the experiment it has been assumed that during the adiabatic expansion of the enclosed air the pressure inside the flask becomes atmospheric when the stopcock is closed. This is far from truth, since actually oscillations of the air set in : on account of the kinetic energy of the outgoing air, more air rushes out than would make the pressure just atmospheric. Consequently, the pressure inside becomes less than the atmospheric pressure. Next some air rushes in, but this too overshoots the mark and the pressure inside becomes greater than the atmospheric. After several surgings forth and back, the atmospheric pressure is attained. This obviously takes considerable time and, as a matter of fact, this to and fro motion of the air has not ceased when the stop-cock is closed. Rigorously speaking, *the stop-cock should be closed at the instant when the pressure just becomes atmospheric.* For reasons of practical difficulty, this source of error cannot be eliminated‡.

---

* Fouss pump oil is ideal for this purpose.

† If $h_1 = 13{\cdot}6$ cms. and $\rho = 0{\cdot}89$, then the pressure due to this $= 13{\cdot}6 \times 0{\cdot}89g$ dynes, which is equal to $\dfrac{13{\cdot}6 \times 0{\cdot}89}{13{\cdot}6}$ $= 0{\cdot}89$ cms. of mercury. If the atmospheric pressure be 75 cms. of mercury, the change in pressure is comparatively very small.

‡ It is in fact due to this practical difficulty that the method in which the variation of temperature with pressure is studied is always preferred to the present one. The stop-cock is not to be closed in this case. Experiments based on this consideration have been performed by Lummer and Pringsheim and later by Partington.

MECHANICAL EQUIVALENT OF HEAT

EXPERIMENT—7

Object. To determine the value of J, the mechanical equivalent of heat, by Searle's friction-cone method.*

Apparatus Required. Searle's apparatus, a sensitive thermometer, a stop-clock, physical balance, and a weight box.

Description of the Apparatus. A sectional diagram of the Searle's apparatus is depicted in the accompanying figure. It consists of two metal cones which fit closely one into the other. The outer one is rotated by attaching it to a vertical spindle driven by a hand-wheel The inner cone tends to rotate with the outer one, but it is prevented from doing so by applying a suitable external force. This is accomplished by means of a wooden disc having two fine holes into which fit the two pins of the inner cone. This disc is kept stationary by a tangential force exerted by a string which passes over a pulley; one end of the string is tied to a pin on the rim of the disc and the other end carries a pan in which suitable weights can be placed. The number of revolutions

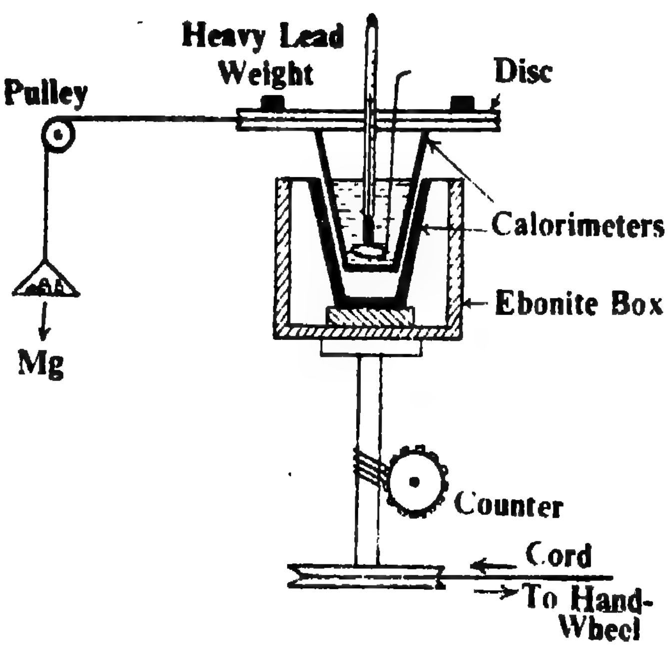

Fig. 10
Friction-cone apparatus

* For electrical methods for the determination of J see Experiments—23 and 24.

performed by the outer cone is given by a counter. Thus the inner surface of the outer rotating cone rubs all over the other surface of the stationary inner cone. Due to this friction heat is generated which goes to heat the two cones and the water kept in the inner cone.

Formula Employed. The value of J is given by the formula

$$J = \frac{\text{Work done}}{\text{Heat generated}}$$

But $$\text{Work done} = 2\pi n \, MgR \text{ ergs}$$

and $$\text{Heat generated} = (m + W)(\theta_2 - \theta_1) \text{ cals.}$$

Thus $$J = \frac{2\pi n \, MgR}{(m + W)(\theta_2 - \theta_1)} \text{ ergs/cal.}$$

where n = No. of revolutions performed by the outer cone.

Mg = Weight hanging from the end of the string.

R = Radius of the wooden disc.

m = Mass of water in the inner cone.

W = Water equivalent of the two cones (= Mass of the two cones $\times$ specific heat).

$\theta_2 - \theta_1$ = Rise in temperature.

PRINCIPLE AND THEORY OF THE EXPERIMENT

It was Dr. Joule who established that when heat was produced by the expenditure of mechanical energy, for the production of each unit of heat a definite number of units of work had to be performed. Thus, there is a constant transformation ratio between W, the work done, and H, the heat generated. This ratio W/H or J is called the *Mechanical Equivalent of Heat*.

Now to evaluate the work done in this experiment, let us refer to fig.-11. Let r be the *mean radius* of the surface of contact of the two cones and if F be taken as the mean value of the force of friction, then the moment of this frictional force is F.r dyne-cm. This must be equal and opposite to the moment of the tension acting in the string when it is tangential to the disc and the inner cone is stationary. If R be the radius of the disc the moment of the tension is Mg R dyne-cm. Thus

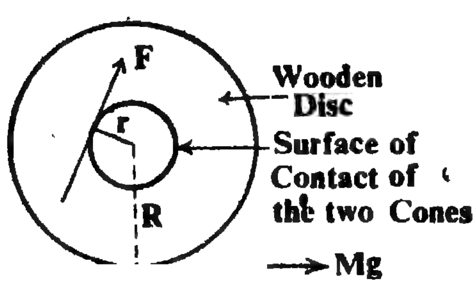

Fig. 11
Calculation of work done

$$F\,r = M\,g\,R$$

where M is the mass suspended from the string. Now when the outer

cone is rotated once round, the work done is equal to $F.2\pi r$ ergs, and if it rotates n times during the experiment, the total work

$$W = n.\,F\,2\pi r$$
$$= 2\pi n R,\, Mg \qquad \text{(since } F r = MgR\text{)}$$

The heat H developed by friction between the two cones raises their temperature as well as that of the water contained in the inner cone. If W be the water equivalent of the two cones and m be the mass of the contained water

$$H = (m + W)(\theta_2 - \theta_1) \text{ cals.}$$

where θ_1 and θ_2 are the initial and final temperatures respectively.

Thus $$J = \frac{W}{H} = \frac{2\pi n R Mg}{(m + W)(\theta_2 - \theta_1)} \text{ ergs/cal.}$$

Method

(i) Before carrying out the actual experiment, it is essential to ascertain the correct amount of friction existent between the two cones, otherwise it will be impossible to keep the hanging weight at a constant level. If the cones are not lubricated, the friction is excessive and the inner cone is seized by the outer cone and begins rotating with it. For this purpose, wipe out the dirty lubrication (if any) and put a single drop of lubricating oil, and then smear it well on the outer surface of the inner cone. Then test the apparatus by turning the driving wheel so as to see whether the load can be maintained approximately at a fixed level,* while turning the wheel at a fair speed.

(ii) Now weigh the two cones with the stirrer, and again weigh them when the inner cone is nearly two-thirds full of water. Replace the cones in the apparatus and insert a sensitive thermometer (reading preferably to one-tenth of a degree). Now by trial, adjust the weight in the pan such that the string remains tangential to the rim of the disc and the suspended mass remains stationary at the same height. When this adjustment is complete, note the initial temperature of water and also the reading of the counter.

(iii) Begin rotating the wheel with a constant speed and immediately start the stop-clock. When the temperature has risen by 6—8°C, stop rotating the wheel as well as stop the stop-clock. Note the final temperature carefully as also the reading of the counter.

(iv) To make the radiation correction, let the apparatus cool for the same time as taken during the experiment and note the fall in temperature ($\delta\theta$) during this interval. To correct for the radia-

* The string must always be tangential to the circumference of the wooden disc when the apparatus is in use. (See fig.-11).

tion loss during the experiment add $\delta\theta/2$ to the observed rise in temperature.*

(v) Now the quantity $2\pi R$ occurs in the numerator of the formula for J. This is equal to the circumference of the wooden disc, to measure which pass a thread once round the disc and measure the length of the thread with a metre scale.

Calculate the value of J with the help of the formula given above.

Observations

S.No.	Determinations	Magnitude	Derived Quantities
1.	Mass of the two cones.	...gm	
2.	Mass of the two cones with water.	...gm	(1) Mass of water = ...gm
3.	Initial temperature of water.	...°C	(2) No. of revolutions = ...
4.	Initial reading of the counter.	...	(3) Radiation correction = ...°C
5.	Final temperature of water.	...°C	
6.	Final reading of the counter.	...	
7.	Temp. of water after cooling for the time during which the expt. is conducted.	...°C	(4) Sp. heat of the material of the cones = ... (given)
8.	Mass suspended.	...gm.	
9.	Circumference of the disc.	...cm.	

Calculations

Corrected rise in temperature = ...°C

Now $$J = \frac{(2\pi R)\, n.\, Mg}{(m + W)\,(\theta_2 - \theta_1)}$$

$$= \ldots \text{ ergs/cal.}$$

---

* This is only an approximate correction. For more accurate work, Regnault's method should always be employed. (See page—22).

Result. The mechanical equivalent of heat = ... ergs/cal.*
[Standard value = $4 \cdot 18 \times 10^7$ ergs/cal. ;
Error = ... %]

Precautions and Sources of Error

(1) Before starting the actual experiment it is essential to test that the two cones are properly lubricated, otherwise it will be impossible to keep the hanging weight at a fixed height. If the lubrication is insufficient, the friction will be excessive and the inner cone will be seized by the outer one and will begin rotating with it.

(2) The string must always be *tangential* to the circumference of the disc when the apparatus is in use, and the mass suspended at the end of the string should remain stationary at a constant height.

(3) The friction at the pulley should be negligible, otherwise the value of the tension will not be equal to the weight hanging at the end of the string, and consequently an error shall be introduced in the result.

(4) A sensitive thermometer reading upto one-tenth of a degree must be employed for the measurement of temperature.

(5) Correction for radiation loss should be applied as explained above. However, it should be noted that this is not an accurate procedure to calculate this loss. Moreover, heat lost by other processes is not accounted for. Consequently, the value of J obtained by this method is always higher than the accepted value.†

* The value of J may also be expressed in joules/calorie, where 1 joule = 10^7 ergs.

† It is for this reason that Callender and Barnes' method is much superior to this method. (See exp.—24).

THERMAL CONDUCTIVITY

EXPERIMENT—8

Object. To determine the coefficient of thermal conductivity of copper with the help of Searle's apparatus.

Apparatus Required. Searle's conductivity apparatus, constant level water flow arrangement, boiler, four thermometers, vernier callipers, beaker, and a graduated cylinder.

Description of the Apparatus. Searle's apparatus for the determination of thermal conductivity of metals is depicted in the figure given below. The metal of which the coefficient of thermal conductivity is to be determined is taken in the form of a cylindrical rod (AB), one end of which is enclosed in a steam jacket C, and

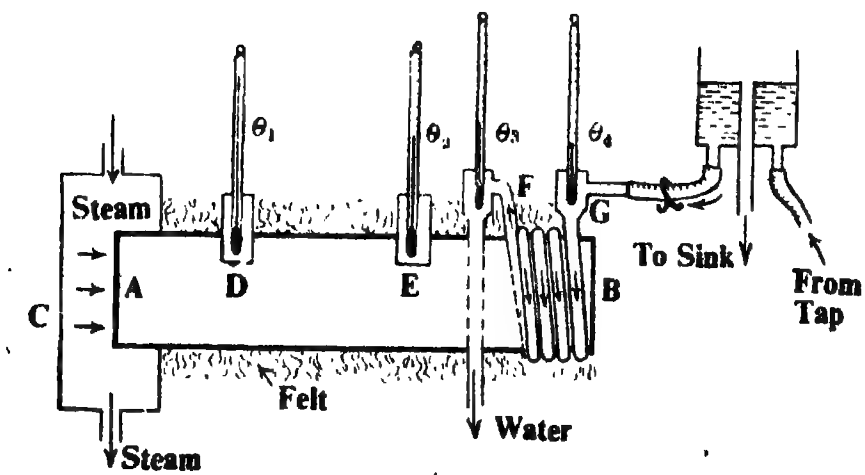

Fig. 12
Searle's conductivity apparatus

over the other end a copper spiral of thin-walled tubing is closely wound. The former end is heated by passing steam into the jacket from a boiler while the other end is cooled by circulating cold water from a constant level tank into the spiral tubing. The temperatures of the incoming and outflowing water are recorded by two sensitive thermometers placed at G and F. The temperatures

at two points D and E along the length of the rod are determined by placing two thermometers in two holes drilled there. To make good thermal contact with the rod a few drops of mercury are poured in each hole. The whole piece of apparatus is lined with a poorly conducting material, such as felt, and is contained in a wooden box having a removable front.

Formula Employed. The coefficient of thermal conductivity k for a material is given by the formula—

$$Q = k\,A\,.\,\frac{\theta_1 - \theta_2}{b}$$

where Q = Quantity of heat flowing *per second* when the steady state is attained by the rod.

A = Area of cross-section of the rod and is given by πr^2 where r is the radius of the rod.

θ_1, θ_2 = Steady temperatures at the points D and E of the rod.

d = Distance apart of the two thermometers.

Now, to calculate the value of Q the following formula is employed—

$$Q = m\,(\theta_3 - \theta_4)$$

where m = Mass of water collected *per second.*

θ_3, θ_4 = Steady temperatures of water at exit and at entrance respectively.

PRINCIPLE AND THEORY OF THE EXPERIMENT

When one end of a rod is heated the heat energy is transmitted from particle to particle to the other end. This kind of transmission of heat is known as *conduction.* Let us consider a section AB which receives a quantity of heat Q, say from the left. The heat transmitted by the adjoining layer CD to the right is, say, Q_2. Thus, the balance of heat $(Q - Q_2)$, gets lodged up in the layer. Part of this heat (Q_1) is absorbed by the layer, its temperature consequently rises. A part of the heat, Q_3, is lost to the surroundings by radiation etc. If the rod is kept surrounded by a poorly conducting substance, Q_3 may be supposed to be zero, and the balance of heat Q_1 goes to increase the temperature of the layer. Thus, the temperature at each point along the length of the rod goes on rising. This state of the rod is known as *variable state.* During this variable state of the rod the rise in the temperature is proportional to the thermal capacity of the layer. Consequently

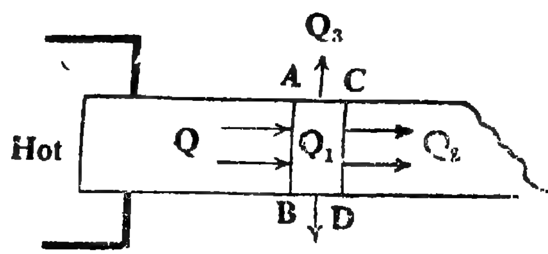

Fig. 13
Flow of Heat through a rod

the heat transmitted by conduction is influenced by the thermal capacity of the rod.

If the heating of the rod be continued for some time, a stage comes when the temperature of the layer becomes constant. Under this circumstance there is no further absorption of heat by the layer ; whatever amount of heat the plane AB receives from the left; it is all transmitted by CD to the right. Now the temperatures at each point along the rod become constant (*not equal*). This state of the rod is known as *steady state*. Thus, in determining the conductivity of the rod *it is essential that the rod acquires a steady state otherwise the result will be affected by thermal capacity*.

When the steady state is reached the quantity of heat conducted by the rod is found to be proportional to its area of cross-section (A), to the time (t) for which heat flows, and also to the temperature-gradient* $(\theta_1 - \theta_2)/d$. Combining all these factors, we have

$$\text{Quantity of heat} \propto A.\frac{\theta_1 - \theta_2}{d}.t$$

Or

$$Q = k\,A\,\frac{\theta_1 - \theta_2}{d}$$

where k is constant known as the *coefficient of thermal conductivity*, which may be defined as *the quantity of heat flowing per second across every sq. cm. of the face of a slab of the material of one cm. thickness when the difference of temperature between the two faces of the slab is one degree centigrade, the flow of heat being normal to the faces.*

If this heat Q transmitted to the other end of the rod be collected by circulating cold water in hollow spirals wrapped round this end, and if m gms. of water be collected per second,

$$Q = m\,(\theta_3 - \theta_4)\text{ cals.}$$

where θ_3, θ_4 are respectively the steady temperatures of water at exit and entrance respectively. Thus

$$m\,(\theta_3 - \theta_4) = k\,A\,\frac{\theta_1 - \theta_2}{d}$$

Further, $A = \pi r^2$, where r is the radius of the rod. Hence,

$$m\,(\theta_3 - \theta_4) = k.\,\pi r^2\,\frac{\theta_1 - \theta_2}{d}.$$

Or

$$k = \frac{m\,(\theta_3 - \theta_4)\,d}{\pi r^2\,(\theta_1 - \theta_2)}.$$

*If, during the steady state, the temperature of the section AB be θ_1, and that of CD be θ_2, and if two sections be separated by a distance d, then $(\theta_1 - \theta_2)/d$ °C per cm. is known as the temperature gradient, which may be defined as the fall of temperature per cm.

[Note—The units in which k is expressed are "*Calories per sec. per sq. cm. per unit temperature gradient*, or simply *C.G.S. units*."]

Method

(i) Connect the inlet tube of the steam chamber to the boiler so that a steady current of steam passes through it. At the other end, pass a steady current of water through the copper spiral by connecting it to the constant level tank. Insert two thermometers, each reading upto half a degree, in the holes on the rod after putting a few drops of mercury in each hole. Insert two more thermometers, each reading upto one-tenth of a degree, at the entrance and exit spots of the circulating water. Wait for the rod to attain a steady state,* which may take from twenty minutes to half an hour.

(ii) Confirm the steady state of the rod by recording the four temperatures every five minutes and observing that there is no variation in their respective values with time. The observations for temperatures recorded last should be employed for the calculation of k.

(iii) After the steady state is attained, collect water (coming from the exit tube of the spiral) in a clean dry measuring cylinder and with the help of an accurate stop-watch note the time during which water is collected. From this determine the mass (m) of water flowing out per second. Repeat this observation several times and calculate the mean value of m.

(iv) Now measure the diameter, with the help of vernier callipers, at several points along the bar and at each point along two mutually perpendicular directions. Also measure the distance between the two thermometers inserted in the rod. Then calculate k from the formula given above.

Observations

[A] *Readings for the determination of* m, θ_1, θ_2, θ_3, *and* θ_4.

S. No	Mass of water collected	Time taken	Mass of water flowing per sec. (m)	θ_1	θ_2	θ_3	θ_4
1.	... gm.	...mins.	... gm/sec.	...°C	...°C	...°C	...°C
...							
...							
...							
...							

* The final steady temperatures recorded will depend on the rate at which water is flowing through the spiral tube. Thus, in order to have a reasonably large difference of temperature for the inflowing and outflowing water, it is necessary to allow the water to flow in a rather slow stream ; in fact little more than a trickle of water should issue from the exit pipe of the spiral.

[B] *Readings for the determination of the diameter of the rod*

S. No.	Reading along any direction	Reading along a perp. direction	Mean diameter (observed)	Remarks
1.	...cm	... cm.	... cm.	1. Vernier constant=...cm.
⋮				2. Zero error =...cm.
⋮				4. Distance between the two thermometers on the rod =...cm.
		Mean	... cm.	

Calculations

Mean corrected diameter = ... cm.

∴ Mean corrected radius = ... cm.

Now
$$k = \frac{m(\theta_3 - \theta_4)\,d}{\pi r^2(\theta_1 - \theta_2)}$$
= ...... C. G. S. units

Result. The coefficient of thermal conductivity of copper = ... cals. per sec. per sq. cm. per unit temp. gradient.

[Standard value = 0·918 units ; Error = ... %]

Precautions and Sources of Error

(1) Water should be allowed to flow under constant pressure and its flow should be so regulated that it issues out in a rather slow stream ; in fact little more than a trickle of water should issue from the exit pipe of the spiral tube. This will ensure a reasonably large difference of temperature for the incoming and outgoing water.

(2) Observations should be recorded only when the rod acquires a steady state. Readings of the thermometers should therefore, be noted after half an hour or so. These should be recorded again after every five minutes, and when no variation in temperature is observed with time, the rod can be supposed to have attained steady state.

(3) Water should be collected only after the steady state is attained. It should be collected in a dry cylinder for four or five minutes so that sufficient quantity is obtained.

(4) The radius of the rod occurs in the second power in the formula for k, hence the diameter should be measured very carefully at a number of places along the rod, and at each place it should be measured along two mutually perpendicular directions.

(5) In the theoretical discussion of the method it has been assumed that the isothermal surfaces are parallel to the faces of the rod and the lines of flow of heat are consequently normal to these isothermal surfaces. Now in the apparatus usually available for this experiment only two thermometers are provided. These are not sufficient to make it certain that the condition of linear flow of heat in the rod prevails, and it is essential that this should be so. To minimise this error, at least four thermometers should be placed along the rod and the conditions must be adjusted (it will be necessary to provide more efficient heat insulation along the rod) until linear flow takes place. A graph should then be drawn between temperature and distance along the rod. From the graph we can easily infer whether the conditions for linear flow have been attained or not. Then this graph should be employed to calculate the value of the temperature gradient existent along the rod.

EXPERIMENT—9

Object. To determine the coefficient of thermal conductivity of glass in the form of a tube.

Apparatus Required. The given glass tube, a wider tube to be used as a steam jacket, constant level water tank, boiler, two sensitive thermometers and a graduated jar.

Description of the Apparatus. The arrangement of the apparatus is schematically represented in figure, 14. The given

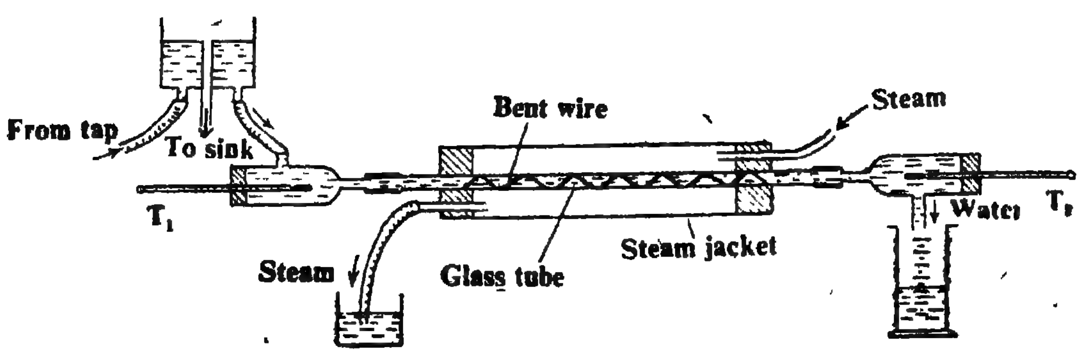

Fig. 14

Apparatus for the conductivity of glass

glass tube is enclosed in a wider jacket in which steam from a boiler is passed. The two ends of the tube are connected, through rubber-tubings, to T-shaped wider tubes which accommodate two sensitive thermometers T_1 and T_2. Cold water is allowed to flow in this tube, the inlet tube of which is connected to a constant level

water tank. The tube is adjusted in a slightly inclined position so that it is always kept full of water.

As the temperature of water at any cross-section of the tube should be uniform, the water is allowed to flow through the tube in a zig-zag manner. This is accomplished by placing along the axis of the experimental tube a bent wire which keeps the water well stirred as it flows along the tube.

Formula Employed. The coefficient of thermal conductivity (k) of glass, which is in the form of a hollow cylinder, is given by the formula—

$$k = \frac{Q}{2\pi l} \cdot \frac{\log_e \frac{r_1}{r_2}}{\theta_2 - \theta_1}$$

where
Q = Quantity of heat transmitted through the walls of the tube in one second.
l = Length of the tube within the steam jacket (excluding portions in the corks).
r_1, r_2 = Internal and external radii of the tube.
θ_1, = Temperature of the inner surface of the tube*.
θ_2, = Temperature of the outer surface of the tube (= temp. of steam).

If in the steady state m gms of water are collected per second, and θ_3, θ_4 be the temperatures of incoming and outflowing water, we have

$$Q = m\,(\theta_4 - \theta_3)$$

Thus $$k = \frac{m\,(\theta_4 - \theta_3)}{2\pi l} \cdot \frac{2 \cdot 303.\ \log_{10} r_1/r_2}{\theta_2 - \theta_1}$$

PRINCIPLE AND THEORY OF THE EXPERIMENT

Let us consider a tube of internal and external radii r_1 and r_2 respectively. If this tube is surrounded by steam, heat will flow radially from the outer surface to the inner one through the thickness of the tube. The isothermal surfaces will obviously be cylindrical in this case. If we consider a thin cylindrical shell of radii r and r + dr. and if $d\theta$ be the difference of temperature between these surfaces in the steady state, the amount of heat Q flowing per second from outside to inside is given by

$$Q = -2\pi r l k \cdot \frac{d\theta}{dr}$$

where l is the length of the tube.

* The temperature θ_1 of the inner side of the experimental tube can be taken equal to the mean temperature of water flowing past it, *i. e.*, $\theta_1 = (\theta_3 + \theta_4)/2$.

Integrating the above expression between the inner and outer surfaces of the tube, (*i. e.*, between the limits r_1 and r_2), having their respective temperatures as θ_1 and θ_2, we have

$$\int_{\theta_1}^{\theta_2} d\theta = -\frac{Q}{2\pi l k}\int_{r_1}^{r_2}\frac{dr}{r}$$

or
$$(\theta_2 - \theta_1) = \frac{Q}{2\pi l k}\log_e \frac{r_1}{r_2}$$

This heat Q is taken by the cold water flowing in the tube, consequently its temperature rises. Let the temperatures, in the steady state, of the flowing and outflowing water be θ_3 and θ_4 respectively, and let m gms of water be collected per second, then

$$Q = m(\theta_4 - \theta_3)$$

Thus
$$k = \frac{m(\theta_4 - \theta_3)}{2\pi l}\cdot\frac{2{\cdot}303.\ \log_1 r_{10}/r_2}{\theta_2 - \theta_1}$$

The temperature θ_1 of the inner surface of the tube can be put down equal to the mean temperature of the water flowing along it, *i. e.*, $\theta_1 = (\theta_3 + \theta_4)/2$. Hence

$$k = \frac{m(\theta_4 - \theta_3)}{2\pi l}\cdot\frac{2{\cdot}303.\ \log_{10} r_1/r_2}{\theta_2 - \dfrac{\theta_3 + \theta_4}{2}}$$

This is the final formula* to be employed for the calculation of k.

Method

(i) Set up the apparatus as shown in the figure. Connect the inlet tube of water (at the lower end) with a constant level water tank, and outlet tube to a sink. Let a steady stream of water be so regulated that the flow is just little more than a trickle.†

Let the tube be inclined a little as shown in the figure. In this case the water will flow up-gradient of height and the tube will always remain full of water.

(ii) Now connect the outer steam jacket with the delivery tube of the boiler so that steam enters through the upper end. Wait till the *steady state* is attained. To confirm this, record the

* It this formula the factor 2·303 is introduced in order to convert the logarithm from the base e to the base 10.

† This kind of flow shall ensure a sufficient difference of temperature at the two ends in the steady state, thereby increasing the accuracy of the result.

temperatures of the two thermometers every five minutes and when the readings do not show any variation, record them.

(iii) For determining the rate of flow of water, collect it in a clean and dry measuring cylinder and note the time with a stop-watch. Repeat this operation several times and calculate the mean value of m.

(iv) Measure the length* of the tube inside the jacket only.

(v) Note the barometric height and evaluate the temperature of steam from the Table of Physical Constants.

(vi) To measure the internal radius (r_1) of the tube, close its one end with a rubber stopper and clamp it vertically.†

Pour a known volume V of water from a burette, and measure the height h of the water column in the tube. Then calculate the value of r_1 with the help of the formula, $V = \pi r_1^2 h$.

To measure r_2, the external radius, use a vernier callipers and measure the diameter at several places along the tube and at every place along two mutually perpendicular directions. Calculate the mean value of r_2.

(vii) Now calculate the value of k with the help of the formula given above.

Observations

[A] *Readings for the determination of m, θ_3, θ_4.*

S. No.	Mass of water collected	Time taken	m	θ_3	θ_4	Remarks
1	...gm.	...mins	... gm/sec	...°C	...°C	(1) Barometric height ...=...cm. ∴ Temp. of steam =...°C (2) Length of the tube exposed to steam =...cm.
		Mean				

* The portions of the tube inside the two end corks should be excluded from measurement, since these are not exposed to steam.

† Test the verticality of the tube with a plumb line.

[B] *Readings for the determination of the internal radius (r_1) of the tube.*

S. No.	Height of wrter column (h)	Mean	Volume of water poured (V)	Mean V
1.				
2.				
3.				

[C] *Readings for the determination of the external radius (r_2) of the tube.*

S. No.	Readings along one direction	Readings along a perp. direction	Mean h	Remarks
				1. Vernier const. = ... cm 2. Zero error = ... cm
		Mean		

Calculations

Mean corrected external diameter = ...cm.

∴ Mean corrected external radius, (r_2) = ...cm.

Again, Internal radius, $r_1 = \sqrt{V/\pi h}$ = ...cm.

Now
$$k = \frac{m(\theta_4 - \theta_3)}{2\pi l} \cdot \frac{2{\cdot}303 \log_{10} r_1/r_2}{\theta_2 - \dfrac{\theta_3 + \theta_4}{2}}$$

= ... C. G. S. units.

Result. The thermal conductivity of glass = ... cals. per sec. per sq. cm. per unit temp. gradient.

[Standard value = 0·0025 units ; Error = ...%]

Precautions and Sources of Error

(1) Water should be allowed to flow in the tube from a constant level tank, the exit tube of which should be connected to the lower tube (inlet) of the experimental glass tube, which should be slightly tilted upwards so that water flows from a lower level to a higher level. This will ensure that the tube is always full of water. When water is flowing in a steady stream, then only pass steam from the boiler.

(2) The water should be allowed to flow in a zig-zag manner so that thorough stirring takes place and therefore constancy of temperature along any section of the tube is ensured. For this purpose a bent wire should be placed along the axis of the glass tube.

(3) To secure a sufficient difference of temperature at the two ends of the tube, the flow of water should be so regulated that it is continuous but slow. To read this difference, sensitive thermometers reading upto one-tenth of a degree should be employed.

(4) Only that length of the tube should be measured which is exposed to steam, hence those portions of the tube which are inside the corks should not be measured.

(5) Readings for the diameters should be taken carefully. Those for the external diameter should be recorded along two mutually perpendicular directions and at several places along the tube.

(6) Readings should be noted only when the steady state is reached. To ensure this note down the temperatures (when steady state is expected) of the two thermometers every five minutes and when two consecutive readings indicate no variation, then only infer that the steady state is reached.

(7) In the theoretical derivation of the formula, it has been assumed that the flow is radial and that the isothermal surfares across which heat is conducted in glass ars coaxial cylinders. Now this is not rigorously true in the present case, since the temperature of internal glass surface towards the exit end is higher than that towards the inlet end. The accuracy of the result is consequently limited.

EXPERIMENT—10

Object. To determine the coefficient of thermal conductivity of rubber in the form of a tube.

Apparatus Required. Rubber tubing, steam generator, a calorimeter, a sensitive thermometer, stop-watch, a physical balance with weight box, and a wooden screen.

Description of the Apparatus. The apparatus consists of a calorimeter of large capacity (500 to 600 c c.) in which a sufficient length of uniform-walled rubber tubing can be coiled. Two pieces

of cotton cord are tied round the rubber at the points where it enters and leaves the water. It facilitates the measurement of the

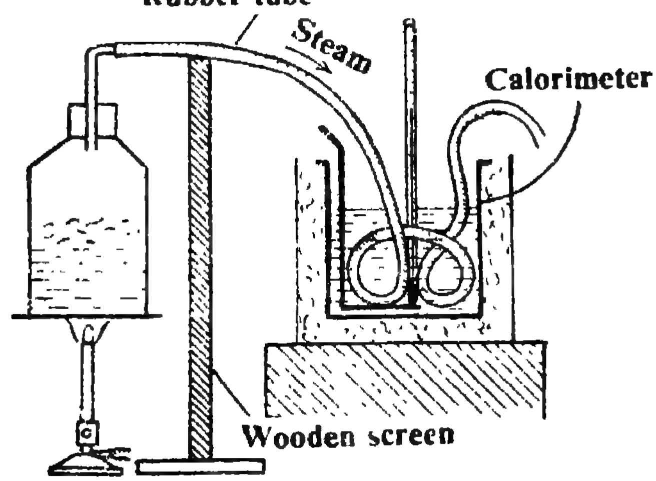

Fig. 15
Apparatus for the conductivity of rubber

length of the portion of the tube lying in the calorimeter at a subsequent stage. Steam is passed from the boiler, circulates through the rubber tube lying in the calorimeter and is then carried away in a beaker containing water where it is condensed. Heat passes through the walls of the tube into the water in the calorimeter whose temperature consequently rises and is read by a sensitive thermometer (reading upto at least one-fifth of a degree). A wooden screen is interposed in between the boiler and the calorimeter in order to protect the latter from direct radiant energy coming from the former.

Formula Employed. The coefficient of thermal conductivity (k) of rubber tube, which is in the form of a hollow cylinder is given by the formula--

$$k = \frac{Q}{2\pi l} \cdot \frac{\log_e r_2/r_1}{\theta_1 - \theta_2}$$

where Q = Quantity of heat transmitted through the walls of the tube into the calorimeter per sec.

l = Length of the rubber tube lying inside the calorimeter.

r_1, r_2 = Internal and external radii of the tube.

θ_1 = Temperature of steam (*i. e.*, temperature of the inner surface of the rubber tube)

θ_2 = Temperature* of the outside surface of the rubber tube.

The temperature θ_2 of the outside surface of the rubber tube can be taken equal to the mean temperature of water outside, *i. e.*, equal to $(\theta_3 + \theta_4)/2$.

If the thermal capacity of the calorimeter be w and m be the mass of water taken, then the quantity of heat taken by the calorimeter is given by

$$Q = (m + w)\,(\theta_4 - \theta_3)/t \text{ cals./sec.}$$

where θ_3, θ_4 are respectively the initial and final temperatures of water and t is the time for which steam is allowed to flow. Thus

$$k = \frac{(m + w)\,(\theta_4 - \theta_3)}{2\pi l t} \cdot \frac{2{\cdot}303 . \log_{10} r_2/r_1}{\theta_1 - \theta_2}$$

PRINCIPLE AND THEORY OF THE EXPERIMENT

Let us consider a uniform-walled rubber tube of internal and external radii r_1 and r_2 respectively. If steam is passed through this tube, heat will flow radially from the inner surface to the outer one through the walls of the tube. The isothermal surfaces will obviously be cylindrical in this case. If we consider a thin cylindrical shell of radii r and (r + dr), and if $d\theta$ be the difference of temperature between these surfaces in the steady state, the amount of heat Q flowing per second from inside is given by

$$Q = -\,2\pi r l k \frac{d\theta}{dr}$$

where l is the length of the tube.

Integrating the above expression between the inner and outer surfaces of the tube (*i. e.*, between the limits r_1 and r_2), having their respective temperatures as θ_1 and θ_2, we have

$$-\int_{\theta_1}^{\theta_2} d\theta = \frac{Q}{2\pi l k} \int_{r_1}^{r_2} \frac{dr}{r}$$

Or
$$(\theta_1 - \theta_2) = \frac{Q}{2\pi l k} \log_e \frac{r_2}{r_1}$$

This heat transmitted by rubber is taken by the calorimeter and water, consequently their temperature rises, say from θ_3 to θ_4. If w be the water equivalent of the calorimeter and m be the mass of water taken, then the amount of heat gained by them is given by

$$\text{Heat gained} = (m + w)\,(\theta_4 - \theta_3)$$

Hence
$$Q = \frac{1}{t} . \;(m + w)\,(\theta_4 - \theta_3)$$

where t is the time in seconds for which heat is allowed to flow in the calorimeter. Thus

$$k = \frac{(m + w)\,(\theta_4 - \theta_3)}{2\pi l t} \cdot \frac{2{\cdot}303 \log_{10} r_2/r_1}{\theta_1 - \theta_2}$$

The temperature θ_2 (of the outside surface of the tube in contact with water) is not a constant quantity, but it continuously changes. Its value may be put equal to the mean temperature, *i. e.*, equal to $(\theta_3 + \theta_4)/2$. Thus, finally we have

$$k = \frac{(m + w)(\theta_4 - \theta_3)}{2\pi l t} \cdot \frac{2{\cdot}303 \log_{10} r_2/r_1}{\theta - \dfrac{\theta_2 + \theta_4}{2}}$$

This is the formula which is employed for the evaluation by measuring quantities occurring on the right-hand side.

Method

(i) Take a large calorimeter of capacity 500—600 c.c. so as to allow a considerable length of the rubber tubing to be coiled in it. Weigh the calorimeter, and after filling two-thirds of it with water weigh it again. This gives the mass of the water taken. Note the initial temperature with a sensitive thermometer.

(ii) Coil up the rubber tube in the water, allowing both ends to project some distance out of the calorimeter. Tie two cotton cord bands at the points where the tube enters and leaves the water. This will facilitate the measurement of length of the tube inside the water.

(iii) Put a wooden screen in between the boiler and the calorimeter which is placed, as usual, in a wooden box packed with a non-conducting material. Now connect one end of the rubber tube with the nozzle of the delivery tube of the steam generator so that a steady current of steam can be passed through it. The other end of the tube may dip in a vessel to catch the condensed steam.

(iv) Allow the steam to pass through the tube for a known length of time so that there is a rise of temperature of 10—15°C. Stir the water and record the final temperature. Note the time with a stop-watch. Note the barometric height and find out the temperature of steam from the Table of Physical Constants.

(v) Measure the length (l) of the tube between the cord-bands. For determining r_2, the external radius of the tube, use a screw gauge and measure the external diameter of several places along the tube and at each place along two mutually perpendicular directions.

In order to find r_1, the internal radius, take a piece of the tube nearly 10 cm. long and immerse it in a measuring cylinder, and note the volume v of water displaced. Then if L is the length of this piece

$$v = \pi L (r_2^2 - r_1^2)$$

All the quantities except r_1 are known, so that its value can be calculated out.*

Observations

[A]

S. No.	Determinations	Magnitudes	Remarks
1.	Mass of the calr. + stirrer	...gm.	(1) Mass of water = ... gm.
2.	,, ,, + water	...gm.	(2) Sp. heat of calr. = ...
3.	Initial temp of water.	...°C.	(3) Time for which steam is passed = ... sec.
4.	Initial reading of the stop-watch	...m...sec.	(4) Temp. of steam (from Tables) = ...°C.
5.	Final temp. of water	...°C.	
6.	Final reading of the stop-watch	...m...sec.	(5) Length of the tube dipped in water = ... cm.
7.	Barometric height	...cm.	

[B] *Readings for the determination of* r_2.

S. No.	Diameter along one direction	Diameter along a perp. direction	Mean observed diameter	Corrected diameter	Remarks
					1. L. C. of the screw guage = ...cm. 2. Zero error = ...cm.
			Mean		

* Another interesting method for measuring r_1 and r_2 is to cut the tube clean, normal to its length and use it as a rubber stamp, pressing it gently on a white sheet of paper. The impression of the inner and outer circumferences of the cross-section will be distinct and the diameters may be measured with a travelling microscope.

[C] *Readings for the determination of* r_1.

(1) Length of the rubber piece = ... cm.

(2) Volume of the water displaced = ... c.c.

[Note. Similar sets can be taken for other pieces]

Calculations

(1) Mean external radius (r_2) = ... cm.

(2) Internal radius. $r_1 = \sqrt{r_2^2 - v/\pi l} =$ ...... cm.

[**Note.** Calculate similarly r_1 for other pieces of the tube and thus determine the mean valve of r_1].

Now $$k = \frac{(m + W)\ (\theta_4 - \theta_3)}{2\pi l t} \cdot \frac{2{\cdot}303 \log_{10} r_2/r_1}{\theta_1 - \dfrac{\theta_3 + \theta_4}{2}}$$

= ...... C. G. S. units.

Result. The coefficient of thermal conductivity of rubber = ... cals. per sec. per sq. cm. per unit temp. gradient.

[Standard value = ... units ; Error = ...%]

Precautions and Sources of Error

(1) Use a calorimeter of large capacity (say, 500 to 600 c.c.) and use sufficient quantity of water (*i. e.*, fill nearly two-thirds of the calorimeter with water) so that a sufficiently long rubber tube may be accommodated in it.

(2) At the places where tie rubber tube enters and leaves the water in the calorimeter, the cord bands so that there is no difficulty in measuring the length of the portion dipped in water.

(3) Calorimeter should be well-protected from losing heat by putting it in a wooden container packed with a non-conducting material. A wooden screen should be used in between the heating apparatus and the calorimeter.

(4) Measure r_1 and r_2 very carefully and avoid back-lash error with a screw gauge.

(5) Sensitive thermometers should be employed for the measurement of temperature, and the rise in temperature should not be more than 15°C.

(6) According to theoretical considerations, the temperature of the outer surface of the rubber tube should be constant, but in this experiment it changes continuously as steam passes through it. This constitutes a serious source of error in this determination.

MAGNETISM

MAGNETIC MEASUREMENTS

EXPERIMENT—11

Object. To determine the absolute value of H, the horizontal component of earth' magnetic field, in the laboratory at......* by using deflection and vibration magnetometers.†

Apparatus Required. Deflection and vibration magnetometers, a magnet, a brass bar, a compass needle, a vernier callipers, a meter scale, a balance with weight box, and a stop-watch.

Formula Employed. If the deflection magnetometer is adjusted in the tan-A position of Gauss, and the magnet produces a deflection θ in the magnetic needle, then by applying tangent law—

$$F = H \cdot \tan\theta$$

or $$\frac{2\,M\,d}{(d^2 - l^2)^2} = H \cdot \tan\theta$$

or $$\frac{M}{H} = \frac{(d^2 - l^2)^2}{2\,d} \cdot \tan\theta \qquad \ldots \qquad (A)$$

If the magnet is oscillated in a vibration magnetometer, then

$$T = 2\pi\sqrt{\frac{I}{M\,H}}$$

or $$M\,H = \frac{4\pi^2 I}{T^2} \qquad \ldots \qquad (B)$$

Dividing (B) by (A), we have

$$H^2 = \frac{4\pi^2 \cdot 2\,I\,d}{T^2\,(d^2 - l^2)^2 \cdot \tan\theta}$$

* Name the place where the experiment is being conducted.

† For more accurate determinations, Kew pattern of magnetometer is employed.

or $$H = \frac{2\pi}{T(d^2 - l^2)} \cdot \sqrt{\frac{2Id}{\tan\theta}}$$

This is the required formula, in which

d = Distance of the magnetic needle from the *centre* of the magnet
l = *Half* of the effective length of the magnet
θ = Angular deflection of the needle
T = Time-period of the magnet
I = Moment of Inertia of the magnet.

The moment of inertia* of rectangular bar magnet is given by—

$$I = w\left(\frac{a^2 + b^2}{12}\right)$$

where w = Mass of the magnet
a = Length of the magnet
b = Breadth of the magnet

PRINCIPLE AND THEORY OF THE EXPERIMENT

(i) *Field due to a magnet in the end-on position.* The intensity of the magnetic field due to the magnet NS is to be calculated at the point P lying on the axial line of the magnet. If a unit positive pole be imagined to be placed at P, the forces exerted by the poles of the magnet on this pole will be oppositely directed. Hence, the resultant force, or the intensity of the magnetic field, is given by—

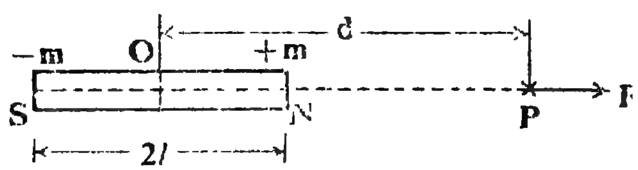

Fig. 16
Field on the axial line of a magnet

$$F = \frac{m}{NP^2} - \frac{m}{Sp^2}$$

* If the given magnet is cylindrical in shape, its moment of inertia is given by the formula—

$$I = w\left(\frac{L^2}{12} + \frac{r^2}{4}\right)$$

where w is the mass, L the length, and r the radius of the cylinder.

$$= \frac{m}{(d-l)^2} - \frac{m}{(d+l)^2}$$

$$= \frac{4\,m\,l\,d}{(d^2 - l^2)^2}$$

$$= \frac{2\,M\,d}{(\,d^2 - l^2\,)} \qquad \dots (1)\dagger$$

since the magnetic moment of the magnet, $M = m.\,(\,2l\,)$

(ii) *Two mutually perpendicular uniform magnetic fields.* Now let the magnet be placed in the east-west direction, and at the point P let there be situated a *small* compass needle. Then the poles of the needle shall experience forces due to H, the earth's horizontal field and F, the field of the magnet. Since the needle is small it can be assumed that the magnetic field due to the magnet in the space in which the needle moves is uniform. Thus, the forces acting at the poles of the needle shall constitute two couples—one deflecting couple (m′F m′F) whose moment is $M'\,F\cos\theta$, the other is the restoring couple (m′H, m′H) whose moment is $M'H\sin\theta$. Here M′ is the magnetic moment of the needle and θ is the deflection when the needle is in equilibrium under the action of these two couples. Thus

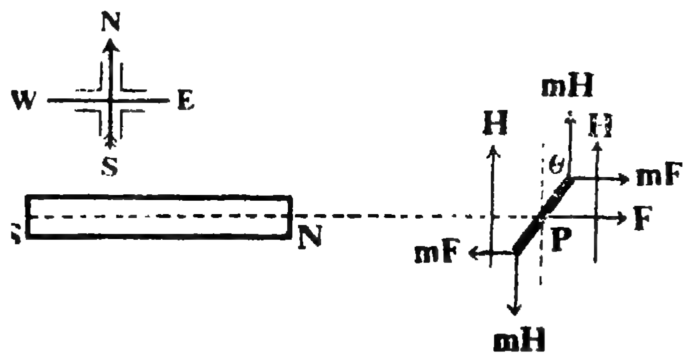

Fig. 17
Tan-A position of Gauss

$$M'\,F\cos\theta = M'\,H\sin\theta$$

or
$$F = H\tan\theta \qquad \dots (2)$$

This is the well-known *Tangent Law* in magnetism. Substituting the value of F from (1) in (2), we have

$$\frac{2\,M\,d}{(\,d^2 - l^2\,)^2} = H\tan\partial$$

or
$$\frac{M}{H} = \frac{(\,d^2 - l^2\,)^2}{2d}\,.\tan\theta \qquad \dots (3)$$

(iii) *Vibration of a magnet in a uniform magnetic field.* When a magnet freely suspended in a uniform magnetic field be

† If d is much greater than l, we have $F = \frac{2M}{d^3}$.

deflected through a small angle θ, the restoring couple acting on it is M H sin θ, which, as the angle of deflection is small, reduces to M H θ. When the magnet is released from its deflected position, the restoring couple tends to set it parallel to the magnetic field. The magnet thus acquires angular acceleration and the couple due to inertial reaction is I $\frac{d^2\theta}{dt^2}$ where I is the moment of inertia of the magnet about the axis of rotation. Since there is no external agency supplying energy, we have—

$$I \frac{d^2\theta}{dt^2} + M H \theta = 0$$

The above equation represents a simple harmonic motion, whose time-period is given by

$$T = 2\pi \sqrt{\frac{I}{M H}}$$

or

$$M H = \frac{4\pi^2 I}{T^2} \qquad \ldots (4)$$

Thus, if we wish to determine the value of H in the laboratory, we can do so by performing the reflection and oscillation experiments.* Now, from relations (3) and (4), we have—

$$H = \frac{2\pi}{T(d^2 - l^2)^2} \sqrt{\frac{2Id}{\tan\theta}} \qquad \ldots (5)$$

Method—(A) Deflection Experiment

(i) Set the magnetometer in tan-A position* of gauss. For this purpose, turn the arms of the magnetometer on the table till they are parallel to the aluminium pointer. Rotate the compass

* The same two experiments can also be employed to find the value of the magnetic moment M of the magnet. Thus, by multiplying (3) and (4), we get

$$M = \frac{\pi(d^2 - l^2)}{T} \sqrt{\frac{2I}{d} \cdot \tan\theta} \qquad \ldots (6)$$

† The field due to a magnet in the end-on position is equal to $2M/d^3$, while in the broadside-on position its value is M/d^3. Hence, if the deflection experiment is conducted in the tan-A position of Gauss, instead of the tan-B position, the deflection of the needle shall be greater (*not double*) and hence will be susceptible of yielding more accurate result.

box without disturbing the arms till the pointer reads zero-zero on the graduated circular scale.

(ii) Now place the magnet on one arm in such a way that its axis is parallel to the arms of the magnetometer and when produced it passes through the centre of the magnet needle. Try to get a deflection* near-about 45°. Note the distance d (from the centre of the magnet to the centre of the needle).

(iii) Note carefully that the deflection θ, which is measured on the deflection magnetometer is subject to the following errors :—

(a) *The pivot of the needle may not pass exactly through the centre of the graduated circular scale.*

To correct for this error, both the ends of the pointer should be read and the mean of the angles should be taken.

(b) It is likely that *the magnetic centre of the bar magnet may not be exactly coincident with its geometrical centre.* (This may be so due to the unsymmetrical magnetisation of the magnet).

To correct for this error, readings should be taken by reversing the polarity of the magnet at the same position. (In this operation, the two poles simply interchange their positions).

(c) *The centre of the linear scale on which the distance d is read may not be coincident with the pivot of the magnetic needle.*

To correct for this error, it is necessary to transfer the magnet on the other arm so that the centre of the magnet lies at the same scale reading.

Thus, in order to eliminate the three errors, take *eight* readings of the deflection θ, the mean of which is free from these errors.

(B) Oscillation Experiment

For conducting this experiment it is essential that when the magnet hangs in the magnetic meridian, the suspension fibre should have no twist, since in deriving the formula for the time-period it has been assumed that the only restoring couple ($=MH \sin \theta$) is

* This gives least error in the measurements provided the distance of the magnet from the needle is large as compared with its own length. The deflection, however, may be kept lower (say, 25°), if it is not possible to get 45° by keeping the magnet at a sufficient distance from the magnetic needle.

due to the earth's magnetic field alone, and that the torsional couple ($= c\theta$, where c is the couple due to unit twist) is non-existent.*

Secondly, the moment of inertia I also includes the moment of inertia of the stirrup, it is difficult to evaluate this quantity and therefore, it should be eliminated.† For this purpose, a paper stirrup or simply a double loop of silk fibre to hold the magnet can be employed.

(i) After drawing the direction of the magnetic meridian on the table, place the vibration box with its longer edge parallel to this line. Now put the compass needle inside the box along the line marked on the plane mirror fixed to the base, and adjust the magnetometer, if necessary, to bring this scratch line exactly in the magnetic meridian.

(ii) Now place the brass bar in the stirrup and wait for the fibre to untwist.‡ If the bar remains motionless when hanging freely from the fibre, it should be set parallel to the scratch line by turning the torsion head.

(iii) Now hold the stirrup tight in position and withdraw the brass rod replacing the magnet used in the deflection experiment.

---

* If torsional couple is not negligible, equation (2) takes the form $(MH+c)=4\pi^2 I/T^2$. In order to eliminate c, if a body of some magnetic material, having a known moment of inertia I′ be vibrated and have the time-period T′, then

$$c = \frac{4\pi^2 I'}{T'^2}, \qquad \text{whence} \qquad MH = 4\pi^2\left(\frac{I}{T^2} - \frac{I'}{T'^2}\right).$$

However, if the suspension in the instrument be a horse hair, there is no initial twist like the one to be found in a silk fibre.

† If the mass of the stirrup provided with the instrument is not negligible, the effect of its moment of inertia can be eliminated by first vibrating the magnet alone, then vibrating the magnet and a brass bar (of moment of inertia I_1) together. If T, T_1 are the respective time-periods, we have—

$$T^2 = \frac{4\pi^2 I}{MH} \qquad \text{and} \qquad T_1^2 = \frac{4\pi^2 (I + I_1)}{MH}$$

whence $\qquad MH = 4\pi^2 I_1/(T_2^2 - T^2)$.

‡ During the process of removal of twist, the motion of the brass rod should be checked after every few revolutions, otherwise, when the fibre is untwisted, the inertia of the rotating bar may cause it to twist in the opposite direction.

The magnet should lie *perfectly horizontal* in the stirrup and its *north pole should point northwards*. Now with the help of a second magnet deflect the suspended magnet through a small angle and take away the second magnet to a safe distance from this oscillating magnet. Count twenty-five oscillations by looking through the slit at the top of the box and note the time with an accurate stop-watch. Repeat the process four times and calculate the time-period of the magnet.

(C) Determination of the Constants of the Magnet

(i) To measure the effective length ($2l$) of the magnet measure its total length (a) with a metre scale. Then 5/6 of this length may be taken as the effective length, half of which gives l of the formula given above.

(ii) Measure the breadth (b) of the magnet with a vernier callipers. Weigh the magnet, and then with the help of a, b, and w calculate the moment of inertia (I) of the magnet.

Finally, calculate H with the help of the formula (5) given above.

Observations

[A] *Readings for the determination of θ*

No S.	(d)	Position of the magnet		Deflection of the pointer		
		Arm	Direction	one end	other end	Mean
1.	...cm.	East	N—pole towards the needle			
2.			S—pole ,, ,, ,,			
3.	...cm.	West	N—pole ,, ,, ,,			
4.			S—pole ,, ,, ,,			
					Mean θ	

[B] *Readings for the determination of* T

No.	Number of oscillations	Time taken	Periodic Time* T
1.	25	...min. ...sec.	
2.	25		
3.	25		
4.	25		

[C] *Readings for the constants of the magnet*

(i) Total length of the magnet, (a) = ...cm.

(ii) Breadth of the magnet, (b) = ...cm. ; ...cm. ; ...cm.

(iii) Mass of the magnet (w) = ...gm.

Calculations

(i) Effective length $(2l)$ of the magnet $= \frac{5}{6}\,a =$ ......cm.

(ii) Moment of Inertia of the magnet

$$I = w\left(\frac{a^2 + b^2}{12}\right)$$

$$= \text{......gm. cm}^2.$$

now

$$H = \frac{2\pi}{T(d^2 - l^2)} \sqrt{\frac{2\,I\,d}{\tan\theta}}$$

$$= \dagger\text{oersted}$$

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* $$T = \frac{\text{Total time taken}}{\text{Total no. of oscillations}} = \frac{\text{...sec}}{100} = \text{...sec.}$$

† Use log tables for the calculation work. For this purpose, factories $(d^2 - l^2)$ and calculate I separately before substituting its value in this expression,

Calculation Work

$$I = w\frac{a^2+b^2}{12} = 40{\cdot}8 \times \frac{10{\cdot}1^2+1{\cdot}20^2}{12}$$

$$= 40{\cdot}8 \times \frac{102{\cdot}0+1{\cdot}44}{12}$$

$$= \frac{40{\cdot}8 \times 103{\cdot}4}{12}$$

log 40·8 = 1·6107   log 12 = 1·0792

log 103·4 = 2·0145

Sum = 3·6252

1·0792

Difference = 2·5460   Antilog = I = 351·5

Now

$$H = \frac{2\pi}{T(d^2-l^2)} \times \sqrt{\frac{2\,I\,d}{\tan\,\theta}}$$

$$= \frac{2 \times 3{\cdot}14}{10{\cdot}2\,(14{\cdot}2^2 - 4{\cdot}2^2)} \times \sqrt{\frac{2 \times 351{\cdot}6 \times 14{\cdot}2}{\tan\,46^\circ}}$$

$$= \frac{2 \times 3{\cdot}14}{10{\cdot}2 \times 18{\cdot}4 \times 10{\cdot}0} \times \sqrt{\frac{28{\cdot}4 \times 351{\cdot}6}{1{\cdot}0355}}$$

Numerator		*Denominator*	
log 2 = 0·3010		log 10·2 = 1·0086	
log 3·14 = 0·4969		log 184 = 2·2648	log 1·036
½ log 351·6 = 1·2730	log 28·4	½ log 1·036 = 0·0077	= 0·0154
½ log 28·4 = 0·7267	= 1·4533		
Sum = 2·7976		Sum = 3·2811	
3·2811			
Diff. = $\bar{1}$·5165		Antilog = H = 0·3285	

Result. The value of the horizontal component of the earth's magnetic field in the laboratory at...... = ...oersted.

Precautions and Sources of Error

(1) All pieces of magnetic materials and current-carrying conductors should be removed to a considerable distance from the magnetometers. Examine your pockets for the presence of magnetic materials, e. g., bunch of keys, which too should be removed.

(2) The deflection magnetometer should be carefully set in the tan-A position of Gauss and the magnet should be so placed on the arms that its axis, when produced, passes through the centre of the magnetometer needle.

(3) The deflection of the needle should be had as nearly equal to 45° as possible, since under the condition the deflection shall be susceptible of yielding maximum accuracy*. Further the value of d should be large, so that the field due to the magnet in the region occupied by the needle is sufficiently uniform. However, if it is not feasible to procure the above two conditions at the same time, a compromise should be effected by making d large so that the deflection falls† in the neighbourhood of 25°.

(4) The pivot of the needle may not pass exactly through the centre of the graduated circular scale. To correct for this error both the ends of the pointer should be read and the mean of the two angles should be taken.

(5) If the magnet is unsymmetrically magnetised, the magnetic centre shall not be coincident with its geometrical centre, hence d, the distance between the magnetic centre of the bar magnet and the centre of the magnetic needle in the compass box, shall not be correctly measured. To correct for this error readings should be taken by reversing the polarity of the magnet at the same position.

(6) The centre of the linear scale on which the distance d is read may not be coincident with the pivot of the needle. To correct for this error the magnet should be transferred on the other arm so that the centre of the magnet lies on the same scale reading.

(7) While reading the deflection of the aluminium pointer on the graduated scale, the error due to parallax should be avoided. For this purpose, use should be made of the plane mirror attached to the base of the compass-box.

* For a reason to this statement see Expt.—20 under "Precautions and sources of Error."

† In no case should be deflection fall below 15°.

(8) Before oscillating the magnet in the vibration box, initial twist in the suspension fibre should be completely removed* with the help of a metallic bar of some non-magnetic material. This bar should preferably be of the same size and shape as the magnet itself.

(9) As the moment of inertia of the stirrup is not taken into account in the derivation of the above formula for T, it should be very light.

(10) In the derivation of the formula for the time-period of the magnet it has been assumed that θ is small so that the restoring couple MH sin θ = MHθ In order to satisfy this condition, the deflection of the oscillating magnet should not be more than 5°.

(11) The oscillations *should not be counted by looking at the magnet from the glass side of the box*, but the eye should be held vertically over the slit made on the top of the box, and the counting should be done with reference to the scratch line made on the glass plate at the bottom of the box.

(12) The main **sources of error** in this experiment are :—

(i) The magnetometer needle is not sufficiently short, hence it cannot be justified that it moves in a uniform field produced by the bar magnet—a condition which is absolutely necessary for the validity of the Tangent Law.

(ii) The friction at the pivot is not totally absent. hence the measurement of the deflection is not very accurate. Moreover, the pointer and scale method is not susceptible of any great accuracy. Furthermore, theoretical considerations demand that the deflection of the pointer should be nearly 45°, and at the same time the distance of the magnetic needle from the magnet should be fairly large. Now the compliance of these two conditions at the same time is not always practicable.

(iii) The effective length of the magnet cannot be accurately determined.

(iv) The amplitude of the vibrating magnet, according to theory, should be infinitely small, but this is not feasible in practice.

* For suspension. a horse hair is preferable. It is very strong and at the same time there is no initial twist in it.

(v) In the derivation of the formula for the time-period it has been assumed that the moment of inertia of the stirrup is negligible. This is not actually so, hence the error.

(vi) The suspension fibre may not be completely free from torsional reaction.

(vii) The graduations of the stop-watch may not be entirely reliable.

ADDITIONAL EXPERIMENT

Expt.—11 (a)

Object. To determine the magnetic moment (M) of a given bar magnet by using deflection and vibration magnetometers.

From equation—(6) given above the value of the magnetic moment (M) can be easily calculated out. The experiment is conducted exactly in the same manner as the one described above.

[**Note.** By dividing M by the effective length of the magnet its pole strength can also be evaluated.]

EXPERIMENT—12

Object. *To verify inverse square law in magnetism by applying Gauss's method.*

Theory. *According to Inverse square law, the force between two magnetic poles varies inversely as the square of the distance between them.*

The most satisfactory, though indirect proof was first given by Gauss, whose method consists of a comparison of the magnetic force at a point lying on the axial line of a magnet with the force at an equidistant point lying on the equatorial line of the same magnet.

Let us assume, for the present, that the force between two magnetic poles of strength m_1 and m_2 separated by a distance d in air is given by the formula.

$$F = \frac{m_1 m_2}{d^n}$$

i. e., the force varies inversely as the n^{th} power of the distance between the poles. With this assumption let us calculate the value of

the intensity of the magnetic field at a point in the end-on and the broadside-on positions of the magnet.

Field in the End-on Position

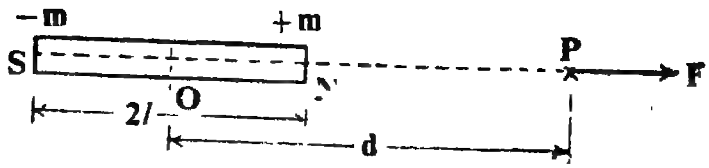


Fig. 18

Field in end-on position

The field at the point P lying on the axial line of the magnet is given by

$$F_1 = \frac{m}{(d - l)^n} - \frac{m}{(d + l)^n}$$

$$= \frac{m}{d^n}\left[\frac{1}{\left(1 - \frac{l}{d}\right)^n} - \frac{1}{\left(1 + \frac{l}{d}\right)^n} \right]$$

The expression inside the brackets can be expanded by the binomial theorem. Neglecting squares and higher powers of l/d we obtain

$$F_1 = \frac{m}{d^n}\left[\left(1 + n \cdot \frac{l}{d}\right) - \left(1 - n \cdot \frac{l}{d}\right) \right]$$

$$= \frac{n \cdot (m \cdot 2l)}{d^{n+1}}$$

$$= \frac{n \cdot M}{d^{n+1}} \qquad \ldots \qquad (1)$$

where M is the magnetic moment of the magnet.

Field in the Broadside-on Position

$$F_2 = \frac{2\,m}{r^n} \cdot \cos\theta$$

Now $r = (d^2 + l^2)^{1/2}$

and $\cos\theta = \dfrac{l}{(d^2 + l^2)^{1/2}}$

$$\therefore \quad F_2 = \frac{2\,ml}{(d^2 + l^2)^{n/2}} \cdot \frac{l}{(d^2 + l^2)^{1/2}}$$

$$= \frac{M}{(d^2 + l^2)^{n+1/2}}$$

or, if l^2 can be neglected in comparison with d^2

$$F_2 = \frac{M}{d^{n+1}} \qquad \ldots \qquad (2)$$

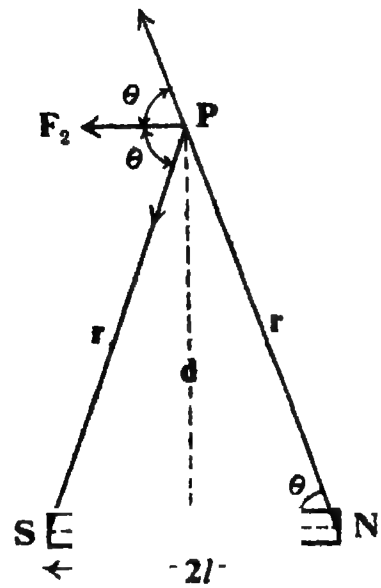


Fig. 19
Field in broadside-on position

Comparing equations (1) and (2), we see that for a *short* magnet

$$\frac{F_1}{F_2} = n$$

The object of the present experiment is to obtain experimentally the numerical value of n.

Method

For this purpose, a sensitive magnetometer (fig.–20) is employed. This consists of a small magnetic needle suspended by a thin silk fibre in a cylindrical brass case, which can be levelled by three levelling screws provided at the base. The frame carrying the needle carries a mirror by which the deflection of the needle is noted by the usual lamp and scale arrangement.

In fig.–20, M is the magnetometer with which two scales S_1 and S_2 are provided, S_1 is in the magnetic meridian while S_2 is perpendicular to it. S is the scale situated nearly a metre away from the mirror of the magnetometer ; L is the lamp, the image of whose filament is focussed on the scale.

To begin with, the magnetometer needle is allowed to come to rest, care being taken that the suspension is entirely free from twist. Now a small bar magnet is placed on the arm S_1 as shown, thereby producing a field at M corresponding to its end-on position. The deflection of light produced on the scale is noted down. Knowing the radius of the cylindrical case of the magnetometer,

the distance d of the magnet is noted down. In order to eliminate the zero reading of the magnetometer, the magnet is reversed and the double reflection of light, left and right, is easily obtained.

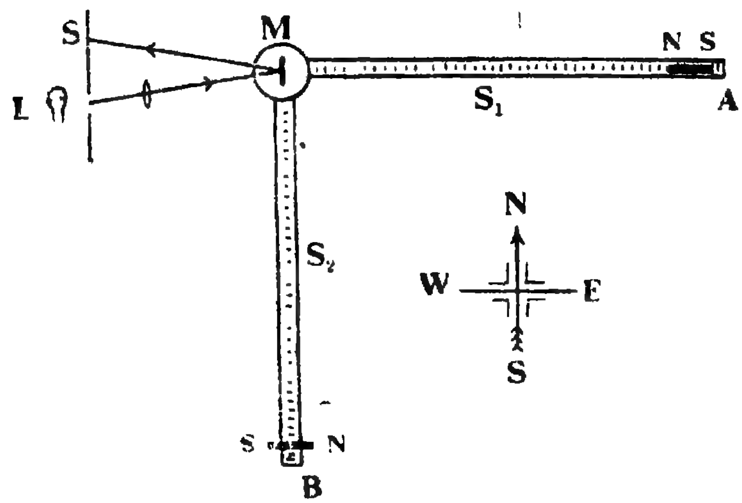


Fig. 20
Verification of the inverse square law in magnetism

Now the magnet is placed on the scale S_2 at an equal distance for the broadside-on position and the deflection noted.

The experiment is repeated for various values of d.

Now, according to tangent law, $F = H \tan \theta$ and we have

$$\frac{F_1}{F_2} = \frac{n}{l} = \frac{\tan \theta_1}{\tan \theta_2}$$

where θ_1 is the deflection when the magnet is at A, and θ_2 is the deflection when the magnet is at B.

If the corresponding displacements of the spot of light be d_1 and d_2, we have

$$\frac{\tan 2\,\theta_1}{\tan 2\,\theta_2} = \frac{d_1}{d_2}$$

since θ_1 and θ_2 are small. Thus

$$n = \frac{d_1}{d_2}$$

If, by actual experiment, the ratio d_1/d_2 comes over to be equal to 2, the Inverse Square Law is verified.

[**Note.** A typical set of observations obtained with a short magnet of length 5 cms. is reproduced below :—

Distance of the magnetometer needle from the centre of the magnet	Position of the spot of light with the magnet in the end-on position, and with the magnet			Position of the spot of light with the magnet in the broadside-on position, and with the magnet			$\frac{d_1}{d_2}$
	Direct	Reversed	Double deflection (d_1)	Direct	Reversed	Double Deflection (d_2)	(=n)
50 cms.	18 8 cm.	−18·2 cm	37·0 cm	9 3 cm	−9·0 cm.	18·3 cm	2·02
60 cms.	11·4 cm	−11·0 cm.	22·4 cm.	5·6 cm.	−5 1 cm.	10 7 cm.	2·09
70 cms.	7·1 cm	−6 6 cm.	13·7 cm.	3·5 cm.	−3·0 cm.	6·5 cm.	2·10
80 cms.	4·6 cm	−4·2 cm.	8 8 cm.	2 4 cm.	−2·1cm.	4·5 cm	1·96
						Mean	2·04

The value of n is very nearly equal to 2, hence Inverse Square Law is verified.]

ELECTRICITY

MEASUREMENT OF RESISTANCES

EXPERIMENT—13

Object. To determine the resistance of a suspended type moving-coil galvanometer by Kelvin's method using a post-office box.

Apparatus* Required—Post-office box, the given moving coil suspended-type galvanometer, Leclanche cell, a variable high resistance, and connecting wires.

Formula Employed—If the four resistances P, Q, R, and G are arranged in the Wheatstone's bridge as shown in the figure, then for a *Constant Deflection* in the galvanometer on operating the usual galvanometer key, the following relation between the four resistances in the arms of the bridge holds good—

$$\frac{P}{Q} = \frac{R}{G}$$

$$G = \frac{Q}{P} . R$$

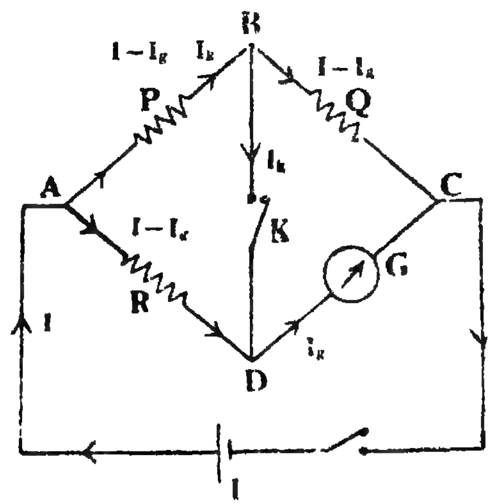


Fig. 21.
Theory of Kelvin's method.

PRINCIPLE AND THEORY OF THE EXPERIMENT

Let I be the main current, I_g the current in the galvanometer and I_k the current in the key branch (BD). Then by applying Kirchhoff's first law, the distribution of the current in the remaining arms of the bridge will be shown as in fig.–21. Now, applying Kir-

* For a detailed study regarding various electrical instruments, read author's book "A Critical Study of Practical Physics and Viva-Voce." (Chapter 12 Page 175).

chhoff's second law in each of the four meshes ABCEA, ADCEA, ABDA, and BCDB, we have respectively the following relations—

$$(I - I_g + I_k)\,P + (I - I_g)\,Q + Ir = E \quad \ldots \quad (1)$$

$$(I_g - I_k)\,R + I_g . G = E \quad \ldots \quad (2)$$

$$(I - I_g + I_k)\,P + I_k . K - (I_g - I_k)\,R = 0 \quad \ldots \quad (3)$$

$$(I - I_g)\,Q - I_g . G - I_k . K = 0 \quad \ldots \quad (4)$$

where K is the resistance of the arm BD (occupied by the key) and r is the resistance of the cell circuit. Now re-arranging the above relations, we have

$$I\,(P + Q + r) + I_k\,P = E + I_g\,(P + Q) \quad \ldots \quad (5)$$

$$-I_k\,R = E - I_g\,(R + G) \quad \ldots \quad (6)$$

$$IP + I_k\,(P + R + K) = I_g\,(P + R) \quad \ldots \quad (7)$$

$$IQ + I_k - K = I_g\,(Q + G) \quad \ldots \quad (8)$$

Now, let the current I_g in the galvanometer be independent of the resistance K, then it follows from relations (5) and (6) that I and I_k are also constant. Now, if $K = \infty$, $I_k = 0$, and hence I_k should always be zero. Then putting $I_k = 0$ in (7) and (8), we have

$$IP = I_g\,(P + R) \quad (9)$$

and

$$IQ = I_g\,(Q + G) \quad (10)$$

whence

$$\frac{P}{Q} = \frac{P + R}{Q + G}$$

or

$$\frac{P + R}{P} = \frac{Q +}{Q}$$

or

$$\frac{R}{P} = \frac{G}{Q}$$

or

$$\frac{P}{Q} = \frac{R}{G} \quad \ldots \quad (11)$$

Thus, when the values of the resistances P, Q, R are so adjusted that they satisfy relation (11), the current in the galvanometer is the same whether the arm BD is joined by a key or not. In this case the initial deflection obtained in the galvanometer remains unaltered when the key in the arm BD is closed. It is due to this reason that the method is known as Kelvin's constant deflection method.

Method

(i) First of all set the galvanometer properly so that its coil swings freely in the clearance space in between the pole-pieces and the soft iron core. Obtain a clear spot of light on the scale. Connect the galvanometer in the unknown resistance arm of the post-

office box and short-circuit the arm, usually occupied by the galvanometer, with a piece of thick copper wire. Make the rest of the connections* as shown in fig.–22 (a).

(ii) Press the cell key K_1 and adjust the high resistance in its series such that the galvanometer gives a suitable deflection. Now

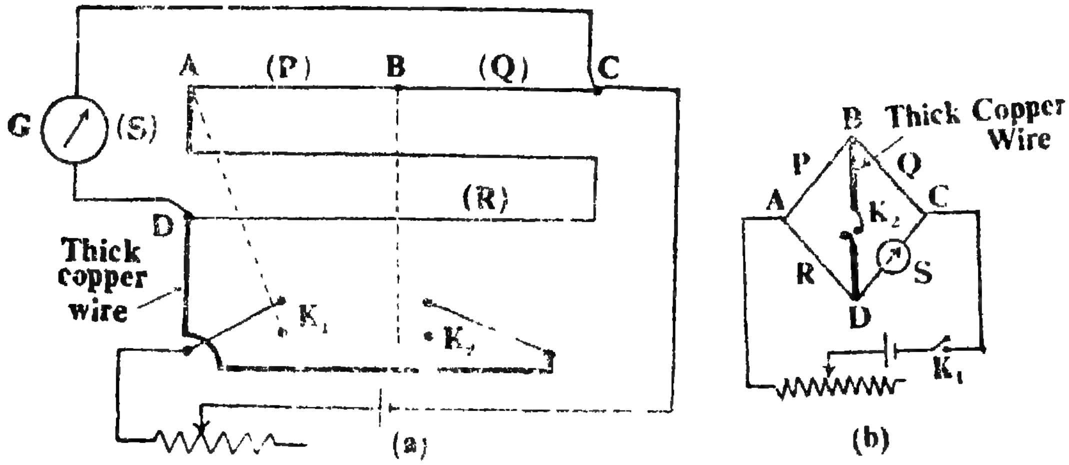


Fig. 22

Connections for Kelvin's method

adjust the ratio arms P and Q to 10 ohms each. Insert a few ohms resistance in the resistance arm R of the post-office box. Press the cell key K_1, and when the deflected spot of light becomes stationary on the scale, press the key K_2. This will normally send a current through the copper wire BD whereby the distribution of current in the network shall change and consequently the position of the spot of light shall change. Note down the direction of deflection in the galvanometer. Now introduce a high resistance in the arm R and observe the direction of deflection in the galvanometer, which will be found to be opposite to the one observed previously.†

(iii) Now determine two consecutive resistances in the arm R such that by introducing them successively the spot of light changes direction. This procedure determines the resistance of the galvanometer within one ohm.

(iv) Now keeping Q = 10 ohms, make P = 100 ohms. Thus the resistance in the arm R shall be ten times the galvanometer resistance in order that a balance may be affected. By trial

* In fig –22 (a) a plug type of post-office box has been employed, but, if available, a dial pattern is always preferable, since it eliminates the uncertain resistances at the contact points. Before making connections, the diagram depicting the usual wheatstone bridge arrangement should be drawn (as shown in fig.–22 b), as it will faciliate the right electrical connections of different components of the circuit.

† Incidentally this procedure ensures that the electrical connections are correctly made.

find out a resistance in this arm which produces no deflection in the spot of light.* Then by the formula given above calculate the galvanometer resistance.

(v) However, if the position of no-change in the deflection of the galvanometer is not attained in the 100 : 10 ratio, the process must be continued for the higher ratio 1000 : 10 and the galvanometer resistance calculated as indicated above.†

Observations

Ratio arm (P)	Ratio arm (Q)	Known resistance arm (R)	Direction of change in the deflection of the galvanometer	Inference regarding the resistance of the galvanometer
10 Ohms	10 Ohms	29 Ohms	Left	Between 29 and 30 Ohms
,,	,,	30 Ohms	Right	
100 Ohms	10 Ohms	293 Ohms	Left	Between 29·3 and 29·4 Ohms
,,	,,	294 Ohms	Right	
1000 Ohms	10 Ohms	2934 Ohms	Left	$G=\frac{1}{100}(29\cdot36\pm1)$
,,	,,	2935 Ohms	No change	$=(2936\pm0\cdot01)$ Ohms
,,	,,	2936 Ohms	No change	
,,	,,	2937 Ohms	No change	
,,	,,	2938 Ohms	Right	

* If at any stage of this process it is found that the bridge has become insensitive, *i. e.*, when the key K_2 is pressed, the deflection of the spot of light is unaffected over a wide range of variation of the resistance in the B arm, increase the current in the galvanometer by adjusting the value of high resistance in the cell circuit. This will result in a greater deflection of the galvanometer coil and the bridge will become more sensitive.

† Sometimes it may be found that the exact balance point is not available even for the ratio 1000 : 10. In that case *the theory of proportional parts* may be employed for the exact evaluation of the unknown resistance. For instance, if it is found that a resistance R in the third arm produces a deflection 'a' towards the right in the spot of light, while a resistance equal to (R+1) produces a deflection of 'b' towards the left, then the resistance required to make the deflection zero is $\left[R+\frac{a}{a+b}\right]$ ohms.

However, it may be added here that there is hardly any justification in going to the third place of decimal, for such a high degree of accuracy cannot be claimed from a post-office box.

Calculations

$$G = \frac{Q}{P} . R$$

$$= \frac{10}{1000} \left(2936 \pm 1 \right)$$

$$= (29{\cdot}36 \pm 0{\cdot}01) \text{ Ohms}$$

Result. The resistance of the given moving-coil galvanometer (suspended type) = (29·36 ± 0·01) Ohms.

Precautions and Sources of Error

(1) The galvanometer should be properly adjusted and levelled so that the coil is free to move in the clearance space in between the pole-pieces and the soft iron core.

(2) The post-office box should be preferably of the dial pattern since in this type uncertain contact resistances are considerably reduced. However, if the plug-type post-office box is the only one available, the sockets should be clean and the plugs should be properly secured in them ; they should be neither too tight nor too loose. To secure a good contact, after inserting the plug in the hole, a slight clockwise screw motion should be given to the head of the plug.

(3) The current in the bridge should be passed momentarily, so that undue heating, and consequent change in the value of the resistances, is avoided.

(4) The battery key should be pressed first so that a constant deflection is produced in the galvanometer. Thereafter, the usual galvanometer key (in the arm BD) should be pressed in order to note the change in the direction of the galvanometer coil. The bridge is balanced when there is no change in the deflected position of the spot of light.

(5) The arm BD should be short-circuited with a *thick copper wire of requisite length* only, since the value of the current flowing in this arm due to a little lack of balance shall depend on its resistance, which, if small, shall facilitate the detection of the balance.

(6) To ensure maximum sensitiveness of the bridge the resistances of the four arms should be of the same order, and if it is not feasible in practice, the battery with its series high resistance should be connected between the junctions of the two higher and the two lower resistances.

A NOTE ON THE SENSITIVENESS OF THE WHEATSTONE'S BRIDGE

In the practical application of the Wheatstone's network for the measurement of resistances of varying magnitudes, the accuracy in the determination of the exact balance point depends on the sensitiveness of the bridge. The sensitiveness of the bridge depends upon the current I_g in the galvanometer, because greater the value of the current flowing in the galvanometer for a given unbalance $\triangle R$ of the bridge, greater is the sensitiveness of the bridge. Hence in order to make the bridge most sensitive, the relative positions of the cell and the galvanometer, and the magnitudes of the resistances in the different arms of the bridge should be so adjusted that a large current flows in the galvanometer for a small unbalance of the bridge.

The conditions for procuring maximum sensitiveness of the bridge are as follows—

(i) When all the resistances except the unknown resistance S can be varied, maximum sensitivity is attained when all the resistances are equal, i. e., when

$$P = Q = R = S = G = B$$

where B is the battery resistance. In actual practice, however, B and G are invariable, and in that case the conditions of maximum sensitivity are—

$$P = GB$$

$$Q^2 = SB\frac{G + S}{B + S}$$

and $$R^2 = SG\frac{B + S}{G + S}$$

But for all practical purposes the bridge is sufficiently sensitive if $P = Q = R = S$.

(ii) Even the last condition just mentioned for sensitivity of the bridge is not always feasible in practice. Hence in cases where it is not possible to make $P = Q = R = S$, maximum sensitivity will depend upon the relative positions of the battery and the galvanometer. Theory shows that the sensitivity of the bridge will be maximum if the battery or the galvanometer (whichever is of higher resistance) is connected between the junctions of two highest and two lowest resistances of the bridge.

Thus, in order to utilise the maximum sensitivity of the bridge in this experiment, *the battery along with its series resistance (the total resistance of the two being greater than that of the galvanometer) should be connected between the junction of the two higher and the two lower resistances.*

EXPERIMENT—14

Object. To determine the internal resistance of a Leclanche cell by Mance's method using a post-office box.

Apparatus Required. Post-office box, Leclanche cell, moving coil (suspended type), galvanometer, a variable high resistance, and connecting wires.

Formula Employed. If the four resistances are arranged in the Wheatstone's bridge as shown in the accompanying figure, then for a *constant deflection* in the galvanometer on operating the usual cell key, the following relation between the four resistances in the arms of the bridge holds good—

$$\frac{P}{Q} = \frac{R}{B}$$

or $$B = \frac{Q}{P} . R$$

where B is the cell resistance.

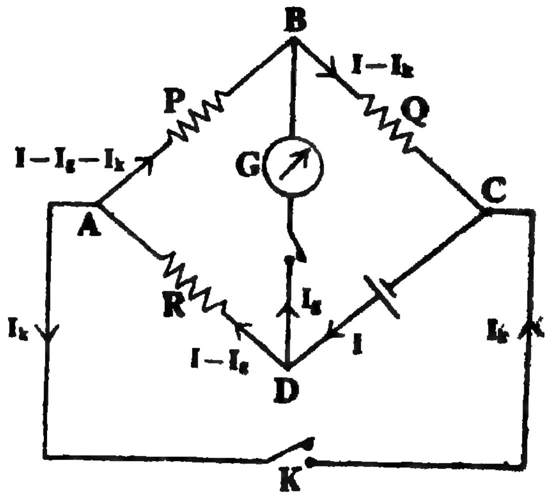


Fig. 23
Theory of mance's Method

PRINCIPLE AND THEORY OF THE EXPERIMENT

Let I be the main current, I_g the current in the galvanometer, and I_k the current in the key branch (arm AC). Then by applying Kirchhoff's first law the distribution of the current in the remaining arms of the net will be as shown in fig.–22. Now applying Kirchhoff's second law to the meshes ABDA and BCDB we have—

$$(I - I_g - I_k)\, P - I_g\, G + (I - I_g)\, R = O$$

and $$(I - I_k)\, Q - IB + I_g . G = E$$

Rearranging the above two equations, we have—

$$I\,(P + R) - I_g\,(G + P + R) = I_k\, P \qquad \ldots (1)$$

and $$I\,(B + Q) + I_g\, G = E + I_k\, Q \qquad \ldots (2)$$

From these two relations we have to determine the value of I_g in terms of I_k only. Thus, by multiplying (1) by (B + Q) and (2) by (P + R), we have—

$$I\,(P + R)\,(B + Q) - I_g\,(G + P + R)\,(B + Q) = I_k\, P\,(B + Q) \qquad \ldots (3)$$

$$\text{and } I\,(B + Q)\,(P + R) + I_g\, G\,(P + R) = (E + I_k\, Q)\,(P + R) \qquad \ldots (4)$$

Subtracting (3) from (4), we get

$$I_g\,[\,G\,(P + R) + (G + P + R)\,(B + Q)\,] = E\,(P + R) + I_k\,[\,Q\,(P + R) - P\,(B + Q)\,]$$

Now let I_g, the current in the galvanometer, be independent of the resistance K in the branch AKC. Thus, it is obvious that the above expression for I_g must be independent of I_k. Since I_k is not equal to zero, we should have—

$$Q(P+R) - P(B+Q) = 0$$

whence
$$\frac{P}{Q} = \frac{R}{B} \qquad \dots \qquad \dots (5)$$

Thus, when the values of the resistances P, Q and R are so adjusted that they satisfy relation (5), the current in the galvanometer is the same whether the arm AKC is joined by a key or not. In this case, the initial deflection obtained in the galvanometer remains unaltered when the key in the arm AKC is closed. It is for this reason that this method is known as Mance's constant deflection method.

Method

(i) First of all adjust the galvanometer so that its coil swings freely in the gap between the pole-pieces and the soft iron core. Obtain a well-defined spot of light on the scale. Connect the cell in the unknown resistance arm (S) of the post-office box, and short-circuit the arm, usually occupied by the cell, with a piece of thick copper wire. Make the rest of the electrical connections as follows :—

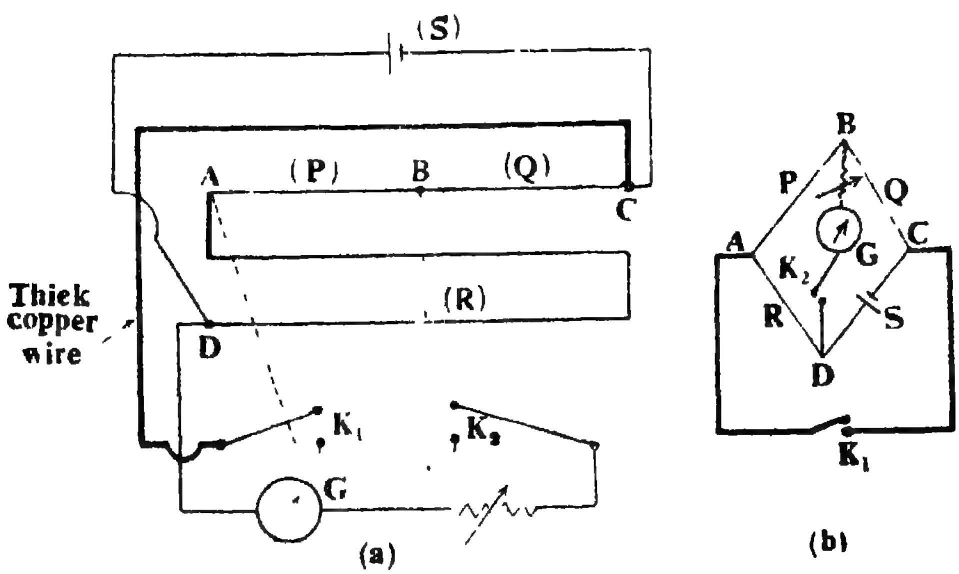


Fig. 24
Connections for Mance's method

[Note. In the above arrangement, a plug-type of post-office box has been employed, but, if available, a dial pattern is always preferable, since it eliminates the uncertain resistances at the contact points. Before making the actual connections, the diagram

depicting the usual Wheatstone bridge arrangement (shown in fig.–24 b) should be drawn, as it will facilitate the correct connections of different components of the circuit.]

(ii) Press the key K_2 and adjust the high resistance in series with the galvanometer so that a suitable deflection is obtained on the scale. Now, adjust the ratio arms P and Q to 10 ohms each. With the key K_2 pressed, find by trial two consecutive resistances in the variable arm R which produce oppositely directed deflections of the spot of light when the key K_1 is pressed. This determines the limits for the resistance of the cell.

(iii) Now, keeping Q = 10 ohms make P = 100 ohms. Thus, the resistance in the arm R shall be ten times the cell resistance in order that a balance may be effected. By trial find out a resistance in this arm which produces *no change in the deflection* in the spot of light.* Then with the help of the formula given above calculate the resistance of the cell.

(iv) However, if the position of no-change in the galvanometer deflection is not attained in the 100 : 10 ratio, the process must be continued† for the higher ratio 1000 : 10 and the galvanometer resistance calculated as indicated above.

Observations

Ratio arms (P)	Ratio arms (Q)	Known resistance arm (R)	Direction of change in the deflection of the galvanometer	Inference regarding the resistance of the cell
10 Ohms	10 Ohms	1 Ohms	Left	Between 1 and 2 Ohms
,,	,,	2 Ohms	Right	
100 Ohms	10 Ohms	11 Ohms	Left	Between 1·1 and 1·2 Ohms
,,	,,	12 Ohms	Right	
1000 Ohms	10 Ohms	113 Ohms	Left	The resistance of the cell = 1·14 Ohms
,,	,,	114 Ohms	No change	
,,	,,	115 Ohms	Right	

* If at any stage of this process it is found that the bridge has become insensitive, *i. e.*, when the key K_1 is pressed, the deflection of the spot of light is uneffected over a wide range of variation of the resistance in the arm R, increase the current in the galvanometer by adjusting the high resistance joined in series with it. This will result in a greater deflection of the galvanometer coil and the bridge will become more sensitive.

† To attain the maximum sensitivity of the bridge, read carefully the remarks given at the end of "Method" of the previous experiment.

Calculations

$$B = \frac{Q}{P} \cdot R$$

$$= \frac{10}{1000} \cdot 114$$

$$= 1{\cdot}14 \text{ ohm.}$$

Result. The internal resistance of the Leclanche cell = 1·14 ohm.

Precautions and Sources of Error

(1) The galvanometer should be properly adjusted and levelled so that the coil is free to move in the gap between the pole-pieces of the magnet and the soft iron core.

(2) The ends of the connection wires should be clean and they should be firmly secured in the binding terminals.

(3) The post-office box should be preferably of the dial type since in this pattern uncertain resistances at the contact points are considerably reduced. However, if only the plug type post-office box is available, the sockets should be clean and the plugs should be properly secured in them ; they should be neither too tight nor too loose. To secure a firm contact, after inserting the plug into the hole, a slight clockwise screw motion should be given to the head of the plug.

(4) The current in the bridge should be passed momentarily, so that undue heating and consequent change in the value of the resistances is avoided.

(5) The galvanometer key should be pressed first so that a constant deflection is produced in the galvanometer. Thereafter, the usual cell key (in the arm AK_1C) should be pressed in order to note the change, if any, in the direction of deflection of the galvanometer coil. The bridge is balanced when there is no change in the deflected position of the spot of light.

(6) The arm AK_1C should be short-circuited with *a thick copper wire of requisite length* only, since the value of the current in this arm due to a little lack of balance shall depend on its resistance on which, if small, shall facilitate the detection of the balance.

(7) To ensure maximum sensitiveness* of the bridge, the resistances of the four arms should be of the same order, and if it is not feasible in practice, the galvanometer with its series high

* Regarding the sensitiveness of the bridge, see the Note given at the end of expt.–13.

resistance should be connected between the junctions of the two higher and the two lower resistances.

Lodge's Modification of Mance's Method

The defect of Mance's method lies in the fact that at the balance point the current flowing through the galvanometer is large and the current flowing through the cell is unknown, whereas the internal resistance of the cell is dependent on the value of the current drawn from the cell, and hence the result obtained by this method is neither consistent nor reproducible. In this sense, the potentiometer method (vide expt.–33) is better as the current flowing through the cell can be known.

In order to prevent the passage of a large current through the galvanometer, Lodge modified Mance's method by using a condenser (of capacity of the order of half a micro-farad) in series with the galvanometer. This does not allow a current to flow through the galvanometer in the steady state and hence there is no initial deflection in the galvanometer. If the key K is pressed and there is a flow of current in the arm AKC, there will be a change of potential difference between B and D This will alter the charge on the condenser and a momentary kick will be produced in the galvanometer. The bridge is balanced when, on pressing the key K, no such kick is observed in the galvanometer.

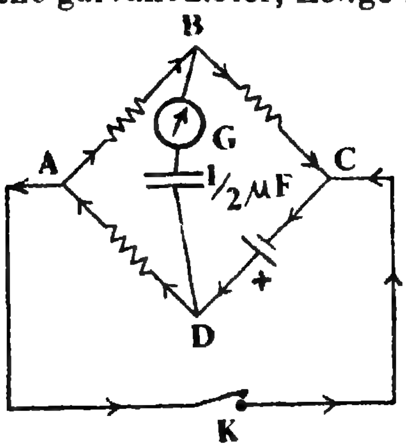


Fig. 25.
Lodge's method

Thus by the use of the condenser the balance point can be accurately determined on account of the increased sensitivity of the arrangement. Further, this method eliminates the possibility of any damage to the galvanometer by the passage of large currents through it.]

EXPERIMENT—15

Object. To determine the resistance per unit length of a Carey Foster's bridge wire and then to compare the resistance of a given one-ohm coil with a standard one-ohm resistance.

Apparatus Required. Carey Foster's bridge, Leclanche cell, Weston galvanometer, interchanging commutator, a sliding rheostat of small value, the given one-ohm coil, a dial pattern decimal ohm box, thick copper strips, plug key, connection wires.

Formula Employed. The resistance (ρ) per unit length of the bridge wire is given by the formula

$$x - y = \rho\,(l_2 - l_1) \qquad \ldots \qquad (A)$$

where x, y are the two resistances connected in the outer gaps

of the bridge, and l_1, l_2 are the readings on the scale of the positions of the balance point on the bridge wire before and after interchanging the resistances. If y = 0 and x = a known resistance R, then we have

$$\rho = \frac{R}{l_2 - l_1} \qquad \ldots \qquad (B)$$

Equation (B) is employed for calculating the resistance per unit length of the bridge wire. Knowing ρ, the value of the unknown resistance x can be calculated with the help of equation (A) provided y be a known resistance.

PRINCIPLE AND THEORY OF THE EXPERIMENT

The ordinary metre bridge, when employed in the usual way, is not susceptible of very great accuracy. With this arrangement it is not possible to find the position of the balance-point within 1 mm. and with a slide wire 1 metre long this uncertainty introduces a possible error* of 1/250 at least. If the balance-point be not situated at the middle of the wire, the uncertainty of the result becomes greater than this. A long bridge wire can be used if desired, and the relative magnitude of an error of 1 mm. in the measurement can be correspondingly diminished, but the use of a wire longer than 1 metre is not convenient.

In the Carey Foster's bridge the effective length of the wire is increased without actually using a wire of more than the usual 1 metre length by introducing resistances in series with the wire, one at each end. The connections are shown in the accompanying figure. The arms P and Q are the usual ratio arms of the Wheatstone's bridge, while the remaining arms R and S are composed of X plus a length l of the bridge wire, and Y plus the remainder of the bridge wire (L—l), where L is the total length of the slide wire.

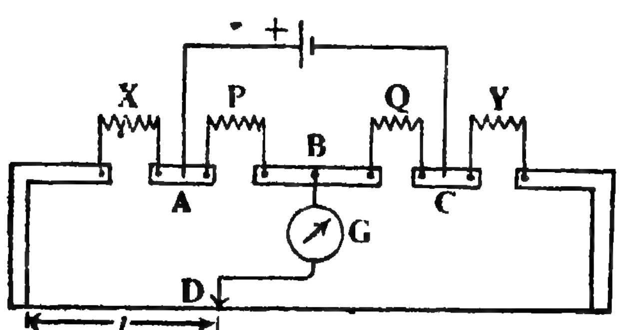


Fig 26.
Theory of Carey Foster's bridge

When the bridge is balanced, the usual Wheatstone bridge relation

$$\frac{P}{Q} \qquad \frac{R}{S}$$

* See the Note given at the end of this experiment.

now stands as
$$\frac{P}{Q} = \frac{X + l\rho}{Y + (L-l)\rho} \quad \dots (1)$$

where ρ is the resistance of the bridge wire per unit length. If X and Y are together equal to ten times the resistance of the bridge wire, the terms $l\rho$ and $(L-l)\rho$ are of the order of ten per cent of X and Y. Any error in reading l is thus reduced to one-tenth of the relative magnitude it has when the bridge is used in the usual manner, an error of one mm. corresponding with an error of 1/2500 in the result instead of 1/250.

Now, let the null-point be obtained at l_1, then equation (1) is put down as

$$\frac{P}{Q} = \frac{X + l_1\rho}{Y + (L-l_1)\rho}$$

which, by adding 1 to both sides, reduces to the equation

$$\frac{P + Q}{P} = \frac{X + Y + L\rho}{X + l_1\rho} \quad \dots (2)$$

Now let the resistances X and Y be interchanged, and a new null-point be obtained at a distance l_2 from the same end of the bridge wire. Then we have similarly

$$\frac{P + Q}{P} = \frac{X + Y + L\rho}{Y + l_2\rho} \quad \dots (3)$$

The left-hand sides of equations (2) and (3) are identical, and the numerators of the fractions on the right-hand side are exactly equal. Hence

$$X + l_1\rho = Y + l_2\rho$$

or
$$X - Y = \rho\,(l_2 - l_1) \quad \dots (4)$$

This result is of great importance because by means of it we can determine the value of ρ by using known resistances X and Y. Further when we have determined ρ for the bridge wire we can use equation (4) to find the difference* between two nearly equal resistances, one of which may be a standard coil.

End-Corrections. If the bridge wire is soldered imperfectly at the ends where it joins the copper strips, the joint may introduce an appreciable resistance into the arms R and S. In this case, the ratio P/Q should be expressed as

$$\frac{P}{Q} = \frac{X + l_1\rho + \alpha\rho}{Y + (L + l)\rho + \beta\rho} \quad \dots (5)$$

α, β being the *end-corrections* expressed as *equivalent lengthenings*

* It is obvious that this difference should be less than the resistance of the bridge wire.

of the two parts of the bridge wire. Now, equations (2) and (3), take respectively the form

$$\frac{P+Q}{P} = \frac{X + Y + (L + \alpha + \beta)\rho}{X + (l_1 + \alpha)\rho} \quad \ldots (6)$$

and

$$\frac{P+Q}{P} = \frac{X + Y\,(L + \alpha + \beta)\,\rho}{Y + (l_2 + \alpha)\,\rho} \quad \ldots (7)$$

The left-hand sides of equations (6) and (7) are equal, hence equating the right-hand sides we have

$$\frac{X + Y + (L + \alpha + \beta)\,\rho}{X + (l_1 + \alpha)\,\rho} = \frac{X + Y + (L + \alpha + \beta)\,\rho}{Y + (l_2 + \alpha)\,\rho}$$

In this relation, the numerators are equal, hence equating the denominators we have

$$X + (l_1 + \alpha)\,\rho = Y + (l_2 + \alpha)\,\rho$$

which, on simplification, gives

$$X - Y = (l_2 - l_1)\,\rho \quad \ldots (8)$$

Thus equation (8) is identical with (4). Consequently, by following the method already described, the effects of these end-corrections can be easily eliminated.*

From the above procedure it is easily seen that the great advantage of the Caray Foster's method lies in the fact that P and Q need not be known accurately ; it is only necessary that they should be approximately equal and absolutely constant. For maximum sensitivity of the bridge they should have nearly the same value as X and Y.

Method

(i) Set up the apparatus as shown in the accompanying figure. To obtain the two ratio arms P and Q, use a sliding

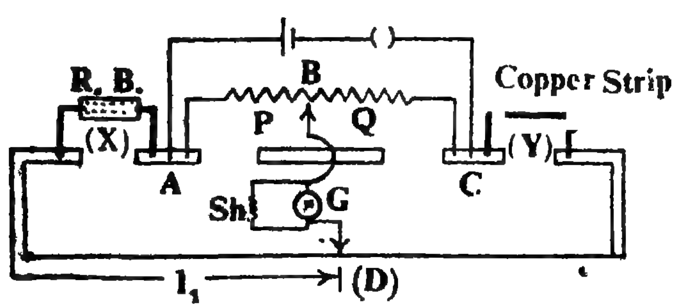


Fig. 27
Carey Foster's Bridge (with electrical connections)

Slight errors due to thermo-electric E. M. F.'s can be eliminated by reversing the battery connections and taking the mean of the readings for l_1 and l_2.

rheostat, whose fixed terminals should be connected to A and C and the variable terminal B to the jockey D through the galvanometer. Adjust the variable point B of the rheostat in the middle so that the two resistances P and Q are nearly equal.*

(ii) Shunt the galvanometer and put the jockey towards the left end of the wire. Introduce a suitable resistance in the decimal ohm box so that an approximate null-point is obtained here. Remove the shunt and determine the exact position of the null-point. Note the reading l_1 *from one end of the wire* (say, the left end).

(iii) Now interchange† the two resistances (X and Y) in the outer gaps of the bridge and note the reading l_2 *from the same* (*i.e.*, the left) *end.*

(iv) Next, alter the value of R slightly and obtain different sets of observation. Calculate the value of ρ separately for each set and obtain the mean value.

(v) Now, to compare the two resistances, replace the copper strip by the given one ohm coil, and introduce a resistance of 1 ohm in the decimal ohm box‡. Move the jockey towards the middle of the wire and obtain the approximate null-point (with the shunt on the galvanometer), and then the exact null-point (with the shunt removed). Note the reading l'_1.

* As stated earlier, it is not essential that the two ratio arms P and Q should be exactly equal. Of course, when high sensitivity of the bridge is desired, P should be equal to Q. Moreover, it is not necessary that the values of P and Q are known. If P = Q and the bridge wire is uniform, the positions of the null-points before and after interchanging the resistances in the two outer gaps will be situated at equal distances from the centre of the bridge wire. If there is too much difference between the values P and Q, then the two null-points may not at all be available on the wire. Moreover, the rheostat is useful inasmuch as it can be employed to obtain the null-points in any part of the wire. Thus with a given set of values for X and Y we can have several sets of readings for $(l_2—l_1)$, This could not have been possible with fixed values of P and Q (*e. g.*, by taking two separate coils and inserting them in the inner gaps (as shown in Fig.–26). Moreover, due to close proximity the two resistances P and Q are equally effected by variations in temperature etc.

† In order to avoid disturbance in the values of X and Y during bodily interchange, an interchanging commutator should preferably be employed.

‡ It is very important to remember that the decimal ohm box and given one ohm coil are connected to the bridge with the help of thick copper strips and not with ordinary connection wires.

(vi) Next interchange the resistances in the outer gaps and determine the value of l'_2 as before. Then slightly vary the position of the sliding contact of the rheostat (thereby varying the relative magnitudes of P and Q) and repeat the observations to obtain different sets for $(l'_2 - l'_1)$. Use this value to obtain the true value of resistance of the given one-ohm coil.

Observations

[A] *Readings for the determination of* ρ.

S. No.	Resistance introduced in the R. Box (R)	Position of null-point with copper strip in the		$l_2 - l_1$	ρ
		right gap (l_1)	left gap (l_2)		
1.					
2.					
3.					
				Mean	...ohm/cm.

[B] *Readings for the comparison of resistances.*

S. No.	Position of null-point with the given one-ohm coil in the		$(l_2'-l_1')$	Calculated resistance of the given coil
	left gap (l'_1)	right gap (l'_2)		
1.				
2.				
3.				
			Mean.	... ohm

Calculations

[A] *Set I*

$$\rho = \frac{R}{l_2 - l_1}$$

$$= \ldots \text{ ohm/cm}$$

etc...etc.

[B] *Set I* $X = Y + \rho\,(l'_2 - l'_1)$

$= 1 + \ldots \quad \ldots \qquad (Y = 1 \text{ ohm})$

$= \ldots \ldots$ ohm.

etc...etc.

Result. (1) The resistance per unit length of the bridge wire = ...ohm/cm.

(2) The resistance of the given one-ohm coil = ...ohm.

Precautions and Sources of Error

(1) The ends of the connection wires should be clean and they should be firmly secured in the binding terminals. However, the decimal ohm box and the given one-ohm coil should be connected to the bridge with the help of *thick copper connection strips* and not with ordinary connection wires.

(2) A plug key should be used in the cell circuit and the current should be allowed to flow only for the time when readings are being taken. This will avoid unnecessary heating of the resistances.

(3) To avoid any induction effects the cell circuit should be completed first and then the galvanometer circuit. While breaking the circuit reverse order should be followed.

(4) The jockey should be pressed gently and momentarily. In no case should the jockey be pressed against the wire when it is being moved along it. This will avoid rubbing of the wire and consequent change in the diameter of the wire.

(5) Use a rheostat to obtain the ratio arms P and Q, whose values should nearly be equal. Moreover, to procure maximum sensitiveness of the bridge, the four resistances (P, Q, X, and Y) should have nearly equal values.

(6) When calibrating the wire, the resistance introduced in the resistance box should be such as enables us to have the two null-points towards the two ends of the bridge wire. This will be so when the resistance in the box is slightly less than the resistance of the entire bridge wire. By getting the null-points towards the ends we are able to get the value of $(l_2 - l_1)$ nearly equal to the length of the bridge wire, thereby reducing to a minimum any error introduced due to the non-uniformity of the wire.

(7) While comparing X and Y attempt should be made to obtain the null-point as near to the centre of the bridge wire as possible. This procedure shall reduce the inaccuracy in the result due to a small error in reading the position of the null-points to a minimum.

(8) To protect the galvanometer from damage, it should be shunted by a low resistance wire, which should be removed only when the null-point is nearly approached.

A NOTE ON THE ACCURACY AND SENSITIVENESS OF THE CAREY FOSTER'S BRIDGE.

In the metre bridge arrangement, if the null-point is obtained at a distance l from one end of the bridge wire whose total length is L, the unknown resistance X is given by

$$X = \frac{l}{L - l} \cdot R$$

$$\therefore \qquad dX = \frac{L \cdot dl}{(L - d)^2} \cdot R$$

Thus, the *relative inaccuracy* dX/X is given by

$$\frac{dX}{X} = \frac{L\ dl}{i(L - l)} = \frac{dl}{l(1 - l/L)}$$

Now, dX/X is minimum when $l = L/2$. Hence the minimum inaccuracy is given by

$$\frac{dX}{X} = \frac{dl}{\frac{L}{2}\left(1 - \frac{1}{2}\right)} = \frac{4}{L} \cdot dl$$

Since $dl = 1$ mm. $= 1/10$ cm. (the least count of the metre scale) and $L = 1$ metre $= 100$ cm., we have

$$\frac{dX}{X} = \frac{4}{100} \cdot \frac{1}{10} = \frac{1}{250}$$

From the above it is clear that the *Accuracy* in the result is directly proportional to L. Hence if we increase the length of the bridge wire, inaccuracy in the result diminishes or accuracy is enhanced. In the Carey Foster's bridge, by introducing extra resistances in the two outer gaps, the length of the bridge wire is apparently increased and consequently this arrangement is more accurate than the ordinary metre bridge.

Let us now examine the *sensitiveness* of the bridge. Sensitiveness can be mathematically represented by $\frac{dl}{dR}$, where dl is a small shift in the position of the null-point due to a small change dR in the known resistance R when the bridge is balanced. Now

$$R = \frac{L - l}{l} \cdot X = \left(\frac{L}{l} - 1\right) \cdot X$$

$$dR = -\frac{L \cdot X}{l^2} \cdot dl$$

or $$\frac{dl}{dR} = -\frac{l^2}{LX}$$

Now l can be put down as some fraction of L, say nL, where n is less than one. Thus

$$\frac{dl}{dR} = -\frac{(nL)^2}{LX} = -\frac{n^2}{X} . L$$

This shows that sensitiveness is directly proportional to the length of the bridge wire. Consequently if we increase the length of the bridge wire, the instrument shall become more sensitive. It is for this reason that *the Carey Foster's bridge is not only more accurate but is also more sensitive than the usual metre bridge.*

EXPERIMENT—16

Object. To determine the temperature coefficient of resistance for platinum using a Carey Foster's bridge and a platinum resistance thermometer.

Apparatus Required. A platinum resistance thermometer, Carey Foster's bridge, Leclanche cell, Weston galvanometer, a decimal ohm box, two resistance coils of equal value (say, 2 ohms), water heating arrangement, and a mercury thermometer.

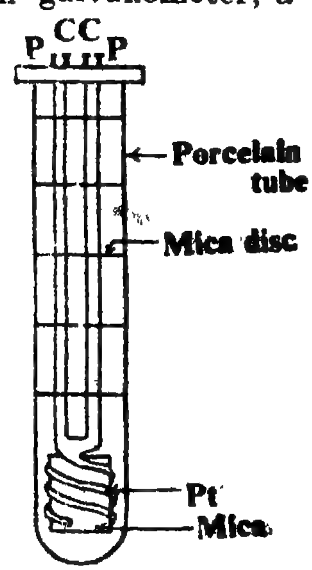


Fig. 28
Platinum resistance thermometer

Description of the Apparatus. The platinum resistance thermometer is a practical application of the temperature variation of resistance for the measurement of temperature. It consists of a fine, exceptionally pure platinum wire wound in a non-inductive manner on a mica frame contained in a long tube made of either hard glass (for use upto temperatures of 700°C) or of glazed porcelain (for higher temperatures). The ends of the wire are brought to the top after passing through holes pierced in mica discs contained in the tube as shown in the figure. The ends of these platinum leads* are lettered PP at the top. The terminals marked CC are connected to an additional platinum wire of exactly the same length and size as the platinum leads. the wire ending in a loop at the lower end. These are called the *compensating leads,* since

* In the cheaper variety of the thermometer these platinum leads are replaced by copper leads.

they nullify the effect of the platinum leads at all temperatures, as they are placed side by side and any change of temperature effects them equally.

Formula Employed.* The temperature coefficient of resistance α is obtained with the help of the following formula—

$$\alpha = \frac{R_2 - R_1}{R_1 t_2 - R_2 t_1}$$

where R_1 = Resistance of the platinum wire at t_1°C

and R_2 = Resistance of the platinum wire at $t°_2$C

Now, $R_1 = r + (l_2 - l_1)\rho$

and $R_2 = r' + (l'_2 - l'_1)\rho$

where ρ = Resistance of the bridge wire per unit length.

r, r' = Respective resistances introduced in the decimal ohm resistance box for obtaining null-point at $t°_1$C and $t°_2$C.

l_1, l_2 = Lengths of the bridge wire from the same end to the balancing points, before and after interchanging the resistances in the outer gaps at $t°_1$C.

$l'_1\ l'_2$ = Same quantities at $t°_2$C.

PRINCIPLE AND THEORY OF THE EXPERIMENT

The ratio of the potential difference between two points of a wire to the current flowing through it is constant only when the temperature is constant.† In other words, the resistance of a wire varies with temperature, and, in general, the resistance at a higher temperature is greater than the resistance at a lower temperature. Thus, if R_0 be the resistance of the wire at 0°C, that at t°C is given by the following formula :—

$$R_t - R_0 (1 + \alpha t) \qquad \ldots \qquad (1)$$

where α is a constant known as the temperature coefficient of resistance for the material of the wire. Relation (1) can be put in the form

$$\alpha = \frac{R_t - R_0}{R_0 \cdot t}$$

Thus, in order to determine α in the laboratory we have to measure the resistance of the platinum thermometer at 0°C, and then at another known temperature t (say, the boiling point of water). However, the use of ice can be avoided if we measure the resistance

* For performing the experiment in a slightly different manner, see the Note given at the end of this experiment.

† This is actually the definition of Ohm' law.

at any two known temperatures (say, the room temperature t_1, and the boiling point of water t_2). Thus, if R_1 and R_2 be the respective resistances at these two temperatures, we have

$$R_1 = R_0 (1 + \alpha t_1)$$

and $$R_2 = R_0 (1 + \alpha t_2)$$

which, on simplification, easily yields

$$\alpha = \frac{R_2 - R_1}{R_1 t_2 - R_2 t_1} \tag{2}$$

Now the accuracy with which α is determined depends on the accuracy with which the *change* in resistance may be measured. Since $(R_2 - R_1)$ is the *small* difference between two large quantities, each resistance must be measured most carefully. An accurate post office box may be employed for measuring the resistances, but when the resistance to be measured is of the order of 1 ohm, Carey Foster's bridge is to be preferred for this determination.

Let the connections be set up as shown in the figure. P and Q are two coils each of 2-ohm resistance. In the left gap platinum leads lettered PP are connected while in the right gap compensating leads lettered CC are connected through a decimal ohm box. Let the resistance introduced in the resistance box to effect a null-point very close to the middle of the wire be r, and l_1 be the length of the wire as indicated If, on interchanging the positions of resistances in the two outer gaps, the balance point shifts to a length l_2, we have for the resistance of the platinum spiral at $t°_1$C.

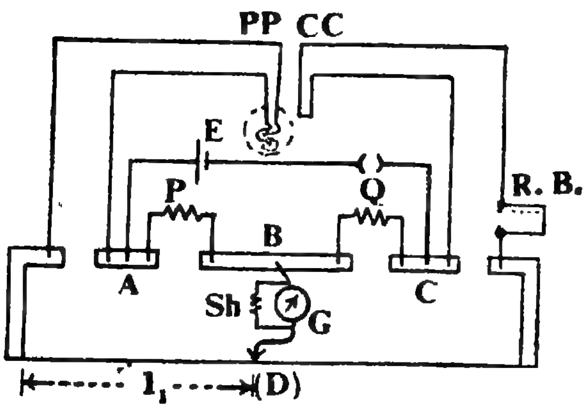


Fig. 29.
Connections with a Pt-resistance thermometer

$$R_1 = r + (l_2 - l_1)\rho \qquad \dots \qquad (3)$$

Let the platinum spiral be now at a temperature t_2, and for a null-point let the resistance introduced in the box be r′, then

$$R_2 = r' + (l_2' - l_1')\,\rho \qquad \dots \qquad (4)$$

Knowing the value of ρ from a previous calibration (as described in expt.-15 the values of R_1 and R_2 can be evaluated. Hence, the value of a can be calculated with the help of equation (2) given above.

Method

(i) First of all carry out the experiment to determine the value of ρ, the resistance of the bridge wire per unit length as described in the previous experiment.

(ii) Now set up the connections as shown in fig.–29. Shunt the galvanometer. Suspend the platinum resistance thermometer in a tumbler of water which can be heated when required. Alongside it suspend also a mercury thermometer.

(iii) By taking a suitable resistance in the decimal ohm box obtain the null-point* near the middle of the bridge wire. The exact position should be ascertained by removing the shunt from the galvanometer. Note the reading of the resistance introduced as also the position of the null-point from one end of the bridge wire.

(iv) With the help of the interchanging commutator, interchange the resistances in the outer gaps, and find the position of the new balance point. Record the value of l_2 from the same end. Calculate the value of the resistance (R_1) of the platinum spiral at temperature t_1 with the help of equation (3) given above.

(v) Now, heat the water in the tumbler till it begins to boil. Wait for at least fifteen minutes so that the platinum spiral acquires the steady temperature of the water bath. Then repeat the steps described in steps (iii) and (iv) and note down the positions (l'_1 and l'_2) of the new balance points before and after interchanging the resistances in the outer gaps. Then calculate the value of R_2 from equation (4). Next calculate the value of the temperature coefficient of resistance† from equation (2).

* The adjustment for balance in the above procedure is correct when there is no *immediate* deflection on depressing the jockey. If the jockey is kept down for a short time, the platinum becomes heated by the passage of the current, its resistance changes, and the balance is no longer correct.

† If observations at several temperatures can be taken, the value of α should be determined by drawing a graph between R (represented on the y-axis) and t (represented on the x-axis). This graph is a straight line.‡ To get the mean value of α from the graph, extend the line both ways and read the resistance (R_0) at 0°C and the resistance (R_{100}) at 100°C, Then

$$\alpha = \frac{R_{100} - R_0}{100 \cdot R_0}$$

From the graph, $R_0 = \ldots\ldots$ ohm ; and $R_{100} = \ldots$ ohm
Hence $\alpha = \ldots$ per °C.

‡ The linear relation between R and t is valid only upto a temperature of 400°C. beyond which the parabolic formula $R_t = R_0 (1 + \alpha t + \beta t^2)$ holds good.

Observations

[A] *Readings for the determination of* ρ.

[**Note.** Make a table as given in the previous experiment.]

[B] *Readings for the determination of* R_1 *and* R_2.

Temperature of the bath	Resistance introduced in the decimal ohm box	Position of the null-point with the platinum spiral in the left gap (l_1)	right gap (l_2)	($l_2 - l_1$)
t°_1C				
t°_2C		(l_1')	(l_2')	($l'_2 - l'_1$)

Calculations

$$\rho = \frac{R}{l_2 - l_1}$$

$$= \ldots \ldots \ldots \text{ ohm per cm.}$$

Now $\quad R_1 = r + (l_2 - l_1)\rho = \ldots \ldots \quad$ ohm

$\qquad\quad R_3 = r' + (l_2' - l_1')\rho = \ldots \quad$ ohm

Again $$\alpha = \frac{R_2 - R_1}{R_1 t_2 - R_2 t_1}$$

$$= \ldots \ldots \text{ per degree C.}$$

Result. The temperature coefficient of resistance for platinum = ... per degree centigrade.

[Standard value* = 0·0038/°C ; Error = ...%]

Precautions and Sources of Error

(1) The ends of the connection wires should be perfectly clean, and they should be firmly secured in the binding terminals.

* This is the mean value between the temperature range 0°—100°C.

Thick copper strips should be used to connect the decimal ohm box which should preferably be of the dial type.

(2) The resistances in the four arms of the bridge should be of the same order of magnitude so that maximum sensitiveness of the bridge is achieved. The null-points should, therefore, be as near to the middle of the wire as possible.

(3) The galvanometer should be shunted by a low resistance wire and the approximate position of the null-point should be determined with its help. For the exact location of the null-point the shunt should be removed.

(4) The jockey should be pressed gently on the bridge wire and it should never be kept pressed while it is being moved along the wire, otherwise the uniformity of the wire will be impaired.

(5) The balance points for the measurement of R_1 and R_2 should be determined only when the temperatures acquired by the platinum thermometer are steady. This will be indicated by the constancy of the balance point on the same point of the bridge wire.

(6) The balance point should be determined when there is no *immediate* deflection in the galvanometer on pressing the jockey. If the jockey is kept pressed for a short time, the platinum spiral gets heated by the passage of current, its resistance changes, and consequently a deflection will be produced in the galvanometer even when the bridge was balanced before depressing the key.

A MODIFIED METHOD FOR THE DETERMINATION OF PLATINUM RESISTANCES

The above procedure for the determination of R_1 and R_2 can be slightly modified so that there is no need of directly determining the value of ρ. For this purpose, two readings for R_1 are taken whereby ρ can be eliminated from the two equations. Thus, let l_1 and l_2 be the readings for the positions of the balance points before and after interchange of the resistances in the outer gaps when a resistance r_1 is introduced in the decimal ohm box, then

$$R_1 - r_1 = (l_2 - l_1) \rho$$

Again, let the resistance in the box be r_2 and the corresponding positions of the balance points be l_1' and l_2', then

$$R_1 - r_2 = (l_2' - l_1') \rho$$

From these two equations we have

$$\frac{R_1 - r_1}{R_1 - r_2} = \frac{l_2 - l_1}{l_2' - l_1'}$$

which, on simplification, yields

$$R_1 = \frac{r_1 (l_2' - l_1') - r_2 (l_2 - l_1)}{(l_2' - l_1') - (l_2 - l_2)}$$

A similar expression can be found for R_2.

EXPERIMENT—17

Object. To determine the internal resistance of an accumulator.

Apparatus Required. Two similar accumulators, a milli-voltmeter, an ammeter, a rheostat, tapping key, and connection wire.

Formula Employed. If a current of i amperes is drawn from an accumulator, whose potential difference consequently falls by v milli-volts, then the internal resistance of the accumulator is given by—

$$r = \frac{v}{i} \cdot 10^{-3} \text{ ohm.}$$

PRINCIPLE AND THEORY OF THE EXPERIMENT

The usual methods applied in the laboratory for the determination of the internal resistance of a cell are unsuitable in the case of an accumulator, for to produce a measurable fall of potential difference across its terminals, the current drawn from the cell shall be excessive which will damage the cell as well as burn the external resistance through which the current is drawn. Hence the following differential method is utilised for this determination.

Let the positive terminals of two similar accumulators be connected through a milli-voltmeter and let the negative terminals be joined together. If the E. M. F.'s of the two accumulators are exactly equal, the milli-voltmeter shall register no reading. However, if the E. M. F. of the cell E_1 is higher than that of E_2, the milli-voltmeter shall record this small difference. Now, let a small current i be drawn from the cell E_2. The potential difference across the terminals of this cell will fall by an amount ir, where r is its internal resistance. Consequently the milli-voltmeter will record a higher reading. The difference between the final and initial readings of the milli-voltmeter will be recorded as v. Hence

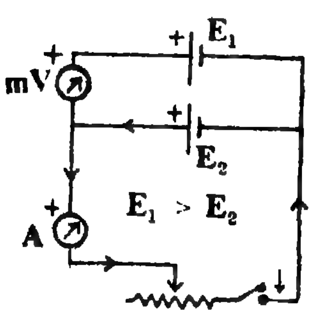


Fig. 30
Connections for r
of an accumulator

$$r = \frac{v \times 10^{-3}}{i} \text{ ohm.}$$

Thus r can be evaluated if v and i are measured.

Method

(i) Take two freshly charged accumulators E_1 and E_2 and connect them in parallel with a milli-voltmeter across their positive terminals. If the E. M. F.'s of the two cells differ slightly, then the positive terminal of the cell with higher E. M. F should be connected to the positively marked terminal of the milli-voltmeter.

(ii) Connect an ammeter, a rheostat (of low resistance, say, of the order of 15 ohms) and a tapping key as shown* in the figure.

(iii) Note the initial reading of the milli-voltmeter. By introducing the full resistance available in the rheostat, depress the key K. A current shall be drawn and the milli-voltmeter reading shall increase. Note down the readings of the milli-voltmeter and the ammeter. Calculate the resistance of the accumulator with these values.

(iv) Wait for some time so that the needle of the milli-voltmeter returns to a steady position which should again be recorded. Now diminish the resistance by a suitable amount and repeat the process as before. In this way by changing the rheostat† in suitable steps take several readings and calculate the mean resistance r of the accumulator.

Observations

S. No.	Reading of the milli-voltmeter with		Fall of potential difference‡ (v)	current flowing (i)	Internal resistance (r)	Remarks
	key open	key closed				
1.	12 mv	25 mv	13×10^{-3} volt	0·12 amp	0·11 ohm	(i) Least count of the milli-voltmeter = ...mv (ii) Least count of the ammeter = ...amp
				Mean	... ohm	

* If the cells have exactly equal E. M. F.'s as shown by the zero reading of the milli-voltmeter, the ammeter, etc., can be connected in series with any of the cells. If the E. M. F.'s slightly differ, then the connections should be made to that accumulator (E_2 in the above figure) *which has a lower e. m. f.*

† In the process of diminishing the resistance by sliding the variable point of the rheostat, be careful that the resistance is not reduced to zero.

‡ Do not forget to convert milli-volts into volts.

Calculations

Set 1 — $$r = \frac{v}{i} \times 10^{-3}$$
$$= \ldots\ldots \text{ohm}$$

[**Note**. Calculate similarly for other sets also.]

Result. The internal resistance of the given accumulator = ... ohm.

Precautions and Sources of Error

(1) The two accumulators chosen should be such that their E. M. F.'s are equal, or if they differ, they should not do so by more than a few milli-volts. Preferably they should be fully charged so that the initial reading of the milli-voltmeter remains constant after each set of observation.

(2) If the E. M. F.'s of the two the cells differ, the positive terminal of the accumulator having a higher E. M. F. should be connected to the positively marked terminal of the milli-voltmeter.

(3) Special precaution should be taken while connecting the positive terminals of the accumulators to the milli-voltmeter, for even if the milli-voltmeter is accidentaly connected to dissimilar terminals it shall be burnt out.

(4) The current through the rheostat should be drawn from that accumulator which has a smaller E. M. F, and, under no circumstances, should the rheostat be completely cut out for drawing the current.

(5) As the current drawn is a small quantity, a sensitive ammeter reading upto one-hundredth of an ampere and of range one ampere should be employed.

(6) After taking one set of observations, wait for sometime before taking the second set, so that the needle of the milli-voltmeter attains a steady position.

(7) When the key is pressed there is a deflection in the milli-voltmeter, thereafter the needle creeps forward. The reading just before the creep should be taken for the calculation of the fall in the potential difference. For this purpose the key should be pressed momentarily and not kept pressed for sometime.

Accumulators

There are two types of accumulators which are employed in laboratory work, namely, the acid accumulator and the alkali accumulator.

(1) Acid Accumulator or Lead Accumulator. This type was invented by Plante, who used electrodes of litharge (PbO) dipped in dilute sulphuric acid. When current is passed in such an accumu-

lator, one plate gets oxidised to PbO_2, and the other plate is reduced to "spongy lead". By repeatedly passing the current in opposite directions, it was found that the thickness of the layer of spongy lead increases and the storage capacity of the accumulator considerably increases. The plates are then said to be *"formed"*.

This process of forming the plates, is, however, a tedious, long and costly one. Faure, therefore, coated the plates, prior to charging with a paste of red lead (Pb_3O_4) and sulphuric acid, the adherence of the paste to the plates being assisted by a covering of paper. Later on Sellon-Volckmar introduced lead antimony plates, which, being constructed in the form of grids, more effectively secured the paste. Thus accumulators follow, in general, two specific types—(i) the Plante or naturally "formed" cells, and (ii) the Faure or "pasted" grid cell. Frequently the cell are of a compound character, having Plante positives and pasted negatives, since such cells have been found particularly suitable for high rates of discharge with comparatively low losses.

In the pasted type of cells the chemical reactions taking place inside the cell during its charging and discharging process can be summarised as follows :—

Before actual charging process a preliminary chemical reaction takes place as follows :—

$$Pb_3O_4 + 2\ H_2SO_4 = \mathbf{PbO_2} + \mathbf{2\ PbSO_4} + 2\ H_2O$$

so that prior to charging both plates contain the peroxide (PbO_2) and the sulphate ($PbSO_4$).

(1) Charging process

(a) *at the positive plate*

$$PbSO_4 + O + H_2O = \mathbf{PbO_2} + H_2SO_4$$

so that the *positive plate contains* $\mathbf{PbO_2}$

(b) *at the negative plate*

$$PbSO_4 + H_2 = \mathbf{Pb} + H_2SO_4$$

and also $\quad PbO_2 + 2\ H_2 = \mathbf{Pb} + 2\ H_2O$

so that the negative plate is coated with spongy metallic lead.

[**Note** It is clearly seen from the above equations that during the charging process, H_2SO_4 is formed, consequently the initial density of the electrolyte rises from 1·18 to 1·25. The E.M.F. of the cell acquires a value of 2·1 volts. The state of charge of the cell can be known with a high resistance voltmeter or with a battery hydrometer.]

(2) Discharging process

(a) *at the positive plate*

$$PbO_2 + H_2 = PbO + H_2O$$
$$PbO + H_2SO_4 = \mathbf{PbSO_4} + H_2O$$

(b) *at the negative plate*

$$Pb + O = PbO$$
$$PbO + H_2SO_4 = \mathbf{PbSO_4} + H_2O$$

Thus the plates return to the initial state.

[**Note.** It will be seen from the above reactions that molecules of sulphuric acid disappear and those of water appear, consequently the specific gravity begins to fall from 1·25 to 1·18 and the E. M. F. from 2·1 to 1 8 volts. However, the voltage should not be allowed to fall below 1·8 volts, otherwise the cell may be permanently damaged due to what is known as the "*Sulphating*" of the plates. In such a case insoluble lead sulphate is formed which is inactive, and consequently does not permit the cell to be recharged. *The cell must, therefore, be recharged as soon as its voltage falls to 1·8 volts*.]

There is a limit to which an accumulator can store electricity, this being reached when the positive plate is covered with a protective layer of lead peroxide. The quantity of electricity so stored is known as the **Capacity** of the accumulator and is measured in *Ampere-hours*, which is equal to the product of the current in amperes and the number of hours for which the current can be drawn from the cell. Thus, if a cell has a capacity of 60 ampere-hours, it means that a current 1 ampere can be drawn for 60 hours, or a current of 0·5 ampere for 120 hours. Theoretically it can also mean that a current of 60 amperes can be drawn for 1 hour, but practically it is not so, since when such excessive currents are drawn, the capacity is considerably reduced. Moreover, the cell is permanently damaged. Hence for the safety and long life of the cell, on no account should the current be drawn at a rate higher than that specified by the makers for charging.

Accumulators having large capacity are provided with a number of positive and negative plates (the number of negative plates is one more than the positive plates) joined in parallel and quite close to one another, the plates being separated by insulating material (*e. g.*, wood, hard rubber, etc.), which prevents the internal short-circuiting of the cell. Due to the large size of the plates and their nearness from each other, the resistance of an accumulator is very low, say, of the order of 0·01 ohm. Hence, if, by accident, the cell is short-circuited, currents of excessive magnitude shall flow, through the cell, resulting in sulphating, disintegration of active material, and buckling of the plates. ***Short-circuiting of an accumulator should, therefore, on no account be done.***

The **Efficiency** of an accumulator is given by the expression

$$\text{Efficiency} = \frac{\text{Watt-hours given out at discharge}}{\text{Watt-hours put in at charge}}$$

and in general it is of the order of 70%.

Accumulators have found applications too numerous to be mentioned here. Their advantages and disadvantages as compared to primary cells can be briefly summarised as follows :—

(a) *Advantages.* (1) They have a high E. M. F. and low resistance and hence they can supply large currents ; (2) When run down they can be recharged ; (3) They can be used for lighting, traction, etc., where primary cells are useless.

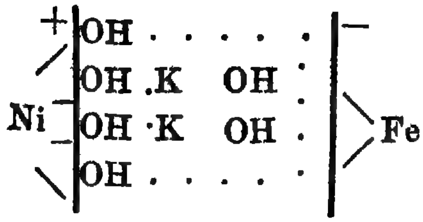
(b) *Disadvantages.* (1) The initial cost of accumulators is very high ; (2) They are heavy, and hence not very portable for laboratory work ; (3) Their efficiency is rather low ; (4) They require careful attention to maintain them in good condition. On slight carelessness or oversight they are subject to sulphating, disintegration, buckling, and short-circuiting.

(2) **The Alkaline Accumulators.** There are two varieties of this type of accumulator, namely, the Edison cell (or the Nickel Iron cell) and the Nife* cell (or the Nickel Cadmium cell). Both the cells use the same electrolyte†, a 20% solution of potassium hydroxide. They use the same type of positive electrode consisting of perforated nickelled-steel tubes containing nickle hydroxide (which is the active material) mixed with nickel flakes (to lessen the resistance of the former). In the Edison cell the negative electrode consists of perforated nickelled-steel pockets containing finely divided iron mixed with a little yellow oxide of mercury (to increase the conductivity of the cell). In the Nife cell the iron of the Edison cell is replaced by cadmium. This is the only difference between the two varieties.

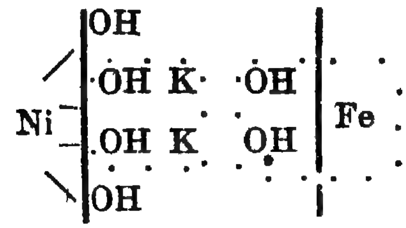
The charging and discharging processes occurring in the alkaline accumulator can be represented as follows—

* The name is slightly a misnomer. "Nife" is derived from Ni (= Nickel) + Fe (= Iron), but it contains one plate of cadmium and not of iron, which is a constituent of the Edison cell.

† The electrolyte also contains small amount of lithium hydroxide, which acts as a catalyst and increases the capacity of the cell by about 10%.



Charge



Discharge

From the above it is clear that there is no change in the composition of the electrolyte and hence its specific gravity remains constant (= 1·19) both during charge and discharge.

The advantages and disadvantages of this type of the accumulator over the lead accumulator can be briefly summarised as follows :—

(a) *Advantages.* (1) This cell is very robust and hence can withstand rough handling, *e. g.*, mechanical vibrations or heavy discharge ; (2) It is not spoiled if it is left idle for sometime without being re-charged ; (3) It is not damaged by over-charge or over-discharge. Moreover, it can be charged and discharged at a high ampearage ; (4) For the same capacity as that of the lead accumulator its weight is lower.

(b) *Disadvantages.* (1) Its E. M. F. (= 1·35 volt) is lower than that of the lead accumulator ; (2) When current is drawn from this cell, its E. M. F. does not remain constant ; (3) Its efficiency is comparatively lower (nearly 50%) ; (4) If exposed to air, its electrolyte absorbs carbon di-oxide,* which lowers the capacity of the cell.

* For this purpose the containers are provided with air-tight stoppers, *which should never be kept loose.*

MAGNETIC, CHEMICAL & HEATING EFFECTS OF ELECTRIC CURRENT

MAGNETIC EFFECT OF ELECTRIC CURRENT

EXPERIMENT—18

Object. To study the variation of the magnetic field, due to an electric current flowing in a straight conductor, with distance by the method of oscillations, and to prove that the magnetic field produced by it varies inversely as the distance.

Apparatus Required. A large rectangular frame work carrying a single coil of copper wire, battery, rheostat, compass needle, connecting wires, a sheet of white paper, board pins, and Searle's oscillating needle.

Formula Employed $$r\left[\frac{1}{T^2} - \frac{1}{T_0^2}\right] = \text{constant}$$

where r = Distance of a point from the straight conductor carrying current.

T = Time-period of the Searle's needle vibrating in the combined field, $(F + H)$, of the current and the earth at a distance r from the conductor.

T_0 = Time-period of the needle in the earth's field alone.

This experiment is studied either by calculation or by graph.

(i) **By Calculation.** The Searle's needle is oscillated at different distances (r) from the current-carrying wire at several points where the two fields assist each other. Then the value of the above expression is found out for various values of r and T. The result is a constant, showing thereby that the magnetic force due to current in a long straight wire varies inversely as the distance from the wire.

(ii) **By Graph.** From the above, $\dfrac{1}{T^2} = \dfrac{k}{r} + \dfrac{1}{T_0^2}$

where k is a constant. Thus if we plot a graph between 1/r as abscissa and $1/T^2$ as ordinate we should get a straight line.

PRINCIPLE AND THEORY OF THE EXPERIMENT

If a long straight conductor carries an electric current, the lines of force due to it are in the form of concentric circles (fig.-31) in a plane perpendicular to the wire, the centres of the circles lying on the axis of the wire. If i be the current the field at a distance r from it will be equal to

$$\frac{2i}{r}$$

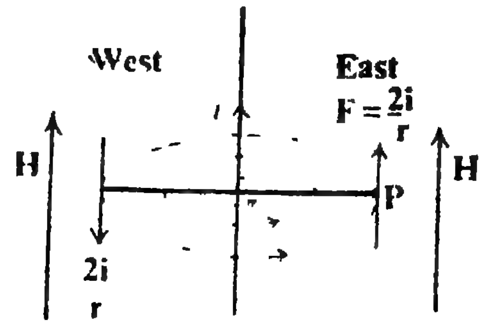


Fig. 31
Lines of force due to a straight conductor

[**Proof.** Let the wire be divided in small elements and let δl be one such element (fig -32). Then according to Laplace's law—

Field at P due to δl

$$= \frac{i \cdot \delta l'}{x^2} = \frac{i \cdot x \cdot d\theta}{x^2} = \frac{i \cdot d\theta}{x}$$

where $\delta l'$ is the apparent length of the element. But

$$\frac{r}{x} = \cos\theta, \text{ or } \frac{1}{x} = \frac{\cos\theta}{r}$$

$\therefore$ Field at P due to $\delta l = \frac{i}{r} \cos\theta \cdot d\theta$

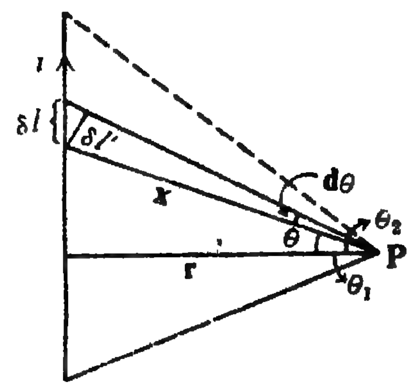


Fig 32
Calculation of field due to a straight conductor

Hence the field due to the whole wire is obtained by integrating this expression between the limits, $\theta = -\theta_1$ and $\theta = \theta_2$, that is—

$$F = \frac{i}{r}\int_{-\theta_1}^{\theta_2} \cos\theta \cdot d\theta = \frac{i}{r}(\sin\theta_1 + \sin\theta_2)$$

In the case of a long conductor $\theta_1 = \theta_2 = \frac{\pi}{2}$, hence $F = \frac{2i}{r}$.]

Imagine a line, passing magnetic east and west, to be drawn through the wire and passing through the point P (fig.-31). The field F at this point is due north and hence the resultant field here

shall be $(F + H)$, while on the other side it will be the difference between F and H.

If now the Searle's needle is placed at the point P and allowed to oscillate *in the earth's field alone,* then its time-period T_0 is given by—

$$T_0^2 = \frac{4\pi^2 K}{M H} \quad \text{or} \quad T_0^2 = \frac{C}{H}$$

where C is a constant. Thus

$$H = \frac{C}{T_0^2} \quad \ldots \quad (1)$$

If now the current is allowed to flow in the wire in the direction as shown in the figure, the time-period T of the needle will be given by—

$$F + H = \frac{C}{T^2} \quad \ldots \quad (2)$$

From (1) and (2), we have

$$F = C\left[\frac{1}{T^2} - \frac{1}{T_0^2}\right] \quad \ldots \quad (3)$$

Now, if F is proportional to $1/r$, we shall have

$$F_1 r_1 = F_2 r_2 = F_3 r_3 \quad \ldots$$

where F_1, F_2, F_3, are the field strengths at distances r_1, r_2, r_3, ... from the wire.

If the corresponding periods of oscillation are T_1, T_2, T_3, ..., we can write

$$F_1 = C\left[\frac{1}{T_1^2} - \frac{1}{T_0^2}\right]; \quad F_2 = C\left[\frac{1}{T_2^2} - \frac{1}{T_0^2}\right]; \text{ etc.}$$

and therefore we can show that $F_1 r_1 = F_2 r_2 = F_3 r_3$, etc., provided we show that

$$C\left[\frac{1}{T_1^2} - \frac{1}{T_0^2}\right] r_1 = C\left[\frac{1}{T_2^2} - \frac{1}{T_0^2}\right] r_2 = \ldots$$

The constant C occurs in each expression, and therefore can be cancelled throughout, and hence F shall be proved proportional to $1/r$ if it is proved that

$$r\left[\frac{1}{T^2} - \frac{1}{T_0^2}\right] = \text{Constant.}$$

This can be done graphically as well as by calculation, the former procedure being more convenient and convincing.

Method

(i) Set up the apparatus as shown in the accompanying figure. With the help of a compass needle mark the direction of the magnetic meridian as well as the east-west line through the vertical wire, and measure off different distances along this (east-west) line, say 5, 6, 8, 10, 15, 20 cm. from the wire.

(ii) Now place the Searle's oscillating needle at some point on this line and determine its time-period before switching on the current, so that the needle executes its oscillations in the earth's horizontal field alone. Let the time-period be T_0.

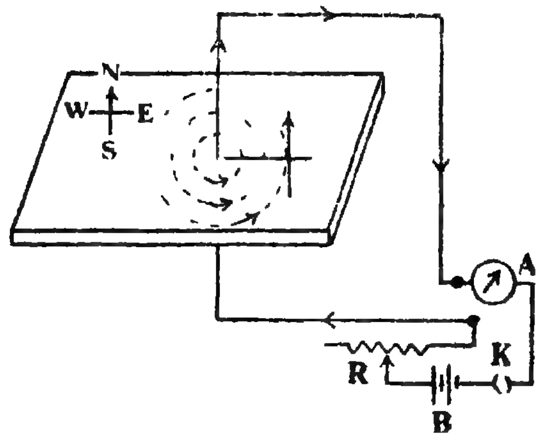


Fig. 33
Field due to a straight conductor

(iii) Switch on the current and study the oscillations of the needle under this condition. If the needle executes its oscillations more rapidly than before, and still points in the same direction, the experiment for the determination of the time-period in the resultant field $(F + H)$ should be continued. If not, the needle should be placed on the line on the opposite side of the wire, where the field of the current and the field of the earth would assist each other. On the side where the fields are in opposition, the swings would be slower than in the earth's field alone, or the needle might be reversed. If H is stronger, than F the needle swings less rapidly, but if F is stronger than H it is turned completely round.*

(iv) Place the needle at each of the points marked along the east-west line on this side of the wire where the two fields assist each other. Observe the time-period in each position. Calculate the value of expression $r\left(\frac{1}{T^2} - \frac{1}{T_0^2}\right)$ which will be found to be practically constant.

* It is a very important precaution that the needle is used on that side of the wire where the fields are added up, because the torsion of the suspension of the needle will have an appreciable effect on the time-period. Since we are taking no account of this torsion, the errors will be large. It was for this reason that in the above theoretical consideration the time-period of the needle in the field $(F + H)$ only was studied.

Or, draw the graph between $1/T^2$ (on the y-axis), and $1/r$ (on the x-axis), when a straight line will be obtained.

Observations

Readings for the determination of the time-periods

S. No.	Time-period in H			Time-period in (F + H)			
	No. of oscillations	Time taken	T_0	Distance from the wire (r)	No. of oscillations	Time taken	T
1	25			5 cm.	25 25	... ...	...
2	25			6 ,,			
3	25			8 ,,			
4	25			10 ,,			
5	...			15 ,,			
6	...			20 ,,			

Calculations

S. No.	r	$\frac{1}{T^2_0}$	$\frac{1}{T^2}$	$\frac{1}{T^2} - \frac{1}{T^2_0}$	$r\left(\frac{1}{T^2} - \frac{1}{T^2_0}\right)$

Result. From the last column of the above table it is clear that within the limits of experimental error the value of the expression, $r\left(\frac{1}{T^2} - \frac{1}{T_0^2}\right)$, is practically constant, hence the

magnetic field due to a straight current is inversely proportional to distance.

Further, the graph between $1/r$ and $1/T^2$ is a straight line which also verifies the above statement.

Precautions and Sources of Error

(1) The experiment should be conducted at a place where there are no disturbing influences, *e. g.*, magnetic materials, current-carrying conductors etc. Connections of the straight conductor with the rest of the circuit should be done with the help of a *twin flex* so that the current flowing in them does not interfere with the field of the straight conductor.

(2) The magnetic east-west line should be carefully marked on the sheet of paper and the oscillations of the Searle's needle should be studied at points where the field of the current and the earth's horizontal field assist each other.

(3) While taking observations with the needle it should be clearly borne in mind that the angular amplitude of the needle is small, say, of the order of 5°.

(4) The current flowing through the wire should be maintained constant throughout the experiment. This should be done with the help of the ammeter and the rheostat included in the electric circuit.

ADDITIONAL EXPERIMENT

Expt.—18 (a)

Object. To plot the resultant magnetic field of a vertical straight conductor carrying current and of the earth in a horizontal plane, and to evaluate the horizontal component (H) of earth's magnetic field by locating the position of the neutral point.

At the neutral point the earth's horizontal field is neutralised by the field due to the current i flowing in the straight conductor consisting of n turns. If the distance of the neutral point is r from the conductor—

$$H = \frac{2\,n\,i}{r} \text{ dyne per unit pole}$$

whence H can be calculated out.

Trace the lines of force as usual. If necessary, tap gently the compass needle for each setting, and soon after drawing a line of force mark its direction with an arrow-head. The current should be adjusted constant and its strength should be one to two amperes, so that the neutral point is obtained at a considerable distance from the conductor. *Express i in electro-magnetic units.*

EXPERIMENT—19

Object. To study with the help of a Stewart and Gee type tangent galvanometer, the variation of magnetic field with distance along the axis of a circular coil carrying current, and to estimate from the graph the radius of the coil.

Apparatus Required. Stewart and Gee type tangent galvanometer, a storage battery, a rheostat, an ammeter, a commutator, and a plug key.

Description of the Apparatus. The apparatus known as the Stewart and Gee type tangent galvanometer consists of a circular

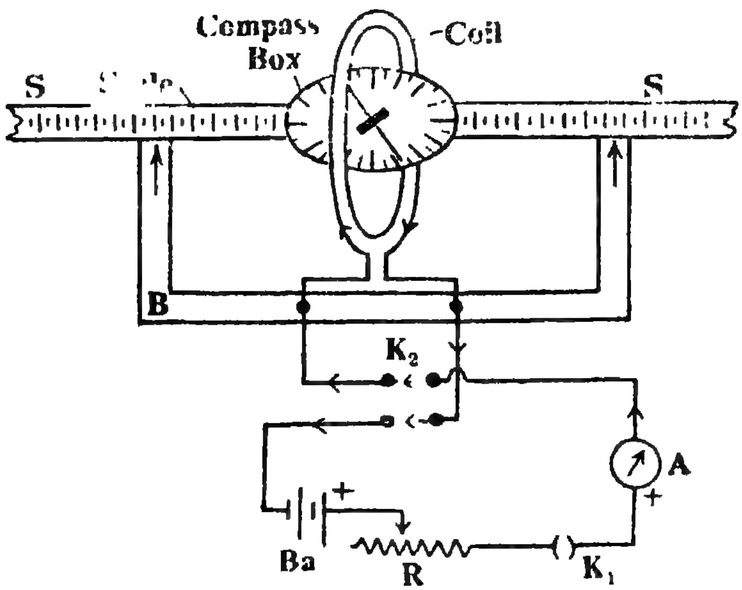


Fig. 34

Stewart and Gee type tangent galvanometer

coil C having a number of turns of insulated copper wire fixed with its plane vertical on a suitable horizontal bench B, and a magnetometer compass box which can slide on the bench such that the centre of the needle always lies on the axis of the coil. The distance of the needle from the centre of the coil can be read off with the help of linear scales S, S provided on the two arms of the magnetometer.

Formula Employed. The field F along the axis of a coil is given by the formula—

$$F = \frac{2\pi n r^2 i}{10\,(x^2+r^2)^{\frac{3}{2}}}$$

where n = No. of turns in the coil

r = Radius of the coil

i = Current strength in amperes

x = Distance of the point (lying on the axis) from the centre of the coil.

If F is made perpendicular to H, the earth's horizontal field, the deflection θ of the needle is given by—

$$F = H \tan \theta$$

Thus
$$\frac{2\pi n r^2 i}{10 (x^2 + r^2)^{\frac{3}{2}}} = H \tan \theta$$

If a graph be plotted with x as abscissa and $\tan \theta$ as ordinate, a curve symmetrical about the y-axis shall be obtained. At the point where the curvature changes sign, *i. e.*, at the point of inflexion,

$$x = \frac{r}{2}$$

Hence the distance between the two points of inflexion lying on the two branches of the curve gives the radius of the coil.

PRINCIPLE AND THEORY OF THE EXPERIMENT

The intensity of the field at a point P lying on the axis of the coil AB. due to a small element ab is in the direction PK, (*i. e.*, perpendicular to AP) and its value is given by (from Laplace's law)—

$$f = \frac{i.\overline{ab}}{d^2}$$

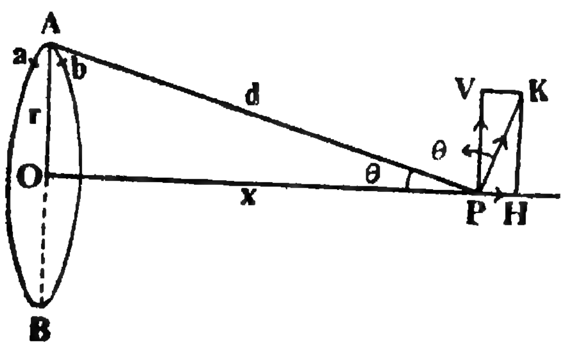


Fig. 35
Calculating of field along the axis of a circular coil

Let PK be resolved into two components, *viz.*, PH along the axis, and PV perpendicular to it. Only the components along the axis need be considered, for when the whole coil is taken into account, the vertical components from corresponding elements of the coil shall fall opposite to each other and hence they shall cancel out. The horizontal component h′ of f is given by—

$$h' = f. \sin. \theta = \frac{i.\ \overline{ab}.\sin \theta}{d^2} = \frac{i.\ \overline{ab}\ r}{d^3}$$

Clearly for the whole ring the intensity H′ at P will be obtained by summing up the above expression for all the elements into which the ring may be divided, *i. e.*,

$$H' = \frac{i.\,r}{d^3}\Sigma\,ab = \frac{i.\,r}{d^3}\,.\,2\pi r = \frac{2\pi\,r^2.\,i}{d^3}$$

If the number of turns in the coil be n, then—

$$H' = \frac{2\pi\,nr^2\,i}{d^3} = \frac{2\pi\,ur^2\,i}{(r^2+x^2)^{\frac{3}{2}}}$$

Replacing H' by F and converting the current from e. m. u. to amperes we have—

$$F = \frac{2\pi\,nr^2\,i}{10\,(x^2+r^2)^{\frac{3}{2}}} \qquad \dots \qquad (1)$$

If the values of the field F and the corresponding values of x for various points lying on the axis of the coil be plotted on a graph, a curve as shown in fig.–36 is obtained. The curve is first concave towards O, the point corresponding to the centre of the coil, but the curvature becomes less and less, and quickly changes sign, the curve becoming convex towards O. At the point of inflexion*, where the curvature changes its sign, $d^2F/dx^2 = 0$.

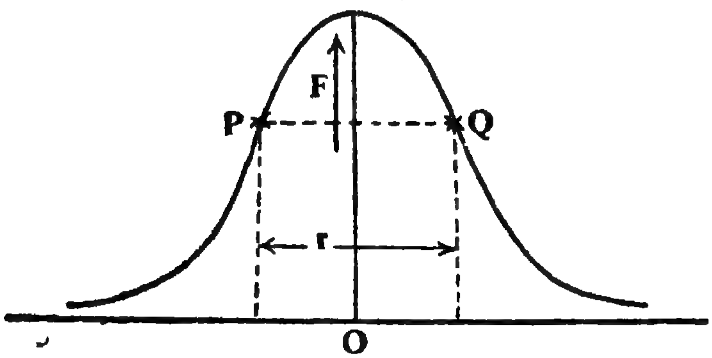


Fig. 36
Variation of field along the axis of a circular coil

Now differentiating equation (1) twice and equating to zero we have—

$$\frac{d^2F}{dx^2} = -\,6\pi\,nr^2\,i\,[\,(r^2+x^2)^{-5/2} + x\,(-\tfrac{5}{2})\,(2x)\,(r^2+x^2)^{-7/2}]$$

$$= -\,6\pi\,nr^2\,i\,(r^2+x^2)^{-5/2}\,[\,(r^2+x^2) - 5x^2\,]$$

* If we have two identical circular coils placed with their axes coincident and at a distance apart equal to the radius of either, then, for the same direction along the common axis, the rate of increase of the field due to one coil at a point midway between the two coils is equal to the rate of decrease of the field due to the other coil at the same point. Thus the field for a fairly large distance on each side of this point will be practically uniform. This condition is utilised in the construction of the Helmholtz tangent galvanometer.

and this quantity is zero when $[(r^2 + x^2) - 5x^2] = 0$, *i. e.*, when

$$x = \pm \frac{r}{2}$$

Method

(i) Place the instrument on the table in such a way that the arms of the sliding magnetometer point roughly east and west and the magnetic needle of the compass box is at the centre of the coil. By placing the eye a little above the coil, rotate the instrument in the horizontal plane in such a way that the coil, the needle, and its image all lie in the same vertical plane. This adjustment puts the coil *roughly* in the magnetic meridian.

(ii) Now to adjust the coil exactly in the magnetic meridian make the electrical connections as shown in fig.–34. With the help of the rheostat adjust the current in the coil so that the deflection in the needle is of the order 75°—80°. After lightly tapping the glass cover of the compass box with a finger, note the deflection at the two ends of the pointer. Then reverse the current in the coil with the help of the commutator and again note the deflection as before. If the mean deflection in the two cases does not agree closely, the coil is not in the magnetic meridian. Slightly turn the coil and repeat the process until the mean deflections with the direct and reverse currents agree as closely as possible.

(iii) Now for $x = 0$ (*i. e.*, when the needle is situated at the centre of the coil) note the deflections* both for direct and reverse currents and calculate the mean deflection θ. Then shift the compass box by 2 cm. and note the deflection† as before $x = 2$ cm. Continue this process‡ till the compass box reaches the end of the bench or the deflection is reduced to 5°.

(iv) Repeat the measurements exactly in the same manner on the other side of the coil.

(v) Plot a graph taking x along the x—axis and tan θ along the y—axis. This curve should be symmetrical and should have a

* When the deflection is maximum the needle should be situated at the centre of the coil, i. e., $x = 0$.

† It is important to note that the current flowing in the coil should remain constant throughout the experiment. For this purpose, an ammeter should be included in the circuit and the current should be adjusted to a constant value with the help of the rheostat.

‡ The nature of the graph reveals that the curve becomes almost vertical in the region $x = r/2$. Hence the deflections in the vicinity of this point should be determined more thoroughly (say, by shifting the compass box in steps of one cm.)

maximum value when the needle is at the centre of the coil itself. Find the two points of inflexion on the two branches of the curve and thus determine the radius of the coil.

Observations

S. No.	Distance along the axis from the centre	Direct current		Current reversed		Mean deflection θ	
		One end of the pointer	Second end of the pointer	One end of the pointer	Second end of the pointer		
1.	0 cm.	75°	75°	75°	75°	75°	
2.	+ 2 cm.	72°	72°	72·5°	72·5°		
⋮	...	...	...	...	...	...	
⋮	+ 20 cm.	5°	5°	6°	6°		
	− 2 cm.	72°	72°	72 5°	72·5°		
⋮	...	...	...	...	...	...	
⋮	− 20 cm.	5·5°	5·5°	6 5°	6·5°		

Calculation

The radius* of the coil as estimated from x — tan θ graph =........cm.

Result. The graph showing the variation of the magnetic field along the axis of the given current-carrying circular coil is attached herewith†. The curve is symmetrical and has a peak value corresponding to the centre of the coil.

Two points of inflexion are situated on the curve. Their distance apart, which is equal to the radius of the coil, is equal to...... cm.

Precautions and Sources of Error

(1) The experiment should be conducted at a place where, in its vicinity, there are no magnetic materials or current-carrying

* Verify this result by measurement of the diameter of the coil with a callipers.

† See the graph given in fig.–37.

conductors. For the same reason the rheostat of the electric circuit should be kept sufficiently far away from the compass box.

(2) So that the magnetic needle obeys the tangent law, the plane of the coil should be carefully adjusted in the magnetic meridian.

(3) The current in the coil should be so adjusted that the deflection in the needle at the centre of the coil is nearly 75°. This current should remain constant throughout the experiment. For

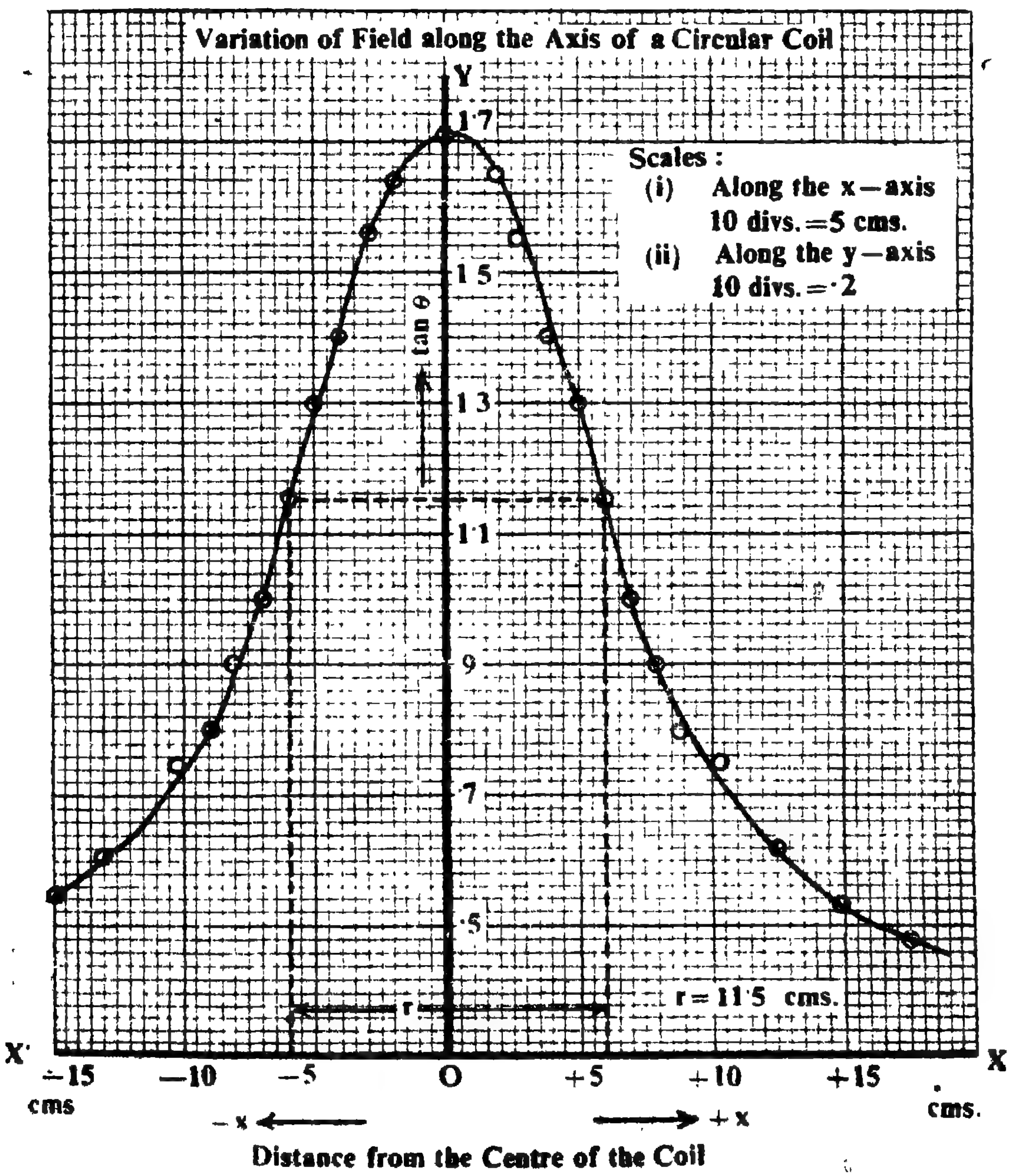


Fig. 37

this purpose a storage battery of large capacity should be employed. Moreover, an ammeter should be included in the circuit and the

current should be adjusted to a constant value with the help of the rheostat.

(4) It is likely that the pivot of the needle does not pass exactly through the centre of the graduated circle. To avoid this error due to ex-centricity of the pivot, both ends of the pointer should be read.

(5) In order to avoid error due to parallax in reading the deflection use should be made of the plane mirror attached to the base of the compass box.

(6) In order to avoid the effects of friction at the pivot the readings should be taken after gently tapping the glass top of the compass-box with finger.

(7) Deflections of the needle should be recorded both for direct and reverse currents.

(8) The curve on the graph should be drawn smooth and the position of the points of inflexion should be located carefully.

(9) The chief *sources of error* arise due to the fact that (a) the coil may not be exactly circular and needle may not be situated exactly at its centre, (b) the plane of the coil may not be exactly in the magnetic meridian, (c) the magnetic needle is not *very* small, so that it does not move in uniform magnetic field at the centre, hence the tangent law is not accurately obeyed, (d) the friction at the pivot may not be totally absent. Moreover, (e) the scale and pointer method for measuring deflections is not very accurate, and (f) the manner of ascertaining the position of the points of inflexion on the graph is not susceptible of any great accuracy.

CHEMICAL EFFECT OF ELECTRIC CURRENT

EXPERIMENT—20

Object. To determine the reduction factor of a tangent galvanometer for two turns of the coil using a copper voltameter for the measurement of current.

Apparatus Required. Tangent galvanometer, a battery of accumulators, rheostat, commutator, copper voltameter, copper plates for deposit of copper, chemical balance, weight box, spirit level, and a stop-watch.

Description of the Apparatus. The tangent galvanometer consists essentially of a small magnetic needle pivoted at the centre

of a circular coil of many turns of insulated copper wire, the plane of the coil being vertical. The needle is small and the radius of the coil large, so that the magnetic field in that region round about the centre of the coil where the needle moves, can be taken as uniform. The deflection of the needle is read with the help of a long aluminium pointer attached at right angles to the needle and moving over a graduated circular scale. To protect the needle-pointer system from draughts, it is placed in a metallic (non-magnetic) box with a glass cover. To avoid errors due to parallax in reading the deflection the base of the box is provided with a plane mirror.

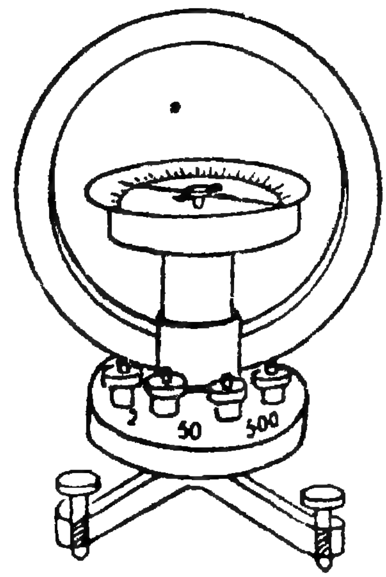


Fig. 38
Tangent
galvanometer

The galvanometer is provided with four binding terminals. Between the first and the second terminals there are two turns of the coil, which have a resistance of nearly ·02 ohms*. This is known as the ammeter coil and is used for strong currents. Between the first and the third terminals there are fifty turns which have a resistance of nearly 1·1 ohms and are used for moderate currents. They are sometimes known as galvanometer coils. Between the first and the fourth terminals there are 500 turns, whose resistance is nearly 225 ohms. These are used for weak currents and are known as voltmeter coils.

The base of the instrument is provided with three levelling screws, with the help of which it can be levelled.

Formula Employed. The current i flowing through the coils of a tangent galvanometer is given by—

$$i = k. \tan \theta \qquad \ldots \qquad (1)$$

where k = Reduction factor of the galvanometer.
θ = Deflection produced in the needle

Now, if the same current is allowed to flow through a copper voltameter—

$$m = izt \qquad \ldots \qquad (2)$$

where m = Mass of copper deposited
z = Electro-chemical equivalent of copper
t = Time for which current flows through the voltameter.

* The constants given in this paragraph are for a Pye pattern tangent galvanometer.

Substituting the value of i from (2) in (1) we have—

$$k = \frac{m}{zt.\ \tan\theta} \qquad \ldots \qquad (3)$$

Equation (3) is employed for the determination of the reduction factor of the tangent galvanometer.

PRINCIPLE AND THEORY OF THE EXPERIMENT

If a current of i units be flowing through l cm. of a wire bent into an arc of r cm. radius, the magnetic force at the centre of the arc is given by—

$$F = \frac{i\ .\ l}{9}$$

If the wire forms one complete circle, $l = 2\pi r$, hence

$$F = \frac{i.\ 2\pi r}{r^2} = \frac{3\pi i}{r}$$

For a circular coil containing n turns

$$F = \frac{2\pi n i}{r} \qquad (4)^*$$

If the coil of the tangent galvanometer is placed in the magnetic meridian, the field due to the current i flowing in the coil is perpendicular to H, the horizontal component of earth's magnetic field, and consequently the needle placed at the centre of the coil will undergo a deflection θ given by the Tangent Law —

$$F = H.\ \tan\theta \qquad \text{or} \qquad \frac{2\pi n i}{10r} = H.\ \tan\theta$$

Hence

$$i = \frac{10\ r\ H}{2\pi n}.\ \tan\theta = k.\ \tan\theta$$

* The absolute *electro-magnetic unit (e. m. u.) of current can be defined* (from eqn.—4) as *that current which, when flowing through a single turn of circular coil of unit radius, exerts a force of 2π dynes on a unit north pole placed at the centre of the coil.*

The practical unit of current is the *Ampere*, which is one-tenth of the e. m. u. It can be defined as *the current which, when flowing through a single turn of circular coil of unit radius, exerts a force of 2π/10 or π/5 dyne on a unit north pole placed at the center of the coil.*

where k is the *reduction factor* of the tangent galvanometer. If $\theta = 45°$, $\tan\theta = 1$, and $i = k$. Thus *the reduction factor of a tangent galvanometer is that current which, when flowing through its coil placed in the magnetic meridian, produces a deflection of 45° in the needle.*

The unit of reduction factor is *amperes.*

If the same current i, which is flowing in the galvanometer coil, also flows through a copper voltameter for t seconds, the mass of copper deposited on the cathode is given by the Faraday's law of electrolysis—

$$m = izt$$

where z is the electro-chemical equivalent* (e. c. e.) of copper. Thus

$$i = \frac{m}{zt} = k.\tan\theta$$

Hence

$$k = \frac{m}{zt.\tan\theta}$$

Thus in this experiment the copper voltameter is being used as a current measurer.

Method

(i) First of all level the compass box of the tangent galvanometer with the help of a spirit level and the three levelling screws provided at the base of the instruments. For this purpose keep the spirit level on the glass cover parallel to the line joining any of the two screws. Now by operating these screws bring the air-bubble of the spirit level in the middle. Then put the spirit level in a direction at right angles to the former, and *by operating the third screw along,* bring the air-bubble again in the middle. Thus the compass box is levelled and the magnetic needle housed inside it is free to move in the horizontal plane.

(ii) Next set the plane of the coil in the magnetic meridian. This can be roughly done by placing the eye above the coil, and rotating it till the coil, the needle, and its image in the mirror all lie in the vertical plane. Under this condition if the pointer does not read zero zero on the scale, rotate the compass box, without disturbing the coil, till the pointer reads so.

---

* The electro-chemical equivalent of an ion is the mass of that ion liberated when an electric current of one ampere flows for one second through an electrolyte containing that ion. Its unit is *"gm per coulomb."*

(iii) Now set up the electrical connections* as given here (fig.-39). With the help of the rheostat R adjust the current so that a deflection of nearly 45° is obtained in the galvanometer. Note the deflection at the two ends of the pointer (if necessary, tap the compass box gently with a finger). Then reverse the current and note the deflection again. If the mean deflection in the two cases is not the same, slightly turn the coil till they agree as closely as possible. If necessary, turn carefully the compass box for the pointer to read zero-zero. Now the coil has been adjusted in the magnetic meridian.†

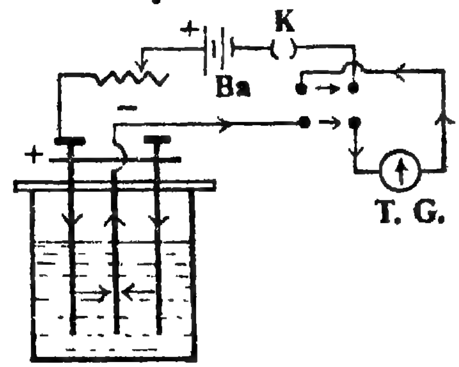


Fig. 39
Reduction factor of a tangent galvanometer

(iv) Switch off the current. Weigh a thoroughly cleaned copper plate, on which the deposit is to be made, with an accurate chemical balance. Replace the test plate by this plate, taking care not to touch its surface with fingers. Switch on the current and immediately start the stop-watch. If necessary, adjust the deflection, which should now be recorded. Have an eye on the deflection ; if it changes, keep it constant‡ with the rheostat.

(v) When the current has passed for sometime, say fifteen to twenty minutes, quickly reverse the current and note the deflection at the two ends of the pointer. After an equal interval of time switch off the current, and note the total time.

(vi) Remove carefully the cathode plate from the voltameter and *immediately* immerse it in a trough of clean tap water placed close at hand. Then transfer the plate to another trough containing distilled water to which two or three drops of sulphuric acid per litre have been added. Now soak the water adhering to the plate by pressing it *lightly* between two sheets of blotting paper. Then dry the plate in warm air from a hot air blower.

(vii) Weigh the plate in the chemical balance and thus find the mass (m) of the deposited copper. Taking $z = 0{\cdot}0003295$ gm. per coulomb calculate the value of k with the help of the formula given above.

* Be careful about the connections at the commutator. The current has to be reversed in the tangent galvanometer, without affecting its direction in the remaining circuit. Secondly, for the cathode in the voltameter connect the clean test-plate.

† Check up the horizontality of the compass box with the spirit level again.

‡ This constancy of the current is to be maintained throughout the whole experiment.

Observations

[A] (1) Mass of the cathode plate before the deposit = ...gm.

(2) Mass of the cathode plate after the deposit = ...gm.

(3) Time for which current was passed = ... mins. = ...secs.

(4) Number of turns of the coil used = 2.

[B] *Readings for the determination of* θ.

Time in mins.	Deflection of the pointer for: Direct current, I end of pointer	Direct current, II end of pointer	Reverse current, I end of pointer	Reverse current, II end of pointer	Mean θ	tan θ
0	45°	45°	45°	45°		
5	...	...	...	...		
10					45°	
15						
20						

[C] E. C. E. of copper (given) = 0·0003295 gm/coulomb.

Calculation

Mass of the copper deposited = ... gm.

$$k = \frac{m}{zt.\tan\theta}$$

$$= \ldots \text{ amp.}$$

Result. The reduction factor of the tangent galvanometer for two turns of the coil = ... amp.

Precautions and Sources of Error

(1) The compass box should be carefully levelled so that the magnetic needle moves freely in a horizontal plane.

(2) For the validity of the tangent law in this case it is essential that the galvanometer coil is set in the magnetic meridian.

(3) The tangent galvanometer should be placed at a place where there are no magnetic materials, current-carrying conductors etc. in its neighbourhood. The rheostat should also be placed at a safe distance from it. The leads connected to the binding terminals of the galvanometer should be of flexible wire twisted together (*twin flex*), so that the magnetic field produced by the passage of current through them has no appreciable effect on the needle.

(4) The copper plate on which the deposit has to be made should be scrupulously clean. Its surface should not be touched by hand, otherwise it will be rendered greasy, and the deposit will not adhere properly and it will not be even.

(5) The accuracy of the result is mainly dependent on the measurement of m, the mass of copper deposited. Hence the cathode plate should be accurately weighed, before as well as after the deposit, with a chemical balance using a rider.

(6) The plate for the deposit of copper should be connected to the negative of the battery, or to the lower potential point in the circuit. For a good deposit of copper on the plate, the current strength should lie between 1 to 2 amp. With a weaker current a long time shall be needed for getting a deposit sufficient for accurate weighing. On the other hand if the current is too strong, a very large area of the plate has to be dipped in the solution to get a firm deposit.*

(7) The strength of the current should be kept constant throughout the duration of the experiment. The deflection of the galvanometer should, therefore, be kept constant with the help of the rheostat.

(8) As far as possible, the deflection should be kept as nearly equal to 45° as possible, since under this circumstance the accuracy in measurement is at a maximum.†

* As a general rule, for each ampere of current flowing through the solution the total area of the cathode plate dipped should be 50 sq. cm.

† This will be evident from the fact that a given variation in the current produces the greatest effect in this region. Thus, if $d\theta$ be a small increase in the deflection produced by a small increase di in the current, we have

$$i = k.\tan\theta, \qquad \therefore di = k.\sec^2\theta \,.\, d\theta$$

Hence $$\frac{di}{i} = \frac{\sec^2\theta . d\theta}{\tan\theta} = \frac{2}{\sin 2\theta} . d\theta$$

Now di/i is the relative change in the current, and for this to be as small as possible for given value of $d\theta$, the factor $2/\sin 2\theta$ must be as small as possible, *i. e.* $\sin 2\theta$ must be as large as possible. This is so when $2\theta = 90°$, *i. e.*, when $\theta = 45°$.

(9) To avoid the error due to ex-centricity of the pivot, both ends of the pointer should be read, and to avoid the error due to parallax in reading the deflection, use should be made of the plane mirror attached to the base of the compass box.

(10) In order to avoid error due to any want of accurate setting of the plane of the coil in the magnetic meridian, readings of the deflection should be recorded both for direct and reverse current. However, the current should be reversed only in the galvanometer, and not in the voltameter circuit.

(11) To avoid oxidation of fine deposit of copper to copper oxide, the cathode plate should immediately be dipped in water, washed and then dried.

(12) The *chief sources of error* in this experiment arise due to the fact that (a) the coil may not be exactly circular and the needle may not be exactly at its centre, (b) the plane of the coil may not be exactly in the magnetic meridian, (c) the magnetic needle is not very small, so that it does not move in a uniform magnetic field at the centre, hence the tangent law is not accurately obeyed, (d) the friction at the pivot may not be totally absent. Moreover, (e) the scale and pointer method for measuring deflections is not susceptible of great accuracy.

ADDITIONAL EXPERIMENTS

Expt.—20 (a)

The above procedure can also be utilised for calculating the value of H, the horizontal component of earth's magnetic field. For this purpose

$$k = \frac{10\, r\, H}{2\pi n} \quad \text{or} \quad H = \frac{2\pi n k}{10\, r} \text{ oersted}$$

Now putting $n = 2$ and using the value of r for two turns as specified by the maker*, the value of H can be calculated.

Expt.—20 (b)

Determination of the Reduction Factor of a Helmholtz galvanometer.

The principle of the Helmholtz galvanometer has been indicated in the theory developed for experiment–19. The field due to one coil at a distance x on the axis is given by—

$$F = \frac{2\pi n r^2 i}{10\,(r^2 + x^2)^{3/2}}$$

* If this value is not available, it can be estimated by noting the diameter with a callipers.

Since there are two coils in this galvanometer carrying current in the same direction and the needle is placed between them at a distance of r/2 from their centres, the total magnetic field is twice the field due to a single coil at x = r/2. Hence the total field is

$$F = 2.\frac{2\pi n r^2 i}{\left[\left(\frac{r}{2}\right)^2 + r^2 \right]^{3/2}} = \frac{32\pi n i}{5\sqrt{5}.r}$$

When the planes of the two coils are set parallel to the magnetic meridian, the field F is also given by—

$$F = H \tan \theta$$

where θ is the deflection produced in the needle,

Thus
$$\frac{32\pi n i}{5\sqrt{5}.r} = H . \tan \theta$$

Hence
$$i = \frac{5\sqrt{5}.\ r\ H}{32\ \pi n} . \tan \theta = k. \tan \theta$$

where k is the reduction factor of the Holmholtz galvanometer.

The method of determining k for this galvanometer is exactly identical to the one described above for the tangent galvanometer. The Helmholts galvanometer replaces the tangent galvanometer in the electric circuit in fig–39.

EXPERIMENT—21

Object. To determine the electro-chemical equivalent of copper using a tangent galvanometer for the measurement of current.

PRINCIPLE AND THEORY OF THE EXPERIMENT

If the plane of the coil of a tangent galvanometer be set in the magnetic meridian, and if a current of strength of i amp. be passed through it, then

$$i = k. \tan \theta \qquad \ldots \qquad (1)$$

where k is the reduction factor of the tangent galvanometer and θ is the angular deflection produced in the needle.

If the same current is allowed to flow in a copper voltameter connected in series with the tangent galvanometer, then from Faraday's law of electrolysis, we have

$$m = i\ z\ t \qquad \ldots \qquad (2)$$

where m is the mass of the copper deposited on the cathode plate and t is the time (in seconds) for which the current flows through

the electrolyte. z is the required electro-chemical equivalent of the ion (copper, in this case) liberated from the electrolyte. From these two equations we have—

$$z = \frac{m}{kt.\tan\theta} \qquad \ldots \qquad (3)$$

The reduction factor k of the tangent galvanometer can be obtained from the formula—

$$k = \frac{10\,r\,H}{2\,\pi\,n}$$

where the constants, r (radius of the coil, as given by the maker), H (the horizontal component of earth's magnetic field at the place of the experiment), and n (number of turns, generally 2) are known. Thus z can be evaluated from equation (3).

[**Note.** The above is really a modification of the previous experiment. Hence for a fuller treatment for method etc., see expt.-20.]

EXPERIMENT—22

Object. To determine the specific conductivity of a given electrolyte (sodium chloride solution) with the help of a metre bridge.

Apparatus Required. A metre bridge, a small induction coil, electrolytic cell, a battery (for operating the induction coil), a telephone (head-piece receiver), a resistance box and a thermostat.

Formula Employed. The specific conductivity (s) of the electrolyte is given by the formula—

$$s = \frac{A}{r} \text{ mhos per cm.}$$

where r is the resistance between the electrodes, and A is a constant depending on the dimensions of the vessel used.

PRINCIPLE AND THEORY OF THE EXPERIMENT

The usual methods employed for the measurement of resistance are inapplicable for the determination of the resistance of an electrolyte. When a direct current is passed through an electrolyte, the resulting polarisation due to the deposition of the products of electrolysis on the electrodes, causes an increase in the resistance of the electrolyte. Further, for continued passage of current through such a liquid, the resulting decomposition also causes a change in the resistance, due to alteration of the concentration of the solution.

The usual special methods adopted to measure such resistances are designed to overcome these difficulties and are either a potentiometer method, or one making use of an alternating current.

The second is the one most generally employed. If the alternating current is small, and the area of the electrodes large, the polarisation effect is reduced to a negligible amount. This is brought out more completely when rapidly alternating current is employed for this purpose. Thus, whereas it is impossible to obtain reliable values for the resistance of electrolytes by the Wheatstone bridge method in the ordinary way, by using alternating current in conjunction with a Wheatstone net, the value of the resistance of the electrolyte may be found. Of course in such an arrangement an ordinary galvanometer is useless as a detector; a telephone replaces it in the usual modification. The scheme of connections is shown in the figure below :—

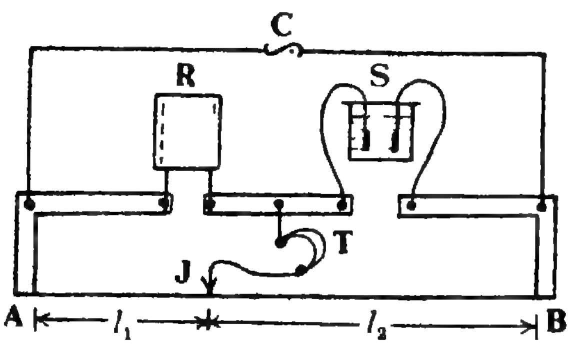


Fig. 40
Specific conductivity of an electrolyte with a metre bridge

(i) C is a small induction coil

(ii) R a resistance box

(iii) S an electrolytic cell

(iv) T a telephone

(v) AB metre bridge wire

(vi) J sliding jockey.

Now, if r be the resistance of the electrolyte as determined by the usual metre bridge method, and 'a' be the cross-sectional area of the electrolytic column and l its length, then the specific conductivity, *which is the reciprocal of specific resistance,* is given by

$$s = \frac{l}{ar}$$

If the distance between the electrodes remains constant, l/a is constant for the given electrolytic cell, and is known as the cell constant. Thus if A be the cell constant the above equation takes the form

$$s = -$$

This constant can be evaluated by taking a standardising solution, such as potassium chloride solution, whose specific conductivity is known and whose resistance can be measured.

[Note. The electrolytic cell should be surrounded by a thermostat, since the resistance of the solution varies rapidly with temperature. The cell should be provided with platinum electrodes, which should be coated with finely divided platinum*, to increase their effective area, and to diminish the back e. m. f. due to polarisation.]

Method

(i) Prepare potassium chloride solution of known strength (say, N/10), and introduce it in the electrolytic cell so that the electrodes are just covered. Place the cell in the thermostat and adjust the temperature (say, to 25°C.)

(ii) Now set up the electrical connections as shown above, and determine the resistance of the electrolytc by the usual Wheatstone Bridge procedure, first putting the electrolytic cell in one gap, and then in the other. Adjust the value of the resistance in the resistance box in such a way that the null-point falls as close to the centre of the bridge wire as possible. Calculate the value of A, the cell constant, by taking the value of specific conductivity from the Table†. Dilute this solution to, say N/100 and then redetermine A. Then calculate the mean value of A which should be employed in subsequent calculation.

(iii) Remove this solution from the cell which should be thoroughly cleaned free of solution and fill the cell with the prepared solution of sodium chloride of known strength and find the resistance of this electrolyte as before. The solution can be diluted to known concentration and the experiment repeated as before.

(iv) Now with the help of the cell constant determined previously with the potassium chloride solution, determine the specific conductivity for sodium chloride solution for each concentration. Note the temperature of the thermostat.

[Note. It is not advisable to introduce any inductance or capacity into the net, so for this reason the wire bridge is preferable to the post-office box, as the single bridge wire has much less self-inductance than the coils of the post-office box.]

* If the electrodes have not already been treated like this, they should be done so by immersing them in a solution made by taking I part platinum chloride, 30 parts water, and 0·008 part lead acetate, and passing a small current first in one direction and then in the reverse. The platinised electrodes should then be raised to dull red heat and allowed to cool.

† See Table—9 given at the end of the book.

Observations

Readings for the determination of resistance of the electrolyte.

Liquid in the cell	Known resistance (R)	Position of the null-point with the unknown resistance in the						Mean
		Left gap			Right gap			
		l_1	l_2	r	l_1	l_2	r	r
N/10 KCl soln.								
N/100 KCl soln.								
NaCl soln.								

(i) Temp. of the cell = ... °C

(ii) Specific conductivity of

(a) $\frac{N}{10}$ KCl soln. = ...

(b) $\frac{N}{100}$ KCl soln. = ...

(at ... °C)

Calculations

[A] *Resistance of the N/10 KCl solution* :—

(i) Cell in the left gap. $r = \frac{l_1}{l_2} \cdot R = ...$ ohm.

(ii) Cell in the right gap $r = \frac{l'_2}{l'_1} \cdot R = \ldots$ ohm.

$\therefore$ Mean r = ... ohm.

Now, Cell constant, $A = s \cdot r = \ldots$

[B] *Resistance of the N/100 KCl solution :—*

[Note. Calculate r and A as above]

$\therefore$ Mean cell constant, $A = \ldots$

[C] *Specific conductivity of N/10 NaCl solution :—*

$$s = \frac{A}{r} = \ldots \quad = \ldots \text{ mhos per cm.}$$

Result. The specific conductivity of the given electrolyte (N/10 sodium chloride solution at......°C = mhos per cm.

Precautions and Sources of Error

(1) The electrolytic cell employed in this experiment should be scrupulously clean and there should be no air-bubbles in between the electrodes when the cell is filled with the electrolytic solution.

(2) In this experiment it is essential that there are introduced no inductance or capacitance into the net, otherwise the telephone will give no point of perfect silence, even if the ordinary formula for the Wheatstone's bridge is satisfied. For this purpose non-inductive resistance with little capacitance should be employed. Hence it is advisable that the connecting wires are not coiled as is usually done for neater connections.

(3) The known resistance in the resistance box should be so adjusted that the point corresponding to no sound in the telephone is obtained at the centre of bridge wire. This will yield maximum accuracy of observation.

(4) The jockey should always be pressed lightly on the bridge wire and it should never be kept pressed while it is being moved along the wire, otherwise by uneven rubbing, the uniformity of the bridge wire shall be impaired.

(5) As the resistance of the electrolytes varies rapidly with temperature*, the electrolyte cell should be kept surrounded by a thermostat and the temperature should be kept constant within one-tenth of a degree.

In contrast to the case of metals the conductivity of electrolytes increases with rise of temperature. At 18°C the increase is about 2·5% per degree centrigrade.

(6) To minimise the error due to end corrections two sets of observations should be carried out with the position of the electrolytic cell and the resistance box interchanged in the two gaps.

(7) If the position of "no sound" in the telephone is not attained, that position where the sound is minimum should be noted down. However, if the position of minimum sound extends over an appreciable length of the bridge wire, the central position of this distance should be recorded.

(8) The chief source of error in this experiment lies in the fact that for no position of the jockey the sound heard in the telephone is reduced to zero. It is primarily due to the use of induction coil, the e. m. f. wave form of which is complex, and hence it is impossible to obtain the condition of complete silence. The balance has to be obtained by reducing the sound in the telephone to a minimum and accuracy is then poor. Whenever possible, the induction coil should be substituted by a valve oscillator of suitable frequency (say, 1000 cycles per second). It generates an e. m. f. of a single frequency without harmonics.

(9) The accuracy of the result is also dependent on the uniformity of the bridge wire. Consequently if the wire is not uniform throughout its length, inaccurate result shall be obtained, The error can be eliminated by accurately calibrating the wire throughout its length.

ADDITIONAL EXPERIMENTS

Expt.—22 (a)

By finding the resistance of several solutions of sodium chloride from a concentration, say, 29·25 gm. per litre, N/2, to 0·2925 gm. per litre, a graph can be plotted showing the relation between specific conductivity and concentration.

Expt.—22 (b)

The variation of the resistance of one of the solutions with temperature may also be investigated by heating the water bath surrounding the electrolytic cell, and the value of the temperature coefficient may be calculated.

HEATING EFFECT OF ELECTRIC CURRENT

EXPERIMENT—23

Object. To determine the mechanical equivalent of heat (J) with the help of a Joule's calorimeter.

Apparatus Required*. A battery of accumulators, rheostat, Joule's calorimeter, an ammeter, a voltmeter, a sensitive thermometer, physical balance, weight box, and a plug key.

Description of the Apparatus. The Joule's calorimeter (see fig.-41) consists of a copper calorimeter, which is fitted with an ebonite lid having holes for the stirrer and the thermometer. The lid supports the heating coil which is suspended inside the calorimeter. The heating wire is made of a material having high resistivity (e.g., eureka or nichrome), and is connected to the lid by thick copper leads. The calorimeter is usually kept in a wooden container, the space in between the two is filled with a non-conducting material, e g., felt, cotton-wool, etc.

Formula Employed. The mechanical equivalent of heat (J) is calculated with the help of the following formula—

$$J = \frac{W}{H} = \frac{VIt.\ 10^7}{(M + W)\,(\theta_2 - \theta_1)} \text{ ergs/cal.}$$

where $W\ (= VIt.\ 10^7)$ = Work done by the electric current

$H\ [= (M + W)\,(\theta_2 - \theta_1)]$ = Heat generated.

Here V = Potential difference between the ends of the coil ; I = Current† flowing through the coil ; t = Time for which current has been flowing in the circuit ; M = Mass of water taken in the calorimeter ; W = Water equivalent of the calorimeter ; θ_1 = Initial temperature of water ; and θ_2 = Final temperature of water.

PRINCIPLE AND THEORY OF THE EXPERIMENT

If a current of I, e. m. u. be flowing for t sec. in a conductor whose ends are maintained at the difference of potential of V e.m.u. the work done by the current is equal to VIt ergs. If this energy

* There can be slight variants of the method for this experiment, for instance, in place of the ammeter, a copper voltameter can be employed as a current measurer. [For this see expt.-20].

Again, instead of a battery the current may be drawn from the D C. mains, in which case a bulb resistance should also be included in the circuit to reduce the current.

Lastly, an A. C. mains may be employed for drawing the current. In that case a step-down transformer yielding 6—8 volts at the ends of the secondary should be employed. The leads from the secondary should be connected to the circuit as usual. However, for this circuit an A.C. ammeter and an A.C. voltmeter should be employed for the measurement of current and the potential difference respectively.

† If a copper voltameter replaces an ammeter, the current flowing in the circuit is calculated with the help of the formula : m = izt, where m is the mass of the copper deposited on the cathode plate and z is the electro-chemical equivalent of copper.

is transformed as heat, we have from the First Law of Thermodynamics that

$$J = \frac{W}{H} = \frac{VIt}{H} \text{ ergs/cal.}$$

However, if the potential difference and the current are expressed in practical units, the above expression for J takes the form—

$$J = \frac{(V.\ 10^8)\ (I.\ 10^{-1}).t}{H} = \frac{VIt\ 10^7}{H} \text{ ergs/cal.}$$

If this heat is absorbed by M gm. of water contained in a calorimeter whose water equivalent is W gm. then

$$H = (M + W)\ (\theta_2 - \theta_1) \text{ cals.}$$

where θ_1 and θ_2 are the initial and final temperatures of water. Thus

$$J = \frac{VIt.\ 10^7}{(M + W)\ (\theta_2 - \theta_1)} \text{ ergs/cal}$$

Method

(i) Weigh the empty calorimeter with its stirrer, and again with two-thirds of water. The difference between these two readings gives the mass of the water taken.

(ii) Set up the electric circuit as shown in the figure. After thoroughly stirring the water, note the initial temperature of the cold water.

(iii) Now switch on the current and immediately stop the stop-watch. Note down the potential difference across the heating coil with the voltmeter, and the current with the ammeter and keep their values constant with the help of the rheostat. Keep the water stirring constantly and efficiently*

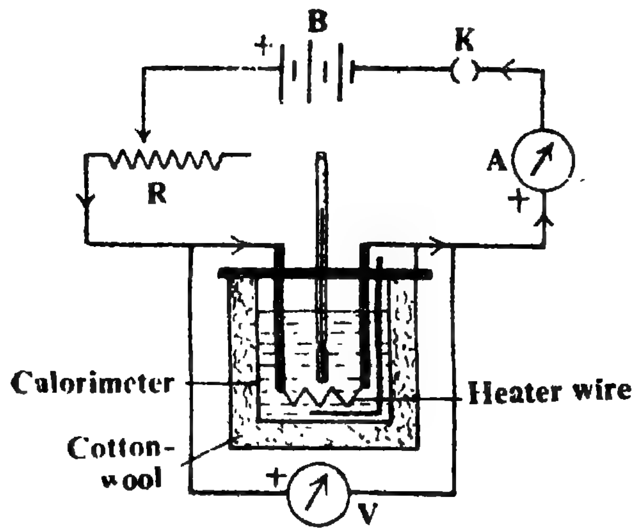


Fig. 41
Joule calorimeter
(with electrical connections)

When the temperature has risen about 6—8°C, switch off the current and immediately note the time. Also note down the final temperature.

*Efficient stirring is very important in this experiment.

(v) Allow the calorimeter and its contents to cool for the same time for which heating was done, and determine the fall in temperature during this interval.*

Add half the fall in temperature to the previously observed final temperature. This will give the final temperature corrected for losses of heat by radiation.†

(v) Finally calculate the value of J from the formula given above.

Observations

S. No.	Determinations	Magnitude	Derived quantities
1	Mass of the calr. + stirrer	...gm.	
2	Mass of the calr. + stirrer + water	...gm.	(1) Mass of the cold water = ...gm.
3	Initial temperature of water	...°C	
4	Initial reading of the watch	...min.	(2) Time taken for the expt. = ...min. = ...sec.
5	Reading of the ammeter	...amp.	(3) Fall in tem. after cooling of water for the same time = ...°C
6	Reading of the voltmeter	...volt.	
7	Final reading of the watch	...min. sec.	(4) Sp. heat of the material of the calr. (taken from the Table of Constant) = ...
8	Final temperature of water	...°C	
9	Last temp. of water after cooling for the same time	...°C	

Calculations

(1) Water equivalent of the calorimeter = .gm

(2) Corrected final temperature =°C

Now
$$J = \frac{V\ It\ 10^7}{(M+W)(\theta_2 - \theta_1)}$$
$$= \ldots\ldots \text{ergs/cal.}$$

* Continue the stirring of water.

† For accurate radiation correction Regnault's Method should be employed (see page—).

Result. The value of the mechanical equivalent of heat = ...ergs per cal.

[Standard value = $4{\cdot}186 \times 10^7$ ergs/cal. ; Error = ...%].

Precautions and Sources of Error

(1) While setting up the electric circuit care should be taken to connect properly the ammeter and the voltmeter. Their positively market terminals should be connected to the higher potential point of the circuit. The ammeter should be connected in series, while the voltmeter should be connected in parallel with the heating coil.

(2) The potential difference across the heating coil should be nearly 6 volts. In any case it should not exceed 8 volts, otherwise electrolysis of water, which sets in, shall effect the result seriously.

(3) The current should be kept constant throughout the duration of the experiment with the help of the rheostat.

(4) The water in the calorimeter should be constantly and efficiently stirred during its heating as well as its cooling when radiation correction is being applied.

(5) The final temperature of the calorimeter and its contents should not go more than 8°C above the room temperature, otherwise heat losses shall be enormous. Moreover, the observed final temperature should be corrected for the radiation losses.

(6) A sensitive thermometer reading upto at least one-tenth of a degree should be employed.

(7) The chief *sources of error* in this experiment are ; (a) heat losses cannot be completely eliminated, (b) the thermal capacity of the heating coil and the thermometer has not been taken into account, (c) loss of heat by evaporation of water has not been considered, (d) part of the total current flowing through the ammeter flows through the voltmeter, so that the current actually flowing through the heating coil is less than the observed value.*

[**Note**. If instead of an ammeter a copper voltameter has been employed for the measurement of current, the precautions pertaining to the use of this instrument should be carefully observed as mentioned in experiment–20.]

EXPERIMENT—24

Object. To determine the mechanical equivalent of heat (J) with the help of Callender and Barne's continuous flow calorimeter.

*Most of these defects are eliminated in Callender and Barne's method which is described and discussed in expt.–24.

Apparatus Required. Callender and Barne's calorimeter, constant level tank, a battery of accumulators, rheostat, voltmeter, ammeter, two sensitive thermometers, and stop watch.

Description of the Apparatus. The apparatus consists essentially of a narrow glass tube along the axis of which a high resistance wire (of, say, nichrome) is laid in the form of a helix. This shape of the wire is necessary to keep the water stirred as it flows along the tube. The water is allowed to flow at a regulated rate by means of the constant level head shown on the left. The coil is

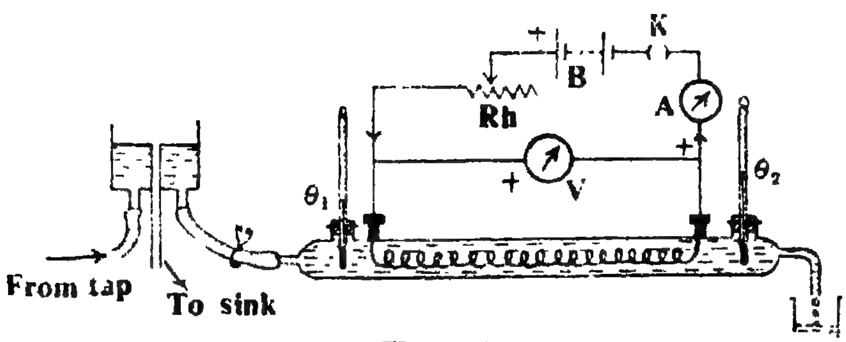


Fig. 42
J by Callender and Barne's calorimeter

heated by allowing a suitable current to flow from a battery of accumulators. The temperature of the incoming and outflowing water is recorded at each end with a sensitive thermometer.

Formula Employed. The value of the Mechanical Equivalent of Heat J is given by—

$$J = \frac{v_1 i_1 - v_2 i_2}{(m_1 - m_2).\,\delta\theta}.\ 10^7 \text{ ergs/cal.}$$

where v_1, v_2 = The potential differences applied successively at the two ends of the heating wire.

i_1, i_2 = The corresponding values of the current for the two sets of observations.

m_1, m_2 = Masses of water collected *per second* for the two sets of observations

$\delta\theta$ = Small rise in temperature of water (equal in both the sets).

PRINCIPLE AND THEORY OF THE EXPERIMENT

If a current of i_1 amp. flows through a wire, at the ends of which a difference of potential of v_1 volts is maintained, then the work done per second by the electric energy is

$$W_1 = v_1 i_1\ 10^7 \text{ ergs.}$$

If the two thermometers show a steady difference of temperature $\delta\theta$ and if the mass of water collected per second is m_1 gm. then the heat produced

$$H_1 = m_1 \,.\, \delta\theta + h$$

where h is the heat lost per second to the surroundings.

$$\text{Hence} \qquad J = \frac{W_1}{H_1} = \frac{\nu_1 \, i_1 . \, 10^7}{m_1 \, \delta\theta + h} \qquad \ldots \qquad (1)$$

$$\text{or} \qquad J\,(m_1 . \, \delta\theta + h) = v_1 \, i_1 . \, 10^7$$

Now let the current be altered to i_2 and let the new potential difference be v_2. Let the rate of flow of water be so adjusted that the steady difference of temperature in the readings of the two thermometers is again $\delta\theta$ as before. The same reasoning then leads to the following equation—

$$J\,(m_2 \,.\, \delta\theta + h) + v_2 i_2 \,.\, 10^7 \qquad \ldots \qquad (2)$$

The loss of heat h is the same as before, since the surface of the tube exposed to the surroundings is the same and the mean difference of temperature is also kept the same. Hence substracting (2) from (1) we have

$$J\,(m_1 - m_2) . \, \delta\theta = (\nu_1 \, i_1 - v_2 \, i_2) . \, 10^7$$

$$\text{or} \qquad J = \frac{v_1 \, i_1 - v_2 i_2}{(m_1 - m_2)\, \delta\theta} \,.\, 10^7 \text{ ergs/cal.}$$

From this relation we can calculate the value of J and no radiation correction is necessary in this case.

Method

(i) Make the electrical connections* as shown in the diagram and insert two sensitive thermometers (each reading upto one-tenth of a degree). Connect the constant level tank to the tap as well as to the calorimeter. Adjust the flow of the water through the tank and adjust its height so that the water issues out of the calorimeter in a steady and continuous stream.

(ii) Now switch on the current and with the help of the rheostat adjust its value to about 2 amp. After some time the thermometers at the two ends will register steady temperatures. If the difference of temperatures at the two ends is less than 5°C, adjust the height of the water tank, thereby regulating the flow of water, till the rise in temperature is at least 5°C.

(iii) Keep the readings of the ammeter and the voltmeter constant with the help of the rheostat and note the readings of the

* Do not switch on the current as yet.

thermometers after every two minutes. When the temperatures are steady for ten minutes, note down their values as well as the readings of the ammeter and the voltmeter. Collect water for a known time (with the help of the stop-watch) in a graduated cylinder and note down the mass of water by knowing the volume collected. Repeat this process thrice and thereby calculate the mean mass m_1 of water collected per second.

(iv) Now alter the value of the heating current, and by raising or lowering the tank, adjust the flow of water in such a way that, when a steady state is attained, *the rise in temperature is exactly the same as before.* As before, determine the mass m_2 of the water collected per second. Record the new values of the current and the potential difference.

(v) Finally, *switch off the current, and thereafter cut off the supply of water.* Calculate the value of J by the above formula.

Observations

Set I

S. No.	Temp. of inflowing water	Temp. of outflowing water	Volume of water collected	Time taken	Remarks
1.					(i) Current (i_1) = ...amp.
2.					(ii) Potential diff. (v_1)=...volt. (iii) Rise in temp. = ...°C
3.					(iv) Rate of flow of water (m_1) = ... gm/sec.

Set II

[**Note**—Make a similar table to record the readings.]

Calculations

$$J = \frac{v_1 i_1 - v_2 i_2}{(m_1 - m_2).\ \delta\theta} \cdot 10^7$$

$$= \text{...ergs/cal.}$$

Result. The value of the mechanical equivalent of heat = ... ergs/cal.

[Standard value = $4{\cdot}186 \times 10^7$ ergs/cal, ∴ Error = ... %]

Precautions and Sources of Error

(1) The heating wire inside the narrow tube should always be taken in a helical form. This helps in exposing a greater surface for the water to come in contact, and secondly, it keeps the water stirred as it moves along.

(2) The level of water in the tank should be maintained constant so that the flow of water through the calorimeter is steady. For this purpose stop-cocks should be employed wherever necessary.

(3) The heating current and the flow of water should be so adjusted that the rise in temperature is of the order of 5°C. For this purpose sensitive thermometers reading upto one-tenth of a degree should be employed.

(4) Before starting the experiment, water should be allowed to flow first, and then the current should be switched on. This order should be reversed at the end of the experiment.

A NOTE ON THE DETERMINATION OF J BY THIS METHOD

This method is susceptible of yielding an accurate value for J. As the temperatures at two ends are steady, the thermal capacity of the calorimeter, of the heating wire, or of the thermometers does not enter into the calculations and hence need not be known. The radiation losses have been eliminated by taking two sets of observations. Moreover, since the conditions are steady, all readings can be taken with utmost accuracy.

However, it must be admitted that in the present experiment the accuracy of the result is limited by the measuring instruments employed *e. g.*, the mercury-in-glass thermometers, the ammeter, the voltmeter, and the stop-watch. In the actual experiment conducted by Callender and Barne, mercury thermometers were replaced by platinum resistance thermometers employed differentially in the opposite arms of a Wheatstone bridge arrangement. The bulb of each thermometer was kept surrounded by a thick copper tube attached to each end of the heating wire. Thus, on account of its high conductivity the tube keeps the bulb at the same temperature as the adjacent water, and due to its negligible resistance does not allow the generation of heat near the bulbs of the platinum thermometers.

Secondly, Callender and Barne measured the potential difference at the ends of the heating wire in terms of a standard Clark cell by means of an accurately calibrated potentiometer. The same potentiometer was also employed for the measurement of current flowing through the heating wire by measuring the potential difference across a standard one-ohm resistance included in the heating circuit.

The time was measured automatically by an electric chronograph.

In order to eliminate the heat losses due to conduction and convection, the narrow tube was surrounded by a hermetically sealed glass jacket from which all air was exhausted by a pump. In order to keep the temperature of the surroundings constant, the vacuum jacket was in turn surrounded by another jacket in which water at the room temperature was kept circulating.

Thus, by employing this elaborate procedure Callender and Barne could get a standard value for J.

EXPERIMENT—25

Object. To calibrate an electric energy-meter with the help of a Joule's calorimeter.

Apparatus Required. Electric energy-meter, Joule's calorimeter, an electric bulb, a voltmeter, an ammeter, a thermometer, a physical balance and a weight box.

Description of the Apparatus. The recording electric energy-meter, or simply an electric meter, is the instrument which fixes over bills for the consumption of electricity in our homes. This instrument gives not the rate at which energy is supplied in our homes (or in any other electric circuit) but it gives the total amount of energy supplied during a given time (*e. g.*, for a month in our homes). This energy is measured in a commercial unit known as kilowatt* hour which is the energy consumed in one hour when the power is one kilowatt.

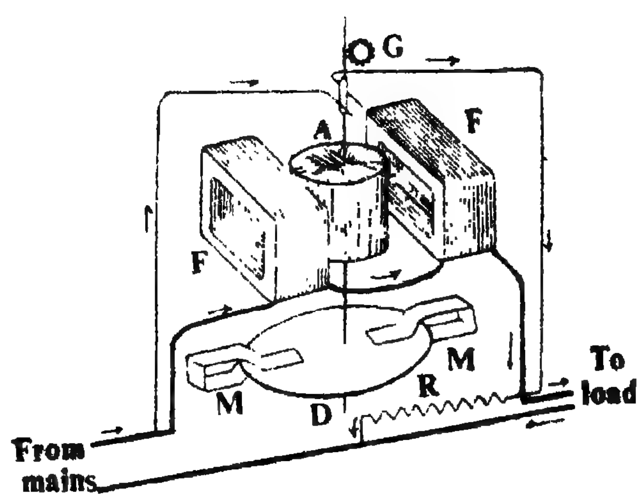


Fig. 43
An electric meter

The electric meter (fig.–43) is essentially an electric motor containing no iron. The field-coils consist of two coils (F, F) of low resistance connected in series, while the armature (A) consists of many turns of fine wire placed in series with a high resistance (R). The armature is carried by an upright spindle, which by means of

$$* \text{ Kilowatt-hours} = \frac{\text{Volts} \times \text{Amperes} \times \text{Hours}}{1000}.$$ Kilowatt-hour is generally written as "kwh".

the worm gear (G) operates the pointers of the counter over the dials. The lower end of the spindle passes through an aluminium disc (D) which rotates between the poles of the two magnets (M, M). As the disc moves between the magnetic poles, it cuts their lines of force, and thus eddy currents are produced in the disc whose speed is thereby slowed down. In this way this device acts like a magnetic brake for the rotating armature of the electric meter.

For measuring the consumption of electric energy in a circuit the field-coils are placed in series with the load, while the armature together with its series resistance is connected in parallel with the supply mains. The driving torque of the motor is proportional to the current in the armature and to the field intensity of the field-coils. The current in the armature is proportional to the potential difference V applied to the load, while the field intensity of the coils (F, F) is proportional to the current I in the main circuit. It is thus clear that the driving torque of the motor is proportional to $V \times I$, *i. e.*. it is proportional to watts consumed in the circuit. Again, the energy consumed in the circuit in a given time is proportional to the number of revolutions made by the armature during that time, *i. e* , the energy consumed is proportional to the reading on the dials of the counter. It is in this way that the electric meter is made to read watt-hours or kilowatt-hours.

Formula Employed. The object of this experiment is to measure the electric energy E consumed in a circuit in a specified time with the help of the electric meter, as also to measure this energy E′ by the other methods and then to calculate the percentage error of the meter with the help of the following formula :—

$$\% \text{ error} = \frac{E - E'}{E} \times 100$$

Now E′ is calculated with the help of the formula—

$$E' = \tfrac{1}{2}(E_1 + E_2)$$

where $E_1 = VIt/3{\cdot}6 \times 10^6$ kwh

and $E_2 = J\,(m + w)\,\theta/3{\cdot}6 \times 10^{13}$ kwh

Here V = Reading of the voltmeter

I = Reading of the ammeter

t = Time (in seconds)

J = Mechanical equivalent of heat

m = Mass of water taken

ω = Water equivalent of the calorimeter

θ = Rise in temperature

PRINCIPLE AND THEORY OF THE EXPERIMENT

The value of electrical energy consumed in a circuit in a given time can be determined in the following three ways :—

(1) By noting the initial reading of the electric meter and again by noting its final reading when the electric energy has been utilised for a time t, we can easily calculate the energy E by taking the difference of the two readings.

(2) Secondly, if the potential difference applied to the circuit be V volts and if a current of I amperes be allowed to flow in the circuit for t seconds, then the energy consumed in the circuit is given by—

$$E_1 = VIt \times 10^7 \text{ ergs} = VIt \text{ Joules.}$$

But 1 kilowatt-hour$= 1000 \times 60 \times 60 = 3{\cdot}6 \times 10^6$ Joule's, hence

$$E_1 = VIt/3{\cdot}6 \times 10^6 \text{ kwh.} \qquad \ldots \qquad (1)$$

(3) If in the electric circuit is included a calorimeter of water equivalent w gm containing m gm of water and if the rise in temperature be θ, then the heat H produced in the circuit is given by—

$$H = (m + w)\,\theta \text{ cals}$$

and the energy consumed is given by

$$E_2 = JH = J\,(m + w)\,\theta \text{ ergs}$$

But 10^7 ergs $= 1$ Joule and 1 kilowatt-hour $= 3{\cdot}6 \times 10^6$ Joule's, hence

$$E_2 = J\,(m + w)\,\theta/3{\cdot}6 \times 10^{13} \text{ kwh} \qquad \ldots \qquad (2)$$

The mean of the two values E_1 and E_2 may be taken as the correct value (say, E′) consumed in the circuit, i. e., E′ may be put equal to $\frac{1}{2}(E_1 + E_2)$. Finally the percentage error in the reading of the electric meter is given by

$$\text{Percentage error} = \frac{E - E'}{E} \times 100 \qquad \ldots \qquad (3)$$

This formula is utilised in the calibration of the given electric meter.

Method

(i) Weigh the empty calorimeter with stirrer, and again weigh it when it is filled nearly two-third with water. The difference between these two readings gives the mass of the water taken. Place a thermometer (reading upto ·1°C) and an electric

bulb (say, of 60 watts) in the calorimeter. After covering the calorimeter with its ebonite laid place it in its container lined with a non-conducting material.

(ii) Make electrical connections* as shown in the figure. A is the ammeter and V is the voltmeter. Record the initial temperature

(iii) Now switch on the current, and simultaneously start the stop-watch. Begin counting the revolutions of the disc of the electric meter. Keep the water thoroughly stirred and note the readings of the ammeter and the voltmeter after every two

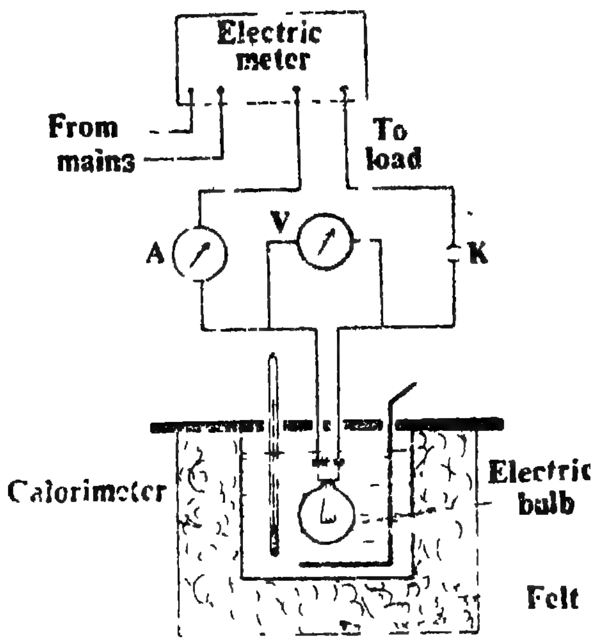


Fig. 44
Electric circuit for calibrating on electric meter

(iv) When the current has passed for some time (say, for 20 minutes), switch off the current stopping simultaneously the stop-watch. Note down the total number of revolutions made by the disc, the final temperature of water and the time for which the current has been passed.

Note down the number of revolutions of the disc which corresponds to the energy consumption of one kilowatt-hour. This information is usually given by the manufacturers on the face of the

(v) Allow the calorimeter and its contents *to cool for the same time* for which they were heated and determine the fall in final temperature during this interval. During this process keep the water stirred. Add half of the fall in temperature to the final temperature to correct it for the loss of heat by radiation.

(vi) Finally, calculate E_1 and E_2 with the help of equations (1) and (2) given above, and the percentage error in the reading of the electric meter from equation (3).

---

* In D. C. circuits connect the ammeter and the voltmeter in such a way that the current enters into them by their positively marked terminals.

Observations

[A] (i) Number of revolutions of the disc of the meter corresponding to 1 kwh of energy consumption (N) = ...

(ii) Number of revolutions made by the disc of the meter during the experiment (n) = ...

[B] *Readings for V and I*

S. No.	Time in mins.	Ammeter readings	Voltmeter readings
1.	0	... amp.	... Volt
2.	2	... amp.	... Volt
⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮
	Mean	... amp.	... Volts

[C] *Readings for the determination of m, w and θ*

S. No.	Determinations	Magnitude	Derived quantities
1.	Mass of the calr. + stirrer	... gm.	(i) Mass of cold
2.	Mass of the calr. + stirrer + water	... gm.	water (m) = ...gm.
3.	Initial temp. of water, (θ_1)	... °C	(ii) Specific heat
4.	Final temp. of water	... °C	of the material of the calorimeter
5.	Time for which current has been passed	... sec.	(s) = ...
6.	Last temperature of water after cooling for the same time	... °C	(iii) Fall in temp. after cooling of water for the same time = ... °C

Calculations

(A) N revolutions of the disc at 220 volts correspond to a consumption of electric energy of 1 kwh

$\therefore$ n revolutions ,, ,, ,, n/N kwh

Since the consumption of electric energy is directly proportional to the operating voltage, hence n revolutions of the disc at V volts correspond to a consumption of electric energy equal to nV/N. 220 kwh.

$$\therefore \quad E = \frac{n\,V}{N \times 220} = \ldots \text{ kwh}$$

(B) (i) Mean ammeter reading, (I) = ... amp.

(ii) Mean voltmeter reading, (V) = ... volts.

(iii) Time taken in the expt., (t) = ... secs.

$$\therefore \quad E_1 = \frac{VIt}{3{\cdot}6 \times 10^6} = \ldots \text{ kwh}$$

(C) (i) Water equivalent of the calr ; (w) = ... gm.

(ii) Corrected final temperature, (θ_2) = ... °C

$$\therefore \quad E_2 = \frac{J\,(m + w)\,(\theta_2 - \theta_1)}{3{\cdot}6 \times 10^{13}} = \ldots \text{ kwh.}$$

Thus average energy consumption in the circuit

$$E' = \tfrac{1}{2}\,(E_1 + E_2) = \ldots \text{ kwh}$$

$\therefore$ % error in the reading of the electric meter

$$= \frac{E - E'}{E} \times 100 = \ldots$$

Result. The percentage error in the reading of the given electric energy meter =........

Precautions and Sources of Error.

(1) This experiment is done on the city supply mains whose operating voltage is 220 volts. *This voltage is dangerous to human body*. Hence instructions given by the teacher in this connection should be strictly followed.

(2) If the experiment is conducted on D. C. mains, care should be taken in connecting the positively marked terminals of the ammeter and the voltmeter to the higher potential point of the circuit. The ammeter should be connected in series in the circuit while the voltmeter should be connected in parallel with the electric bulb.

(3) The readings of the ammeter and the voltmeter should be recorded after every two minutes and their mean values should be employed in the calculation.

(4) The thermometer employed in this experiment should be a sensitive one. It should preferably read upto ·1°C.

(5) The water in the calorimeter should be constantly and efficiently stirred during its heating as well as its cooling when radiation correction is being applied.

(6) The final temperature of the calorimeter and its contents should not go more than, say, 8°C above the room temperature, otherwise heat losses shall be enormous. Moreover, the observed final temperature should be corrected for heat loss due to radiation. Further, in order to minimise heat losses by other processes the outer surface of the calorimeter should be made shining and it should be surrounded by some non-conducting material.

[Note—For *sources of error* see expt.-23].

CAPACITANCE

EXPERIMENT—26

Object. To determine the ballistic constant of a moving coil ballistic galvanometer with the help of a condenser of known capacity.

Apparatus Required. Ballistic galvanometer, an accumulator, a condenser* of known capacity, a high resistance voltmeter, a Morse key, and a tapping key.

Formula Employed. The ballistic constant (k) of the galvanometer is calculated with the help of the formula—

$$k = \frac{CE}{\theta_1\left(1 + \frac{\lambda}{2}\right)}$$

where C = capacity of the condenser.

E = Voltage of the cell which is used for charging the condenser.

θ_1 = First observed throw of the galvanometer when the condenser is discharged through it.

λ = Logarithmic decrement.

The logarithmic decrement can be calculated with the help of the formula—

$$\lambda = 2{\cdot}303.\ \frac{1}{10} \log_{10} \frac{\theta_1}{\theta_{11}}$$

where θ_1 and θ_{11} are the first and the eleventh throws of the galvanometer.

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* If available, a *capacity box*, should preferably be employed. With its help several readings can be taken.

PRINCIPLE AND THEORY OF THE EXPERIMENT

The Ballistic Galvanometer

A galvanometer suitable for measuring a quantity of electricity (*e. g.*, the discharge of a condenser) is called a ballistic galvanometer and has the following essential features :—

(i) The periodic time of the moving part of the galvanometer is fairly large. It may be from 10 to 15 seconds.

(ii) Damping of the moving part is negligibly small. Now, the time-period (T) of the moving system is given by :—

$$T = 2\pi\sqrt{I/c}$$

where I is the moment of inertia of the moving part, and c is the restoring couple per unit angular displacement. Thus, by increasing the moment of inertia and decreasing the restoring forces the time-period of the galvanometer can be suitably increased, so that the first condition laid down above for a ballistic galvanometer is fulfilled. The moment of inertia is increased by loading the movable part while the controlling forces are reduced by the use of a phosphor-bronze suspension strip which possesses small torsional rigidity.

The damping of the movable part of the galvanometer, apart from external artificial agency, may be considered to be due to two causes :—

(a) **The damping due to the viscosity of air.** This is present in the moving coil as well as the moving needle type instruments and is approximately proportional to the angular velocity of the moving system. It is always present, but is usually small.

(b) **Electro-magnetic damping.** In the case of a moving magnet the amount of damping due to this cause is very slight when the magnet is in a non-metallic case and when the coils are wound on wood or ebonite.

In the case of a moving-coil instrument, the suspended coil moves in a strong magnetic field. When the circuit is closed, the movement of the coil is opposed by the current induced. This electro-magnetic damping is reduced by winding the coil on a non-conducting frame, so that eddy currents which are responsible for this type of damping are eliminated.

The value of the damping current depends on the magnitude of the external resistance, and may become very great for a low series resistance.

(iii) **A third condition for the fulfilment of a ballistic galvanometer is, that when used to measure a quantity of electricity, the whole of the transient current should pass**

out before the coil or the needle of the instrument shifts from the zero position. Should there arise a case in which the quantity of electricity to be measured takes a longer time to traverse the instrument (due, for example, to the presence of inductance in the circuit), the time-period of the moving system should be increased by loading it, so that this third condition is fulfilled.

As indicated above the galvanometer may be of the moving needle or moving-coil type. We shall develop a relation between the throw or angular deflection in each type, and the quantity of electricity which passes through the galvanometer.

[A] Moving-Needle Type

This ballistic galvanometer consists of a coil through which the charge Q passes. At the centre of the coil hangs a magnetic needle suspended by a fine quartz or unspun silk fibre. The controlling force in this case is provided either by the earth's horizontal field or by an externally placed control magnet. In its initial or zero position the magnetic needle is adjusted at right angles to the axis of the coil, so that when the transient current flows through the coil, a field at right angles to the control field is set up.

Suppose the instantaneous value of the current at any instant is i, which remains sensibly constant for a very small time dt. Since the instrument is so constructed that the whole of the transient current passes before the magnetic needle moves from its zero position consequently the needle will be perpendicularly situated to a field of strength iG, and will experience a turning moment iGM. Here M is the magnetic moment of the needle and G is the galvanometer constant (*i. e*, it is the field produced by the coil when a unit current circulates through it). This couple will produce an angular acceleration, $d^2\theta$ dt^2 in the needle. Hence, if I be the moment of inertia of the magnetic needle, we have

$$I\frac{d^2\theta}{dt^2} = iGM$$

Integrating $$I.\ \frac{d\theta}{dt} = GM\int_0^t i.\ dt = GMQ \qquad \dots (1)$$

Now, the kinetic energy of the needle at any instant is given by $\frac{1}{2}$ I $(d\theta/dt)^2$, which is reduced to zero in doing work against the restoring force (mH) acting at each pole. If θ_0 be the first angular

swing of the needle, the value of this work* is equal to $MH(1-\cos\theta_0)$.

Hence $$\frac{1}{2} I \left(\frac{d\theta}{dt}\right)^2 = MH(1-\cos\theta_0) = 2\,MH.\sin^2\frac{\theta_0}{2}$$

or $$I\left(\frac{d\theta}{dt}\right)^2 = 4\,MH\sin^2\frac{\theta_0}{2} \quad \dots (2)$$

Hence, squaring (1) and dividing by (2), we have

$$I = \frac{G^2 M^2 Q^2}{4\,MH\sin^2\frac{\theta_0}{2}} \quad \dots (3)$$

Again, the time-period T of the needle is given by

$$T = 2\pi\sqrt{\frac{I}{MH}}$$

or $$I = \frac{T^2.\,MH}{4\,\pi^2} \quad \dots (4)$$

Equating (3) and (4) we have

$$\frac{G^2 M^2 Q^2}{4\,MH\sin^2\frac{\theta_0}{2}} = \frac{T^2\,MH}{4\,\pi^2}$$

Hence $$Q = \frac{TH}{\pi G}.\sin\frac{\theta_0}{2} \quad \dots (5)$$

[B] Moving-Coil Type

As stated earlier, in this type of ballistic galvanometer the electro-magnetic damping is reduced by using a coil wound on a non-conducting frame; the time-period is increased by using a fine phosphor-bronze strip suspension.

Suppose the instantaneous value of the current at any instant is i, which may be supposed to remain sensibly constant for a very small time dt. Then

* The work done against the restoring couple $MH\sin\theta$ for a small additional deflection $d\theta$ is equal to $MH\sin\theta.\,d\theta$. Hence the total work done is given by $\int_0^{\theta_0} MH\sin\theta\,.\,d\theta$, which is equal to $MH(1-\cos\theta_0)$.

$$I\left(\frac{d^2\theta}{dt^2}\right) = i\,G \qquad \dots (6)$$

where I is the moment of inertia of the coil, and G is the galvanometer constant. Integrating the above expression we have

$$I \, . \frac{d\theta}{dt} = G\int_0^t i.\,dt = GQ \qquad \dots (7)$$

If $\frac{d\theta}{dt}$ is the original angular velocity imparted to the coil by the impulsive force due to the charge Q, the initial kinetic energy of the coil is $\frac{1}{2}\, I\left(\frac{d\theta}{dt}\right)^2$. This energy is spent up in doing work against the torsional couple in the suspension which is twisted in the process. If the torsional couple per unit twist be c, that due to an angular displacement θ will be equal to $c\theta$. To twist it further through an angle $d\theta$, the work done is $c\theta.\ d\theta$. Hence the total work done in deflecting the coil is given by

$$\int_0^{\theta_0} c\theta.\,d\theta = \frac{I}{2}\; c\theta_0{}^2 \qquad \dots (8)$$

Hence $$\frac{I}{2}\, I.\left(\frac{d\theta}{dt}\right)^3 = \frac{1}{2}\, c\theta_0{}^2$$

or $$I\left(\frac{d\theta}{dt}\right)^2 = c\theta_0{}^2 \qquad \dots (9)$$

squaring equation (7) and dividing by (9) we have

$$I = \frac{G^2\,Q^2}{c\theta_0{}^2} \qquad \dots (10)$$

Now, the periodic time of the coil is given by

$$T = 2\pi\sqrt{I/c}$$

or $$I = \frac{cT^2}{4\pi^2} \qquad \dots (11)$$

Equating (10) and (11) we have

$$\frac{G^2\,Q^2}{c\theta_0{}^2} = \frac{cT^2}{4\,\pi^2}$$

or $$Q = \frac{cT}{2\pi G} \cdot \theta_0 \qquad \dots (12)$$

Thus, in this case the quantity of charge flowing through the coil of the ballistic galvanometer is proportional to the first angular throw of the coil. Thus

$$Q = k\,\theta_0 \qquad \dots (13)$$

where k is known as the ballistic constant, which may be defined as *the quantity of charge required to produce a unit angular deflection of the coil.*

Correction due to Damping

In the above formulation it has been assumed that the damping experienced by the moving system is negligibly small. Hence for any damping which still exists a correction has to be made. If $\theta_1, \theta_2, \theta_3, \dots\dots$ be the successive swings to left and right, it is found that—

$$\frac{\theta_1}{\theta_2} = \frac{\theta_2}{\theta_3} = \dots\dots\dots = \text{a constant} = d \text{ (say)}.$$

This constant d is called the *decrement,* and $\log_e d$ is called the *logarithmic decrement* and is denoted by λ. Thus

$$\log_e d = \lambda, \qquad \text{or} \qquad d = e^{\lambda}$$

Hence $$\frac{\theta_1}{\theta_2} = \frac{\theta_2}{\theta_3} = \dots\dots\dots = e^{\lambda}$$

Now the decrease from θ_1 to θ_2 takes place in half a complete vibration, and $\theta_1/\theta_2 = e^{\lambda}$. Clearly for a whole vibration $\theta_1/\theta_3 = e^{2\lambda}$, and so on. Thus if θ_0 be the observed first swing and θ_0' what it would have been if damping would have been absent, then, as the period in question is a quarter vibration,

$$\frac{\theta_0'}{\theta_0} = e^{\lambda/2} = 1 + \frac{\lambda}{2} + \text{terms in higher powers of } \lambda$$

$$= 1 + \frac{\lambda}{2} \qquad \text{(since } \lambda \text{ is small)}$$

$$\therefore \quad \theta_0' = \theta_0\left(1 + \frac{\lambda}{2}\right) \qquad \dots (14)$$

Thus, to correct for damping the observed first swing (θ_0) of the moving system should be multiplied by the factor $(1 + \lambda/2)$.

Hence relations (5) and (12) respectively take the forms—

$$Q = \frac{TH}{\pi G} \sin\left[\frac{\theta_0}{2}\left(1 + \frac{\lambda}{2}\right)\right] \quad \ldots \quad (15)$$

and

$$Q = \frac{cT}{2\pi G} \cdot \theta_0\left(1 + \frac{\lambda}{2}\right) \quad \ldots \quad (16)$$

Equation (16) consequently takes the form

$$Q = k\,\theta_0\left(1 + \frac{\lambda}{2}\right) \quad \ldots \quad (17)$$

In order to determine the ballistic constant k of the galvanometer, a condenser of known capacitance (C) is first charged to a potential difference of E volts (by connecting it across an accumulator) and then discharging it through the ballistic galvanometer. Thus the charge on the condenser

$$Q = EC = k\,\theta_0\left(1 + \frac{\lambda}{2}\right)$$

or

$$k = \frac{EC}{\theta_0\,(1 + \lambda/2)} \quad \ldots \quad (18)$$

which is the required relation.

Now, to evaluate the logarithmic decrement we see that

$$\frac{\theta_1}{\theta_2} = \frac{\theta_2}{\theta_3} = \ldots\ldots\ldots = \frac{\theta_{10}}{\theta_{11}} = e^{\lambda}$$

Hence by multiplying the successive terms we have

$$e^{10\lambda} = \frac{\theta_1}{\theta_{11}}$$

or

$$10\lambda = \log_e \frac{\theta_1}{\theta_{11}}$$

$$\therefore\ \lambda = 2{\cdot}303.\ \frac{1}{10}\log_{10}\frac{\theta_1}{\theta_{11}} \quad (19)^{*}$$

---

* In order to convert the logarithm from the base e to the base 10, the factor 2·303 ($= \log_e{}^{10}$) comes in.

Method

(i) Before starting the experiment, level the galvanometer and see that the coil moves freely in the clearance gap provided for its movement. Make this adjustment with a little caution so that the suspension of the galvanometer coil does not get broken. Focus the telescope on the central division of the scale, so that the line of demarcation of the red and black semi-circles coincides with the vertical cross-wire.

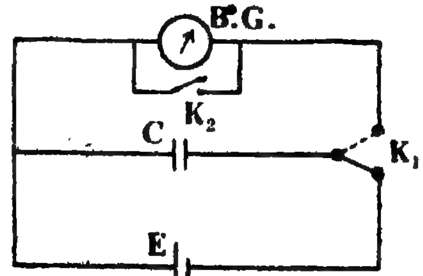

Fig. 45
Determination of ballistic constant

(ii) Now make the electrical connections as shown in the figure. K_2 is the damping key (to arrest the motion of the swinging coil). K_1 is the two way key.

(iii) Adjust the capacity box to any suitable value of capacitance. Connect the two-way key* in such a way that the condenser is connected to the cell and thereby it is charged for about 15 seconds.

Now release the key so that the condenser coatings are connected to the ballistic galvanometer. The condenser is thereby discharged. Note the successive deflections of the galvanometer coil. †

(iv) Repeat the process for different values of the capacity. Measure the e. m. f. (E) of the cell with a high resistance voltmeter. Calculate the mean value of k (the ballistic constant).

Observations

S. No.	Capacity of the condenser	Deflection of the coil.				Remark
		θ_1	θ_2	θ_3	... ...	
1	...μf					E. M. F. of the accumulator = ... volts
⋮						
⋮						

Before pressing the key for the charging of the condenser, be careful that by mistake the cell is not directly connected with the galvanometer, which will show an excessive deflection.

If with the accumulator the throw of the galvanometer is excessive, a fraction of the e. m. f. should be employed by using a potential-divider arrangement.

Set I

Calculations

$$\lambda = 2{\cdot}303.\ \frac{1}{10} \log \frac{\theta_1}{\theta_{11}}$$

$$= \ldots \ldots \ldots \ldots$$

Now, $$k = \frac{EC}{\theta_0 (1 + \lambda/2)}$$

$$= \ldots \ldots \ldots \text{ coulombs/ m m.}$$

[Note. Calculate k for other sets in the same way].

Result. The ballistic constant of the given galvanometer ... coulombs/ mm.*

Precautions and Sources of Error

(1) The galvanometer should be properly levelled and the coil adjusted in such a way that it is free to move in the clearance space provided in between the magnetic poles and the iron core.

(2) A damping key should always be employed across the galvanometer terminals so that undue swinging motion of the coil is immediately arrested.†

(3) An accumulator, which gives a constant e. m. f. should be employed for charging the condenser. However, if on discharging the condenser, the throw of the coil is excessive, this should be reduced by using only a part of the voltage of the cell by using a potential-divider arrangement.

(4) While charging the condenser see that the galvanometer is not directly connected with the cell terminals. To avoid this difficulty a suitable two way key should be employed. Before charging the condenser check up the connections once again.

(5) The capacitance of the condenser should be accurately known. Moreover, the condenser employed in this experiment should be dielectric-loss free. It should be charged nearly for fifteen seconds so that its coatings acquire the voltage of the cell.

(6) The e. m. f. of the cell should be measured with a *high resistance* voltmeter.

* Convert the observed deflections in mms. to get the result in the above units.

† This is due to Faraday's law of electro-magnetic induction, whereby induced currents are produced in the coil due to its vibratory motion and consequent cutting of the lines of force of the field of the magnet. According to Lenz's law these currents oppose the cause (i. e., the motion of the coil) which produces them. This helps considerably in bringing the coil quickly to rest.

ADDITIONAL EXPERIMENTS

Expt-26 (a)

To determine the ballistic constant with the Constant Deflection Method

According to equation (12) given above

$$Q = \frac{cT}{2\pi G}\,\theta_0$$

Now, if a steady current i produces a steady deflection ϕ in the coil, we have

Couple acting on the coil = couple due to twist

Or $$iG = c\phi \qquad \ldots \qquad (20)$$

Thus $$Q = \frac{T}{2\pi} \cdot \frac{i}{\theta}\,\theta_0 \qquad \ldots \qquad (21)^*$$

Hence the ballistic constant

$$k = \frac{T}{2\pi} \cdot \frac{i}{\phi} \qquad \ldots \qquad (22)$$

Equation (22) can be employed for the ballistic constant of the galvanometer in the following manner :—

For producing a steady current, a potentiometer arrangement is employed as shown in the accompanying figure. If the e. m. f. (E) of the cell is distributed over the total resistance $(r_1 + r_2)$, the fraction of the e. m. f. over the resistance r_1 is given by $E.\,\frac{r_1}{r_1 + r_2}$.

Now, this e. m. f. is utilised to send the steady current i through the galvanometer through a high resistance R (of the order, say, 10,000 ohms). Thus the galvanometer resistance can be neglected in comparison with such a high resistance.

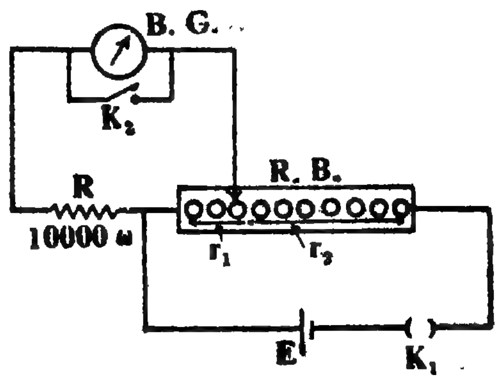

Fig. 46
Ballistic constant by constant deflection method

* The quantity Q/θ_0 is known as the *quantity sensitivity* (Q_s) of the galvanometer, while i/θ is known as the *current sensitivity* (I_s). Thus the above result leads to the important conclusion—

$$Q_s = \frac{T}{2\pi} \cdot I_s$$

i. e., ***the quantity sensitivity of the ballistic galvanometer is*** **T/2π** ***times the current sensitivity.***

From Ohm's law the steady current i flowing in the galvanometer circuit is given by—

$$\text{Steady current} = \frac{\text{e. m. f. applied}}{\text{Resistance of the circuit}}$$

$$= \frac{E.\, r_1}{(r_1 + r_2).\, R}$$

This value of i and the corresponding deflection produced by it are substituted in (22) to get the value of k. The time-period is observed in the usual manner by means of an accurate stop-watch.

[**Example**—Suppose $(r_1 + r_2)$ as obtained from the resistance box is 5,000 ohms, and $r_1 = 10$ ohms. If the e. m. f. of the cell be 2 volts,

(i) The e. m. f. applied $= \frac{2 \times 10}{5000}$ volts.

(ii) The total resistance of
the galvanometer circuit $= 10000 + 90$ (galv. res.) $+ 10$ ohm
$= 10000$ ohms (approximately)

$\therefore \qquad i = \frac{2 \times 10}{5000 \times 10000}$ amps.

(iii) Deflection $\theta = 10{\cdot}0$ mm.

(iv) Period $T = 5{\cdot}0$ sec.

Hence $\qquad k = \frac{5\,0}{2 \times 3{\cdot}14} \times \frac{2 \times 10}{5000 \times 10000 \times 10}$

$= 32 \times 10^{-8}$ coulombs per mm.]

Expt. 26 (b)

To determine the constant of the ballistic galvanometer by employing a standard solenoidal inductor.

The solenoidal inductor consists of a primary coil P of several turns of silk-covered copper wire wound uniformly on a glass tube nearly a metre in length and nearly 4 cm. in diameter. The secondary S consists of a few turns of silk covered copper wire wound uniformly over the central region of the primary.

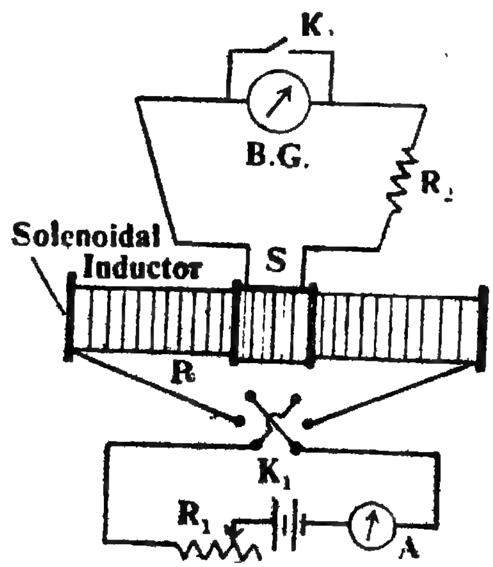

Fig. 47
Ballistic constant with a solenoidal inductor

The arrangement is indicated in the accompanying figure where B. G. is the ballistic galvanometer

under test. Allow a steady current of I amp. (as indicated by the ammeter A) to pass through the primary coil P. This produces a field of strength $4\pi nI/10$ oersteds at the centre of the primary coil, where n is number of *turns per cm* of the primary. If n′ be the *total number of turns* in the secondary and A be the face area of the secondary, then the magnetic flux linked with the secondary is equal to $4\pi nIn'A/10$. When the ballistic galvanometer is quite steady, reverse the current with the help of the commutator K_1. Evidently the change of flux in the secondary is equal to $8\pi nIn'A/10$. This produces a flow of charge Q through the galvanometer which consequently gives a throw. If θ_1 be the first throw of the galvanometer, we have

$$Q = \frac{\text{Change of flux}}{\text{Resistance of the circuit}} = \frac{8\pi nIn'A}{10^9 . R} = k \; Q_1 \left(1 + \frac{\lambda}{2}\right)$$

where R is the *total resistance* of the secondary circuit.

[**Note**—Take care with the units employed.]

Thus
$$k = \frac{8\pi nIn'A}{10^9 . R\theta_1 \left(1 + \frac{\lambda}{2}\right)}$$

[**Note**—In order to avoid excessive throw in the ballistic galvanometer, a high resistance of the order 10,000 ohms (R_2 in the figure) is included in the secondary circuit.]

Expt —26 (c)

Object—*To compare the capacities of two condensers* by using a ballistic galvanometer.*

Let the two condensers of capacity C_1 and C_2 be successively charged so that the coatings are raised to the same difference of potential of E volts, then their respective charges Q_1 and Q_2 are given by

$$Q_1 = EC_1 \text{ and } Q_2 = EC_2$$

or
$$\frac{Q_1}{Q_2} = \frac{C_1}{C_2}$$

Now let the two condensers be discharged successively through a ballistic galvanometer giving rise to the respective throws θ_1 and θ_2, then

$$Q_1 = k\,\theta_1 \left(1 + \frac{\lambda}{2}\right)$$

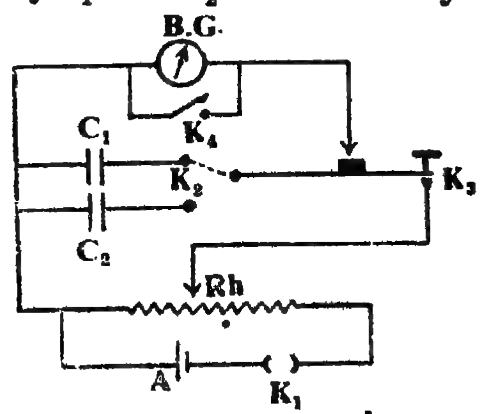

Fig. 48
Comparison of capacities of two condensers

* Capacities can be measured (or compared) with the help of an electric vibrator also. For this experiment see Chapter–13.

and $$Q_2 = k\,\theta_2\left(1 + \frac{\lambda}{2}\right)$$

or $$\frac{Q_1}{Q_2} = \frac{\theta_1}{\theta_2}$$

Thus $$\frac{C_1}{C_2} = \frac{\theta_1}{\theta_2}$$

The experimental arrangement is exactly the same as described in the main experiment (No. 25) given above.

Expt.—26 (d)

Object—*To compare the E. M. F.'s of two given cells by means of a ballistic galvanometer.*

It is a slight variation of the experiment just given above. For this purpose a condenser of capacity C is taken. It is first charged by connecting it to a cell of e. m. f E_1 and then discharge through the galvanometer giving rise to the throw θ_1. Now the condenser is similarly charged by connecting it to the other cell E_2 and then discharged through the galvanometer giving rise to the throw θ_2. Then obviously

$$\frac{E_1}{E_2} = \frac{\theta_1}{\theta_2}.$$

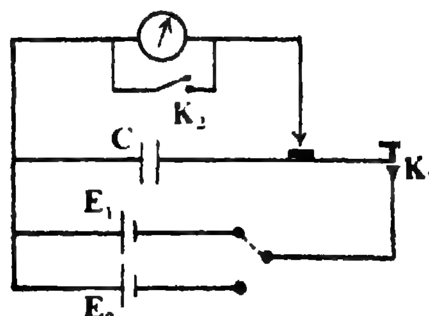

Fig. 49
Comparison of E.M.F.'s of two cells

Expt.—26 (e)

Object—*To determine the internal resistance of a cell by means of a ballistic galvanometer.*

Make the connections as shown in the accompanying figure. Remove the infinity plug of the resistance box (R. B.), and charge the standard condenser of capacity C. The coatings of the condenser are raised to a potential difference equal to the e. m. f. (E) of the cell, since no current is drawn from the cell. Discharge the condenser through the ballistic galvanometer and observe the first throw θ_1. Now

$$Q_1 = EC = k\theta_1$$

Now insert the infinity plug and introduce a resistance of 2 or 3 ohms in the resistance box. Again charge the condenser and discharge it through the ballistic galvanometer and note the first throw θ_2. This time the condenser is

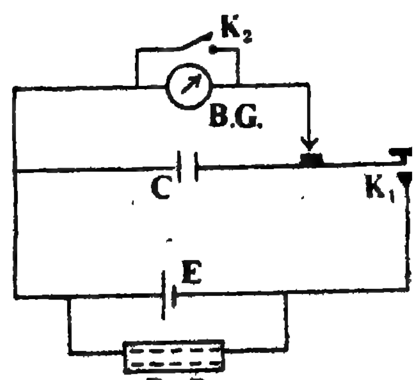

Fig. 50
r of a cell with a ballistic galvanometer

charged to a potential difference of V volts. This time the charge on the condenser coatings

$$Q_2 = VC = k\,\theta_2$$

Thus
$$\frac{E}{V} = \frac{\theta_1}{\theta_2}$$

But
$$\frac{E}{V} = \frac{R + r}{R}$$

where R is the resistance introduced in the box and r is the internal resistance of the cell. Hence

$$\frac{R + r}{R} = \frac{\theta_1}{\theta_2}$$

whence
$$r = \frac{\theta_1 - \theta_2}{\theta_2} \,.\, R.$$

This equation can be employed for the evaluation of the internal resistance of the cell*.

Expt.—26 (f)

Object. To determine the capacity of a condenser by Maxwell's method using a vibrator with A. C. mains.

[**Note.** For its detailed discussion see expt.—38].

If a condenser of capacitance C be charged with an accumulator having an e m. f. of E volts and discharged through a micro-ammeter (or through a sensitive galvanometer whose figure of merit is known) n times per second, the charge flowing through the microammeter per second (i. e. the value of current) is given by—

$$I = nCE \quad \text{or} \quad C = \frac{I}{nE} = \frac{\text{Current}}{\text{frequency} \times \text{voltage}}$$

Example

(1) Figure of merit of the galvanometer = 20 μA.
(2) Steady deflection in the galvanometer = 10 divs.
(3) E. M. F. of the accumulator = 2 volts
(4) Frequency of the A. C. mains = 50 cycles/sec.

Now, current in the galvanometer, $I = 10 \times 20 \times 10^{-6}$ amp.

$$\therefore 50 \times c \times 2 = 10 \times 20 \times 10^{-6}$$

$$\therefore C = 2 \times 10^{-6} \text{ farad, or } = 2\mu f$$

* This method is unsuitable for the determination of the resistance of an accumulator. See exp.—17.

INDUCTANCE

EXPERIMENT—27.

Object. To determine the self-inductance of a given coil by Rayleigh's method.

Apparatus Required. A post-office box, suspended type moving-coil ballistic galvanometer, the given inductance-coil, a decimal ohm box, a piece of resistance wire, a rheostat, a double key, a tapping key, an accumulator, and connection wires.

Formula Employed. The self-inductance (L) of a coil is given by the following formula—

$$L = \frac{r\,T}{2\pi\phi}\ \theta\left(1 + \frac{\lambda}{2}\right)$$

where r = A small resistance (of the order of 0·1, 0·01, etc. ohm).

ϕ = Constant deflection produced in the ballistic galvanometer when r is introduced in the circuit.

T = Time-period of the coil of the galvanometer.

θ = First throw of the galvanometer coil when the inductance (L) is employed in the circuit.

λ = Logarithmic decrement.

The logarithmic decrement is obtained by the formula—

$$\lambda = 2{\cdot}303 \cdot \frac{1}{10} \log_{10} \frac{\theta_1}{\theta_{11}}.$$

where θ_{11} is the eleventh observed swing of the galvanometer.

PRINCIPLE AND THEORY OF THE EXPERIMENT

Self-Inductance.

In 1831 Farady showed that an electric current was produced in a closed circuit whenever the number of lines of magnetic

induction passing through the circuit was changed. This phenomenon is known as *Electro-magnetic Induction* and the current induced thereby is momentary, lasting only so long as the change is taking place.

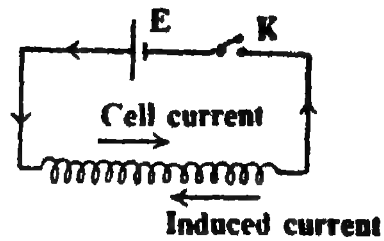

Fig. 51
Self-induction of a coil

Let us consider a circuit as depicted in the accompanying figure. When the key K is closed, the current rises from zero to its maximum value. Thus, there is a change of current in the coil accompanied with a consequential change of flux.* The total flux N linked with the coil is proportional to the current I flowing through it, *i e.*,

$$N = LI$$

and hence the magnitude of the induced e. m. f. (e), which is defined as the rate of change of flux, is given by

$$e = -\frac{dN}{dt} = -L\frac{dI}{dt}$$

In these expressions L is a constant for the circuit and is known as the *coefficient of self-induction* of the circuit, which can be defined in the following two ways :—

(i) The coefficient of self-induction is equal to the effective magnetic flux linked with the circuit when unit current flows through it ; or

(ii) It is numerically equal to the induced e. m. f. in the circuit when the rate of change of the current is unity.

The practical unit of self-inductance is *henry*. A circuit possesses a self-inductance of one henry if the magnetic flux is 10^8 when one ampere current passes through it, or if the e. m. f. induced in the circuit is one volt when the current is changing at the rate of one ampere per second.

Now let the unknown inductance L be connected in one arm

* The *total* number of lines of magnetic induction threading the circuit is called the *magnetic flux* (or simply flux) through the circuit.

of a Wheatstone's bridge as shown in fig.-51, G is a ballistic galvanometer. Let the resistances of the various arms be so adjusted that the bridge is balanced in the usual way for steady currents. Keeping the galvanometer key k_2 still closed, open the cell key K_1. During the decay of the cell current an e. m. f. equal to $-L\frac{dI}{dt}$ will be induced in the coil in the arm CD, where I is the instantaneous value of the current in this arm. This induced e. m. f. will send an instantaneous current in the galvanometer which will consequently give a momentary throw given by*

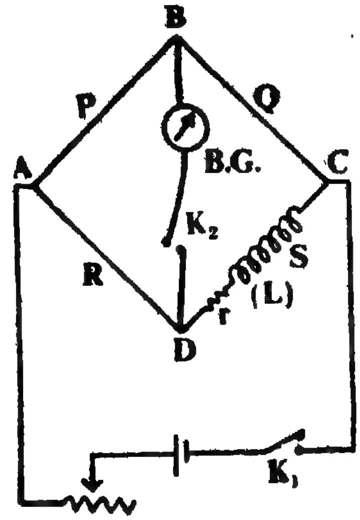

Fig. 52
Principle of Rayleigh's method

$$Q = \frac{cT}{2\pi AH}\,\theta\left(1+\frac{\lambda}{2}\right) \qquad \ldots (1)$$

Let the current in the galvanometer branch be k. $\left(L\frac{dI}{dt}\right)$, where k is a constant which depends on the value of the resistances. Under such circumstances the total quantity of electricity which passes through the galvanometer due to this cause is

$$Q = \int_0^t k\,L\frac{dI}{dt}\,.\,dt = k\,L\int_0^{I_0} dI = k\,L\,I_0 \qquad \ldots (2)$$

where I_0 is the maximum steady current flowing through the arm CD. Thus from (1) and (2) we have

$$\frac{cT}{2\pi AH}\,\theta\left(1+\frac{\lambda}{2}\right) = k\,L\,I_0 \qquad \ldots (3)$$

To eliminate k and I_0, a measurable small potential change is introduced into the arm CD. This can be brought about by adding a small resistance r (of the order of 0·1 ohm) in the arm CD. Assuming that the current I_0 will not be appreciably affected by this small change, the potential difference introduced in the arm CD is I_0 r. This causes a current k I_0r to flow in the galvanometer producing a steady deflection ϕr. Now for a steady current k I_0r the couple on the coil is AH . k I_0, so that

$$AH\,.\,k\,I_0 r = c\phi \qquad (4)$$

* The theory of the ballistic galvanometer has been thoroughly discussed in expt.—26. For the derivation of this formula see page 152.

where c is restoring couple in the suspension per unit angular displacement. Combining the equations (3) and (4) we have

$$L = \frac{rT}{2\pi\phi}\,\theta.\left(1 + \frac{\lambda}{2}\right) \qquad \ldots \qquad (5)$$

Equation (5) is employed for the evaluation of L.

Method

(i) Before starting the experiment set up the ballistic galvanometer properly so that the coil moves freely in the clearance space provided for it. Throw light on the mirror of the galvanometer and get a well-defined, bright spot of light on the scale.

(ii) Connect the given coil (see fig.–53), whose self-inductance is to be determined, in the fourth arm S of the post-office box. In this arm introduce also a decimal ohm box capable of giving resistances of the order of 0·1, 0·01, etc ohm.

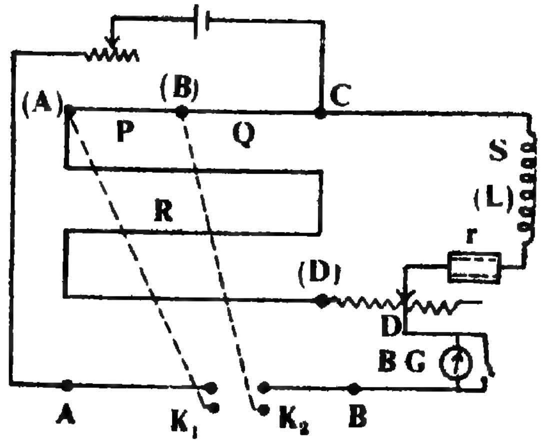

Fig. 53
Rayleigh's method for the determination of L.

(iii) Fix the ratio P : Q at 1 : 1 or 10 : 10 (i. e., P=Q) whichever order is nearer to the resistance of the inductance coil. Thus, to secure an exact balance, put a platinoid wire in series with the resistance arm R such that its length can be varied by slipping it through a binding screw.

(iv) Make other connections as shown in the figure. If the deflection in the galvanometer is excessive, use a rheostat in the cell circuit. Adjust the decimal-ohm box to zero value and obtain a perfect balance in the usual way for steady currents.*

Having obtained perfect balance for steady currents, break first the cell circuit and then immediately after it the galvanometer circuit. Note down the successive throws of the galvanometer for the evaluation of λ. Thereafter determine the time-period of the galvanometer with an accurate stop-watch.

* If available, a special double key can be employed for this experiment. In that case the keys provided with the P. O. box can be dispensed with. Study the connections with this double key.

(v) Bring the galvanometer coil to rest with the help of the tapping key. Now test the bridge again, and if found imperfect, restore perfectness by altering the length of the platinoid wire. Next introduce a small resistance, say 0·01 ohm, in the decimal ohm box, and by operating the keys in the order K_1, K_2, obtain a steady deflection of the spot of light.*

(vi) Calculate the value of the logarithmic decrement and then the value of the self-inductance L of the coil.†

Observations

[A] *Readings for the determination of θ, ϕ and λ.*

S. No.	Successive throws of the galvanometer				r	Constant deflection φ	Logarithmic decrement λ
	θ_1	θ_2	θ_3	....			
1.							
2.							
3.							

[B] *Readings for the determination of the time-period of the galvanometer.*

S. No.	No. of oscillations	Time taken	Time-period
1.	25	...min. ...sec.	
2.	25	,,	.........sec.
3.	25	,,	
4.	25	,,	

* The experiment may be repeated thrice by taking different values of r.

† The unit of L will be henries if r is in ohms, T in seconds, and θ and ϕ in the same units, *i. e.*, cm. or mm.

Calculations

$$\lambda = 2{\cdot}303 \frac{1}{10} \log_{10} \frac{\theta_1}{\theta_{11}}$$

Set I

$$L = \frac{r\,T}{2\,\pi\,\phi} \cdot \theta \left(1 + \frac{\lambda}{2}\right)$$

$$= \ldots\ldots\ldots\ldots \text{ henry}$$

[**Note.** Calculate similarly for other sets also.]

$\therefore$ Mean value of $L = \ldots$henry.

Result. The self-inductance of the given coil $= \ldots$ henry.

Precautions and Sources of Error

(1) Level the galvanometer properly so that the coil moves freely in the clearance space provided in between the pole-pieces. Adjust the scale normal to the light beam by seeing that the deflections on both sides of the mean position are equal when the current is reversed.

(2) All resistances used in the experiment should be non-inductive. *Connection wires should not be coiled* but they should be short and straight. This would avoid spurious inductive effects.

(3) To secure maximum sensitiveness of the bridge all the four arms of the bridge should have nearly equal resistances. For this purpose, make ratio arms equal and choose P : Q such that this order is nearer the resistance of the inductance coil.

(4) If a moving-coil type of ballistic galvanometer is used and P, Q, R and S, are small, the galvanometer may give but a small deflection as it is shunted by these resistances. This is specially so when the resistance of the inductance coil is small. Measurable and reliable results can be obtained if the galvanometer circuit is broken the moment the discharge is passed through it. To ensure this use the special double key.

(5) Use an accumulator for the constancy of the e. m. f. and if the deflection with this is excessive, use a rheostat in the cell circuit.

(6) The balance point with the steady currents should be accurately determined. For this purpose use a resistance wire (say, of platinoid) in series with the resistance arm R of the bridge.

(7) Since the inductance coil is generally made of copper, whose temperature coefficient of resistance is large, the current should be passed for short duration only, otherwise the balance point shall continually vary.

(8) In the derivation of the formula given above, it has been assumed that the maximum current flowing in the L-arm is constant both for steady and transient currents. To achieve this condition the *resistance r should* be sufficiently small.

(9) While taking readings for the determination of the time-period of galvanometer, its circuit should be kept *open*.

EXPERIMENT—28

Object. To determine the mutual inductance of two coils with the help of a ballistic galvanometer.

Apparatus Required. An accumulator, moving-coil ballistic galvanometer, the two given coils, resistance box, rheostat, plug type reversing commutator, and tapping keys.

Formula Employed. The mutual inductance (M) of two coils is given by the following formula—

$$M = \frac{rT}{2\pi\phi} \cdot \theta \left(1 + \frac{\lambda}{2}\right)$$

where r = Small resistance to get a steady deflection ϕ in the galvanometer.

T = Time-period of the galvanometer.

θ = First throw of the galvanometer due to the operation of the mutual inductance of the two coils.

λ = Logarithmic decrement.

PRINCIPLE AND THEORY OF THE EXPERIMENT

The coefficient of mutual induction of two coils may be defined as the magnetic flux which is linked with one coil when unit current circulates through the other. Thus, if a current of maximum strength I_0 passes through one of the coils, whose mutual inductance is M, MI_0 lines of magnetic force pass through the second coil, and while the current grows in the primary, the flux threading the secondary is changing. Therefore an induced e. m. f. is set up in the secondary during the time of growth of the primary current. This e. m. f. is numerically equal to the rate of

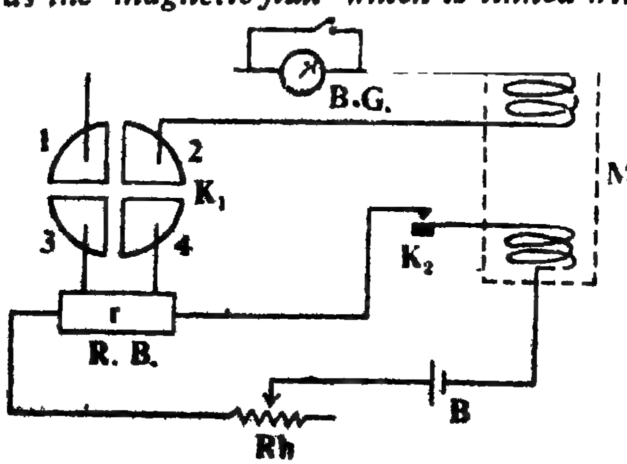

Fig. 54
Determination of the mutual inductance

change of magnetic flux in the secondary*, i. e., it is equal to $\frac{d}{dt}$ (MI), where I is the instantaneous value of the current in the primary coil during the growth of that current.

If L be the coefficient of self-inductance of the secondary coil, and I′ be the current in the secondary corresponding to a current I in the primary, we have a further e. m. f. in the secondary due to the self-inductance of the former. This e. m. f. is numerically equal to $L\frac{dI'}{dt}$. Hence if R be the total resistance of the secondary coil circuit, we have

$$I'R = L\frac{dI'}{dt} \pm M\frac{dI}{dt} \qquad \ldots \qquad (1)†$$

Now Q, the quantity of electricity passing through the secondary is $\int I'.\,dt$ where the integration is carried out over the whole time during which I rises to the steady value I_0. Thus

$$Q = \int I'.\,dt = \int \frac{L}{R}.\,dI' \pm \int \frac{M}{R}dI \qquad \ldots \qquad (2)$$

The value of I′ at the commencement and at the end of this integration is zero, hence

$$\int \frac{L}{R}.\,dI' = 0$$

and

$$Q = \int_0^{I_0} \frac{M}{R}.\;dI = \frac{MI_0}{R} \qquad \ldots \qquad (3)$$

But for a moving-coil ballistic galvanometer‡

$$q = \frac{cT}{2\pi AH}\,\theta\left(1 + \frac{\lambda}{2}\right)$$

Thus

$$\frac{MI_0}{R} = \frac{cT}{2\pi AH}\theta\left(1 + \frac{\lambda}{2}\right) \qquad \ldots \qquad (4)$$

* This statement is valid for those coils only, which have non-magnetic cores. If there is an iron core the value of the flux is not proportional to the current.

† The ± sign indicates that the direction of the e. m. f. due to the coupling of the coils depends on the manner in which they are wound in either sense.

‡ For the derivation of this formula see the Theory of exp.—26.

Now let the secondary coil in series with the ballistic galvanometer be connected across a small resistance r (fig.–54) already included in the primary circuit. The potential drop established at the ends of r is $I_0 r$, and hence the current through the galvanometer is $I_0 r/R$ since r is very small compared with the resistance of the galvanometer. If this causes a steady deflection ϕ we have

$$\frac{I_0 r}{R} = \frac{c}{AH}\phi \qquad \ldots \qquad (5)$$

From (4) and (5) we have

$$M = \frac{rT}{2\pi\phi}\,\theta\left(1 + \frac{\lambda}{2}\right) \qquad \ldots \qquad (6)$$

This is the required formula for the evaluation of M.

Method

(i) First of all adjust properly the ballistic galvanometer so that the coil hangs symmetrically in the clearance space provided for it and does not touch the pole-pieces when it is oscillating. Adjust the electric lamp so that a bright image of a vertical part of the lamp filament is focussed on the scale. Adust the scale normal* to the beam.

(ii) Set up the apparatus as shown in fig.–54. Adjust the resistance box to a small resistance r (which may be of the order of 0·1 or 0·01 ohm), Connect the segments 1 and 2 of the reversing commutator, so that the ballistic galvanometer is in direct circuit with the secondary. Press the key K, and observe the successive throws of the galvanometer coil.

(iii) Bring the galvanometer coil to rest with the help of the tapping key connected across it. Now make the connection between 1 and 3, 2 and 4 and observe the steady deflection produced in the galvanometer.

(iv) Determine the time-period T of the galvanometer and calculate† M with the help of equation (6) given above.

Observations

[Note. For this and subsequent items see the previous experiment on the determination of the self-inductance of a coil.]

* The normality of the scale to the beam can be tested by seeing that the deflections on both sides of the mean position of the spot of light are equal when the current is reversed.

† The experiment may be repeated with different values of r.

EXPERIMENT—29

Object. To determine the angle of dip in the laboratory at ....* by means of an earth inductor.

Apparatus Required. An earth-inductor, a ballistic galvanometer, compass needle, tapping key, and connecting wires.

Description of the Apparatus. An earth-inductor consists of a coil of several hundred turns of insulated copper wire wound on a ring having a large face area. It is so mounted that it can be rotated about an axis in its plane, the rotation being confined to 180° by means of stops provided in the outer frame. The ends of the coil are connected to two semicircular metallic strips separated from each other and fixed on the axis They are called commutators. There are two springs called the collectors which lightly press upon the commutating strips from opposite sides. The coil is set in rotation through 180° by releasing it from a catch which is provided for the purpose.

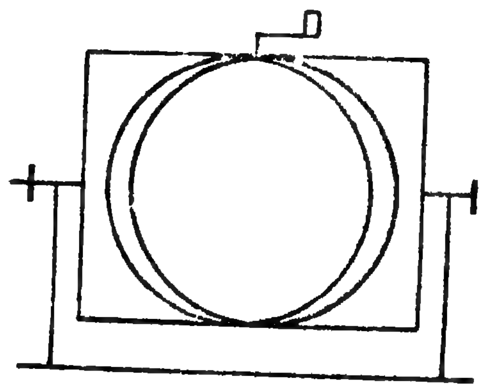

Fig. 55
Earth-inductor

Formula Employed. The angle of dip (ϕ) is calculated with the help of the formula—

$$\tan \phi = \frac{d_2}{d_1}$$

where d_2 = Deflection as read on the scale when the earth-inductor is rotated in the earth's *Vertical* field (V) alone.

d_1 = Deflection on the scale when the earth-inductor is rotated in the earth's *Horizontal* field (H) alone.

PRINCIPLE AND THEORY OF THE EXPERIMENT

If a closed coil of wire is rotated in a magnetic field (say, the earth's magnetic field), the number of lines of force embraced by the coil continually varies, and consequently an induced current shall circulate in the coil.

Name the place where the experiment is conducted.

Let A be the effective face area of the coil rotating in the uniform field of intensity F, and let AB (fig.-55) be the original position (perpendicular to the lines of force due to F) of the coil. The effective flux through the coil in this position is nAF where n is the number of turns in the coil. The effective flux N through the coil when it has rotated through an angle θ into the position CD is given by—

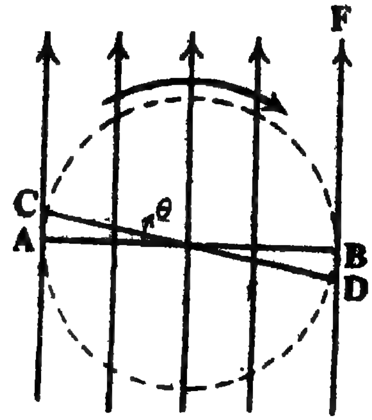

Fig. 56
Induction in a rotating coil

$$N = n\ AF \cos\theta$$

$$\therefore \text{ Induced e.m.f., } e = -\frac{dN}{dt}$$

$= n\ AF\ \sin\theta \dfrac{d\theta}{dt}$, and since $d\theta/dt$ is the angular velocity ω—

$$e = n\ AF\ \omega \sin\theta \qquad \ldots \qquad (1)$$

From equation (1) it is clear that during the first half revolution of the coil the induced e. m. f. is in one direction, and during the second half of the revolution it is in the opposite direction, i. e., the e. m. f. is alternating in nature.*

If the ends of the coil are connected to a ballistic galvanometer, the charge set in motion is given by

$$Q = \sum_{o}^{T/2} I \,.\, dt = \sum_{o}^{T/2} \frac{e}{R}\, dt = \sum_{o}^{T/2} \frac{n\ A\ F}{R} \,.\, \sin\ \theta \,.\, \frac{d\theta}{dt} . dt$$

or $$Q = \frac{n\ A\ F}{R} \int_{o}^{\pi} \sin\theta \,.\, d\,\theta = \frac{2\,n\,A\,F}{R} \qquad \ldots \qquad (2)$$

If this charge produces a throw θ in the ballistic galvanometer

$$Q = k\,\theta \left(1 + \frac{\lambda}{2} \right) = \frac{2\,n\,A\,F}{R} \qquad \ldots \qquad (3)$$

If the earth-inductor is successively rotated in the horizontal (H) and the vertical (V) magnetic field of the earth and the

* This simple principle of the earth-inductor is the basis of huge alternators employed for the production of electricity on a commercial scale.

corresponding throws obtained in the ballistic galvanometer are respectively θ_1 and θ_2, then

$$\frac{2\,n\,A\,H}{R} = k\theta_1\left(1 + \frac{\lambda}{2}\right)$$

and $$\frac{2\,n\,A\,V}{R} = k\theta_2\left(1 + \frac{\lambda}{2}\right)$$

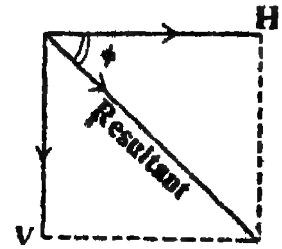

Fig. 57
Earth's total field

whence $$\tan\phi = \frac{V}{H} = \frac{\theta_2}{\theta_1} = \frac{d_2}{d_1} \qquad \ldots \qquad (4)$$

where ϕ is the angle of dip and d_1, d_2 are the respective deflections observed on the galvanometer scale. Thus by observing the values of d_1 and d_2 the angle of dip can be calculated with the help of equation (4).

Method

(i) Set the ballistic galvanometer properly so that its coil moves freely in the clearance space provided for it. Form a sharp image of the lamp filament on the scale. Adjust the scale normal to the beam of light by seeing that the deflections on both sides of the mean position are equal when the current is reversed*. Connect a tapping key across the galvanometer so that its coil can be quickly brought to rest. Also connect the galvanometer by means of a twin flexible cord to the binding terminals of the earth-inductor.

(ii) Now with the help of a compass needle draw, with a piece of chalk-stick, two lines on the table parallel and perpendicular to the magnetic meridian. Then arrange the earth-inductor, when against one of the stops, so that *its axis of rotation is vertical and its face is perpendicular to the magnetic meridian.*† By releasing the catch rotate the coil through 180° and note the deflec-

* If, instead of the lamp and scale arrangement, the galvanometer is provided with a telescope-scale arrangement, then focus the telescope on the reflected image of the scale and set the vertical cross-wire on the zero mark of the scale. If the cross-wire is set on any other convenient scale division, this reading should be invariably noted down.

† It should be carefully borne in mind that the two successive adjustments of the earth-inductor have been so accomplished that it embraces the flux due to the horizontal and vertical components of the earth's magnetic field respectively. Before noting the deflections make sure that they are due to single fields. A faulty adjustment shall result in an intermingled field.

tion d_1 of the spot of light on the scale (or, in the case of telescope-scale arrangement, note the division coincident in the deflected position with the cross-wire of the telescope).

(iii) Bring the galvanometer coil to rest with the help of the tapping key. Now set up the earth inductor *with its axis of rotation lying in the magnetic meridian and its face horizontal.* Quickly rotate the coil as before and note the first throw d_2 on the scale. Calculate the angle of dip ϕ. Repeat the process several times.

Observations

[A] *Readings for the determination of deflection due to H.*

S. no.	Initial reading	Final reading	Throw (d_1)
		Mean	... ... cm.

[B] *Readings for the determination of deflection due to V.*

[Note. Make a similar table.]

Calculations

$$\text{Angle of dip, } \phi = \tan^{-1} \frac{V}{H}$$

$$= \tan^{-1} \frac{d_2}{d_1}$$

Result. The angle of dip as determined in the laboratory at ... ... = ...

[Standard value = ... Error = ...]

Precautions and Sources of Error

(1) The galvanometer should be properly set so that the coil moves freely in the clearance space between the pole-pieces of the magnet. The scale should be adjusted normal to the beam of light.

(2) To damp the unnecessary oscillations of the galvanometer coil a tapping key should be connected across its terminals.

(3) The earth inductor should be placed a bit away from the galvanometer, connections to which should be made with the help of a twin flexible wire.

(4) While observing the deflections, be careful that the earth-inductor first embraces the flux due to the horizontal component and second time due to the vertical component of the earth's magnetic field. *During a single rotation there should be no intermingling of the two magnetic fields.*

(5) The time of rotation of the earth-inductor should be small compared to the time-period of the ballistic galvanometer.

ADDITIONAL EXPERIMENTS

Expt.—29 (a)

Object. To determine the value of H in the laboratory with the help of an earth-inductor and a Hibbert's magnetic standard.

[**Note.** *Hibbert's Magnetic Standard* is shown in the diagram given below. It is a special type of magnet, which is made from a steel block in which a cylindrical groove is cut in the centre. The central part is the north pole and the outer one is the south pole. The lines of force run radially in the space wherein the field is uniform. A coil of insulated copper wire is wound on a brass cylinder, the two ends being connected to two terminals at the top. When the cylinder is dropped in the standard field, it cuts the lines of force normally. An induced e. m. f. is generated in the circuit which produces a throw in the ballistic galvanometer. The value of the flux embraced by the coil is known by calibrating it with a standard solenoidal inductor and is given by the manufacturers and is written on the body of the instrument.]

As shown in the figure, the earth-inductor, Hibbert's standard and the ballistic galvanometer are all connected in series. Rh is a rheostat which is used to cut down the deflection if it is excessive*. The Hibbert's standard and the earth-inductor are placed at a considerable distance from each other and also from the ballistic galvanometer. The earth-inductor is operated in the earth's field H, as explained above. The throw θ_1 is observed. Thus

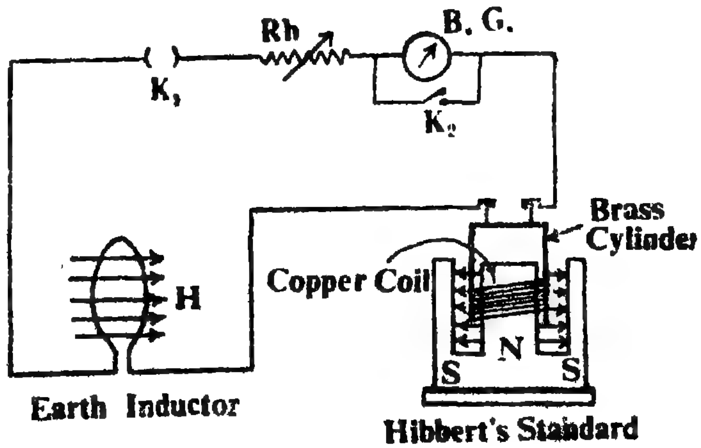

Fig. 58
H with a Hibbert's magnetic standard

*In the above diagram K_1 is a plug key which is unplugged when the adjustments with either the earth-inductor or with the Hibbert's standard are being done, otherwise the galvanometer coil will be unnecessarily deflected to and fro.

$$Q = k\,\theta_1 \left(1 + \frac{\lambda}{2}\right) = \frac{2\,n\,A\,H}{R}$$

where R is the resistance of *entire* circuit.

Now the Hibbert's standard is operated and the deflection θ_2 is noted. If F be the flux given by the standard

$$k\,\theta_2 \left(1 + \frac{\lambda}{2}\right) = \frac{F}{R}$$

From these two equations we have

$$H = \frac{F}{2\,n\,A} \cdot \frac{\theta_1}{\theta_2}$$

All the quantities on the right are known, hence H can be evaluated.

Expt.—29 (b)

Object. To determine the ballistic constant of a suspended type moving-coil ballistic galvanometer by means of an earth-inductor.

From the above it is clear that

$$k\,\theta \left(1 + \frac{\lambda}{2}\right) = \frac{2\,n\,A\,H}{R}$$

From this expression k, the ballistic constant, can be evaluated if all the other quantities are known. If a resistance is at all necessary in this case, the rheostat should be replaced by a resistance box so that a known resistance can be inserted. R in the above formula is the total resistance of the entire circuit.

EXPERIMENT—30

Object. To determine the magnetic field between the pole-piece of an electro-magnet with the help of a search coil and a ballistic galvanometer, using an earth inductor* for the calibration of the galvanometer.

Apparatus Required. The given electro-magnet, battery, am. meter, rheostat, a ballistic galvanometer, earth-inductor, plug keys, and a search coil.

Description of the Apparatus. *A search coil* is merely a small flat circular coil consisting of a large number of turns of fine insulated copper wire wound on an ebonite bobbin attached to a handle.

* For performing this experiment the earth-inductor can be replaced either by (i) *Hibbert's magnetic standard* (see page 175), or by a (ii) *standard solenoidal inductor* (see page 158). The actual method of carrying out the experiment can be easily worked out by following the procedure given in this experiment.

The ends of the copper coil are attached to two binding terminals provided at the top of the handle The coil is made narrow for mainly two reasons—firstly, most of the fields to be measured are un-uniform, secondly, the coil is often inserted into narrow gaps for such measurements. The exact number of turns and the face area of the coil are given by the makers.

Formula Employed. The magnetic field (F) between the pole-pieces of an electro-magnet is given by—

$$F = \frac{2\,na}{n'\,a'} \cdot \frac{\theta'}{\theta} \cdot H$$

where
- n = Number of turns in the earth-inductor
- a = Face area of the earth-inductor
- n' = Number of turns in the search coil
- a' = Face area of the search coil
- θ = Throw of the ballistic galvanometer with the inductor
- θ' = Throw of the galvanometer with the search coil
- H = Horizontal component of earth's magnetic field.

PRINCIPLE AND THEORY OF THE EXPERIMENT

Electrical connections for various components of the apparatus are depicted in fig.-59. Let the electro-magnet be energised, and

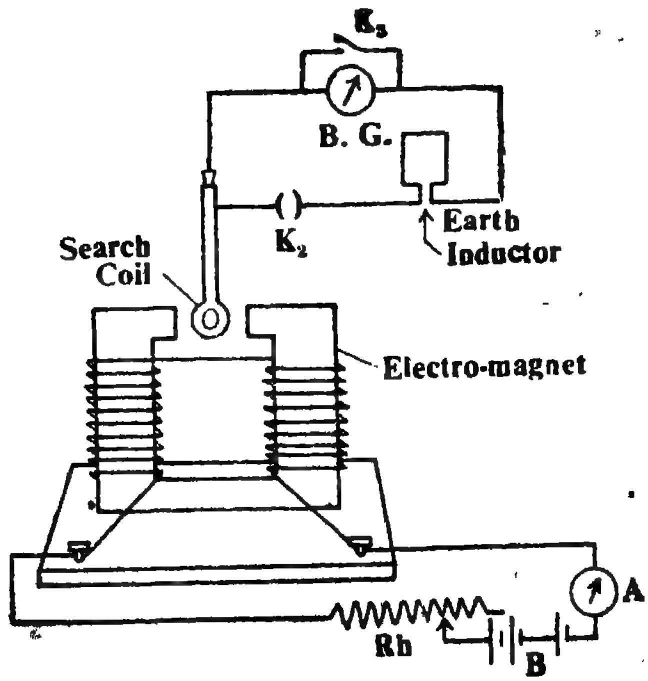

Fig. 59
Measurement of field of a horse-shoe magnet.

let the search coil be inserted between the pole-pieces with its face

perpendicular to the magnetic lines of force. The search coil is in series with the earth inductor and the ballistic galvanometer (B. G.) Let the search coil be rapidly withdrawn. The change of magnetic flux N is given by

$$N = n'a'F \qquad \dots \quad (1)$$

where F is the intensity of the magnetic field, and n', a' are respectively the number of turns and the face area of the search coil. If I be the instantaneous value of the current, the charge circulating in the coil is given by

$$Q = \sum_{o}^{t} I.\, dt = \frac{1}{R}\sum_{o}^{t}\frac{dN}{dt}.\ dt = \frac{N}{R} = \frac{n'a'F}{R} \qquad \dots \quad (2)$$

where R is the total resistance of the search coil circuit. If the throw produced in the galvanometer be θ', then

$$Q = k\theta'\left(1 + \frac{\lambda}{2}\right) = \frac{n'\,a'\,F}{R} \qquad \dots \quad (3)$$

Now, to eliminate k the earth-inductor is employed. For this purpose the earth-inductor is placed so that the axis of rotation is vertical and its face is perpendicular to the magnetic meridian. Under this circumstance the earth-inductor embraces magnetic flux due to earth's horizontal field (H) alone. Now the earth-inductor is rapidly rotated through 180°. If the corresponding throw in the ballistic galvanometer be θ, we have as above

$$k\,\theta\left(1 + \frac{\lambda}{P}\right) = \frac{2naH}{R} \qquad \dots \quad (4)$$

since flux changes, in this case, from naH to $-$naH, the faces of the inductor being reversed due to rotation.

From (3) and (4) we have

$$F = \frac{2\,na}{n'\,a'}\ .\ \frac{\theta'}{\theta}.\ H \qquad \dots \quad (5)$$

This equation can be utilised to evaluate F, the magnetic field between the pole-pieces of the electro-magnet, if the constants of the earth-inductor and the search coil are known and other quantities are measured.

Method

(i) First of all adjust the ballistic galvanometer so that the coil hangs freely in the clearance space. Secure a bright image of the straight portion of the filament on the scale. Now set up the apparatus* as shown in fig -59.

---

* Insert a plug-key (K_2) in the search coil circuit as shown in the figure. This may be unplugged when the adjustments are being made, so that the galvanometer coil may not be unnecessarily get deflected to and fro during this process. The key may be plugged in when the apparatus is ready for the recording of observations.

(ii) Now by means of a compass needle set the earth-inductor in such a way *that its axis of rotation is vertical and its plane is normal to the magnetic meridian.* Under this circumstance the coil, during its revolution, shall embrace magnetic flux due to H.

(iii) Allow a suitable current to flow in the electro-magnet coils Place the search coil in between the pole-pieces of the electro-magnet such that its plane is parallel to the plane of the pole-pieces and the magnetic lines of force traverse the face of the coil normally.

(iv) Now close the key K_2, and rapidly withdraw the search coil. Note the throw of the ballistic galvanometer.

(v) Bring the galvanometer coil to rest with the help of the tapping key. By releasing the catch of the earth-inductor rotate it through 180° and note the first throw of the galvanometer. Then calculate the value of F with the help of equation (5) given above.

(vi) Now after the current strength I in suitable steps and determine the corresponding field-strength F. Then draw a graph between F (depicted on the y—axis) and I (depicted on the x—axis).

Observations

Readings for the determination of θ and θ'.

(a) Constants of the search coil.

(i) No. of turns = ...
(ii) Face area = ... sq. cm.

(b) Constants of the earth-inductor.

(i) No. of turns = ...
(ii) Face area = ... sq. cm.

(c) H — ... oersted

S. No.	Current in the field coils (I)	Readings with the search coil			Readings with the earth-inductor		
		First position	Second position	θ'	First position	Second position	θ
1.	...amp.	...cm. ...cm. ...cm.	...cm. ...cm. ...cm.				

Calculations

Set I

(i) Mean $\theta' =$ ....cm. ; (ii) Mean $\theta =$ .... cm.

$$F_1 = \frac{2\,n\,a}{n'\,a'} \cdot \frac{\theta'}{\theta} \cdot H$$

$=$ oersted.

[**Note.** Make similar calculations for other sets. Draw a graph between F and I.]

Result. The strength of the magnetic field between the pole-pieces of the given electro-magnet for different values of current flowing in the field-coils is given below—

S. No.	Current strength (I)	Field strength (F)
1.	......amp.	......oersted
⋮		
⋮		

The variation of field strength with current in the field-coils is depicted in the graph* attached herewith.

Precautions and Sources of Error

(1) Before using the ballistic galvanometer release its coil and level the instrument in such a way that the coil is free to move in the clearance space provided for it. The beam of light falling on the scale should be adjusted normal to it. This can be achieved by seeing that the deflections of the spot of light on both sides of the mean position are equal when the current is reversed.

(2) In order to bring the coil quickly to rest insert a tapping key across the terminal of the galvanometer. A plug key should also be included in the search coil—earth inductor circuit. This should be kept unplugged when the search coil and earth inductor

* This graph will be a straight line.

are being adjusted, thereby eliminating unnecessary oscillations of the galvanometer coil.

(3) The search coil should be so placed in the gap of the pole-pieces that its face is parallel to the face of the poles. In order to utilise the induction effects due to the earth's horizontal field alone, the earth inductor should be so placed that its axis of rotation is vertical and its face is normal to the magnetic meridian.

(4) The ballistic galvanometer and the earth-inductor should be situated at a fairly large distance from each other and from the electro-magnet. For this purpose it is advisable to use twin flexible cord to make connections with the ballistic galvanometer.

(5) The search-coil should be rapidly withdrawn from the gap in between the pole-pieces and removed away from them.

(6) The time of rotation of the earth-inductor should be small in comparison to the time-period of the galvanometer.

11

MEASUREMENT OF POTENTIAL DIFFERENCE

POTENTIOMETER*

A potentiometer is essentially a piece of apparatus by means of which e. m. f.'s are compared. If one of the two e. m. f's is known, the other may be determined by comparison with the known one, and thus the potentiometer is used for the measurement of e. m. f.'s by comparison with a standard e. m. f. It may also be applied to the measurement of current and resistance by methods which are described and discussed below.

The principle of the potentiometer is illustrated in fig.–60, which shows the connections of the most elementary form. A battery B sends a current through a slide-wire AC of uniform cross-section, R being a variable resistance to limit the slide-wire current. B_1 is a cell whose e. m. f is to be measured. This is connected in series with a galvanometer and a jockey J which slides along the wire. The polarity of B_1 is as shown in the figure.

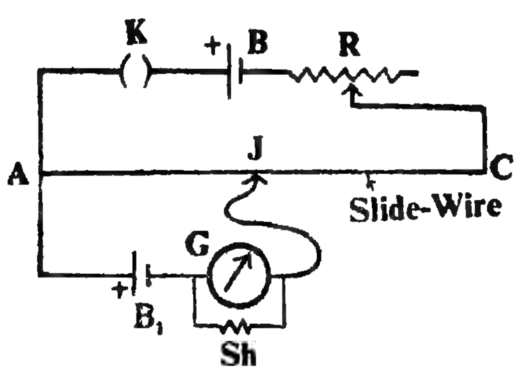

Fig. 60
Principle of a potentiometer

Suppose that ρ is the resistance *per unit length* of the wire, and that i is the current when the jockey is not pressed. Then if the length AJ is l, the voltage drop across AJ is $i \rho l$.

If the jockey J is now pressed, a current will flow through the galvanometer in the direction AGJ if the voltage drop across the length l of the slide-wire is greater than the e. m. f. of the cell B_1. If the e. m. f. of the cell is greater than the potential difference between A and J, the current in the galvanometer will flow in the

* For further study of this apparatus read author's book "A Critical Study of Practical Physics and Viva-Voce."

reverse direction. If these two are equal no current will flow through the galvanometer.*

Suppose now that the e. m. f. of two cells B_1 and B_2 are to be compared. Then, the first cell B_1 is inserted, as shown in fig.-60, in series with the galvanometer, and the jockey J is adjusted on the slide-wire until no current flows through the galvanometer. Let that balancing length be l_1. B_1 is then replaced by B_2 and the jockey again adjusted until no current flows through the galvanometer. Let this new length be l_2.

Then, if E_1 = e. m. f. of cell B_1

E_2 = e. m. f. of cell B_2

we have† $E_1 = i\,\rho\,l_1$ and $E_2 = i\,\rho\,l_2$

so that
$$\frac{E_1}{E_2} = \frac{l_1}{l_2}$$

A scale is provided in this ordinary form of the potentiometer, so that l_1 and l_2 may be read off, and the ratio l_1/l_2 gives the ratio of the two e. m. f.'s as shown above.

If one of the cells (say B_1) is a standard cell‡ of known e. m. f., the e. m. f. of the cell B_2 given by
$$E_2 = \frac{l_2}{l_1} . E_1$$

In the above experiment it is essential that the supply battery B is of ample capacity so that the current i in the slide-wire may remain constant throughout the test. A resistance should be connected in series with the galvanometer—or a shunt used—for protection during the initial stages of adjustments of the jockey J, this shunt being cut out as the position of zero deflection is reached. Such a resistance is also necessary in order that no appreciable current shall be taken from the standard cell, when inserted in the galvanometer branch, during the preliminary adjustment of the jockey. *The e. m. f. of the standard cell cannot be relied upon if it is allowed to give any appreciable current.*

It should be noted that when the potentiometer is balanced no current is passing through the cell under test, so that the e. m. f. measured is the open circuit e. m. f. of the cell.

---

* The cell B_1 is connected in such a way that it *opposes* the passage of the current due to the potential difference between A and J.

† Obviously, both E_1 and E_2 must be less than the e. m. f. of the supply battery B.

‡ For instance, it may be a Weston cadmium cell whose e. m. f. at 20°C is equal to 1·0184 volts.

Obviously, in the above ordinary form of the potentiometer the accuracy of measurement depends to a large extent upon the accuracy with which ratio l_1/l_2 can be determined. For making such a comparison, the accuracy of the determination depends on the accuracy of obtaining the balance point. If instead of using a 1 metre potentiometer (as in the above case), a wire of 10 metre length be used, then each cm. of wire has a potential drop equal to one-tenth the drop in the simpler potentiometer, *i. e.*, a movement of 1 mm. in the single wire would correspond to 1 cm. movement in the 10 metre instrument. Hence, by using a 10 metre potentiometer the true balance point can be very easily and more accurately located.

But the use of many wires involves two serious difficulties, (i) the apparatus becomes cumbersome, and (ii) it is difficult to get a very long wire of absolutely uniform cross-section throughout its entire length—a condition which is essential for the precise performance of the instrument as demanded by theoretical considerations given above. In the modern forms of the potentiometer designed for precise measurements, these difficulties have been overcome and the effect of a very long wire is obtained by connecting a number of resistance coils in series with a comparatively short slide wire, as given below.

This pattern (fig.–61) of the potentiometer consists of ten coils arranged in line with one stretched wire of platinoid, 50 cms. in length and of uniform cross-section, and its resistance is adjusted to 1 ohm. The resistance of each coil is equal to that of the wire. On the scale provided along the slide wire each cm. is indicated as two.

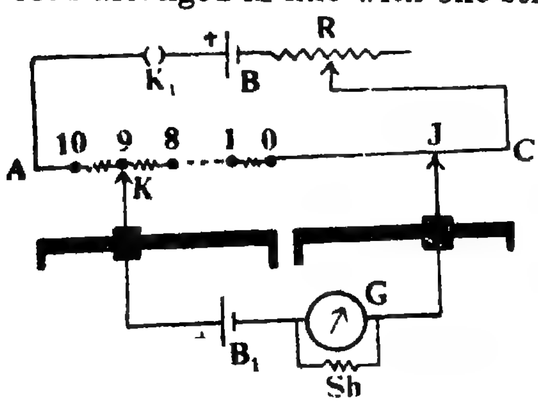

Fig. 61
Connections with 11-wire potentiometer

The special feature of the instrument is that not only the contact maker J, connected with the negative terminal of B_1, but its positive terminal connected to another contact maker K, moving over the studs of the coils, is also movable. By taking 10 coils and 18·4 cms. on the wire, against a Weston cadmium standard cell the potentiometer wire is accurately calibrated* and then it indicates 1 millivolt per cm.

* If only a Daniell cell is available in the laboratory as a standard cell (e. m. f. = 1·1 volt), its e. m. f. can be balanced on all the eleven resistances including the wire. Thus, the fall of potential across one coil will be very nearly equal to $1 \cdot 1/11 = 0 \cdot 1$ volt, and the wire indicates as before 1 millivolt per cm.

Crompton's Potentiometer

It is a compact and precision type of potentiometer in which the sensitivity of the instrument is considerably increased, and at the same time its accuracy is not sacrificed. A simplified figure, depicting the essential features, is depicted in fig -62.

A graduated slide wire is connected in series with fourteen (or more) coils, each of which has a resistance exactly equal to that of the slide wire (of the order of 10 ohms). There are two contact

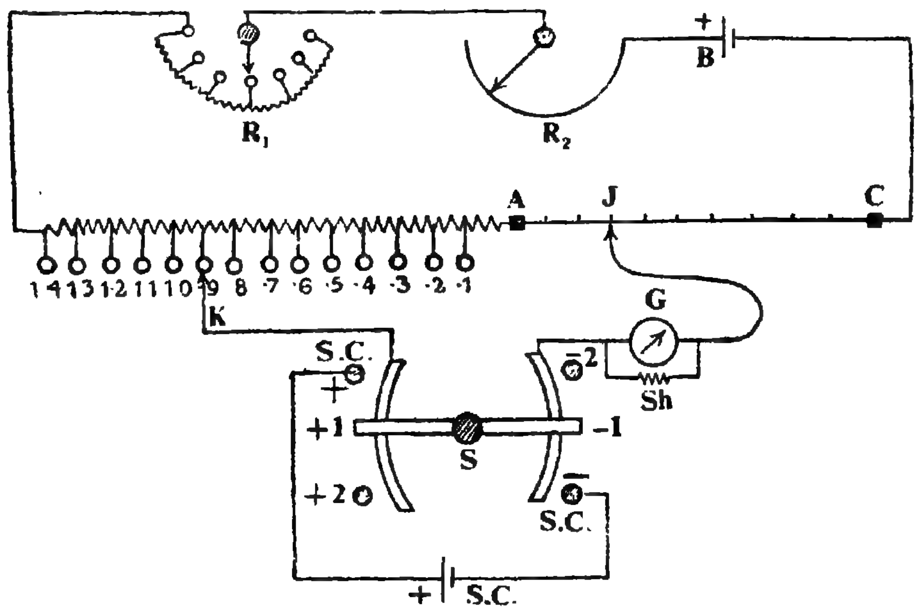

Fig. 62
Crompton's potentiometer

makers J and K, sliding along the slide wire, and the studs of the coils respectively, R_1 and R_2 are two variable resistances, the former consisting of a number of coils used for coarse adjustment of the potentiometer current and the latter taking the form of a slide wire for fine adjustment.

The galvanometer G is connected to a multiple circuit switch S, with the help of which either the standard cell (S. C.), or other E. M. F.'s to be measured, can be connected in the galvanometer circuit. The terminals to which the source of unknown E. M. F. is connected are marked positive (+) and negative (—) to avoid the possiblity of damage to the potentiometer due to the wrong polarity being used. The standard cell as well as supply battery terminals are also marked similarly.

[Note—It is very important that there shall be no appreciable thermo-electric E M.F.'s within the potentiometer itself, since such extraneous E. M. F.'s shall affect the readings. For this reason, manganin, which has a very low thermo-electric E. M. F. with copper, is usually chosen as the material for the slide-wire as well as

the resistance coils. To ensure further that all parts are at a uniform temperature, all contacts and joints in the potentiometer circuit are included in the case of the instrument. This procedure also ensures the protection of the joints and contacts from the atmosphere. This is essential since any acidity of the atmosphere causes corrosion of the contacts and may set up small voltaic E. M. F.'s at the joints. To avoid corrosion the contacts are often made of a special gold-silver alloy.

Further, in order to avoid leakage between adjacent parts of the potentiometer circuit, it is essential that insulation is perfect. It is for this reason that the working parts of the instrument are mounted on an ebonite board and the internal connections are spaced so as to be as far apart as possible. A bakelite cover is also fitted above the ebonite board for protection of the instrument from light and dirt. The knobs operating the moving parts project through holes in this cover, which also carries the graduation marks.]

Student's Potentiometer

Crompton's potentiometer is a comparatively costly instrument. Moreover, it requires skill for its proper operation and careful handling. For this reason, less expensive and easy-to-operate student's potentiometers are available in various patterns, one of which is depicted in fig–63.

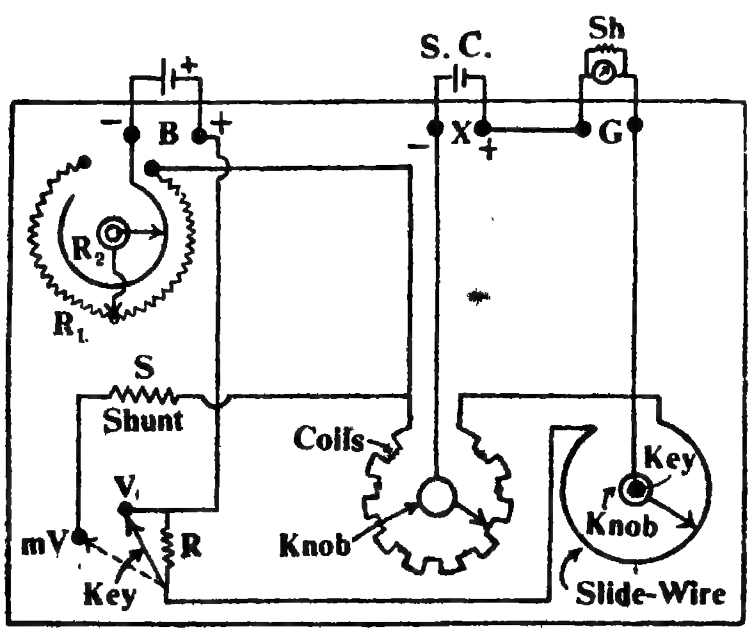

Fig. 63
Student's potentiometer

It consists of a number of coils of manganin wire arranged in the form of a circle and connected in series with a manganin slide-wire also in a circle form. The wire can be rotated with the help

of a knob, at the centre of which is provided a key by depressing which contact at any point on the slide-wire is affected, the corresponding reading being given by a circular graduated scale. R_1 and R_2 are two rheostats which are put in series with the main battery circuit (marked B in the figure). R_1 being for coarse adjustment while R_2 is for finer one. The source of unknown E.M.F. is connected at X with the polarity as marked, and the galvanometer is connected at G.

Provision is made in the instrument to read directly either volts or millivolts. For this purpose, a key is provided in the left, which normally makes contact with a stud marked V, meaning thereby that under this condition the scale provided with the instrument shall indicate volts. When the instrument is desired to read millivolts, the key is swung towards the left, as shown by the dotted line, and now it makes contact with the stud marked mV (meaning millivolts). This operation results in including a resistance R in the battery circuit, and a shunt is put in parallel with the potentiometer wire with the result that the instrument standardised to read volts can give readings in millivolts by shifting this key only.

[Note—It is easy to see that the resistance R should be equal to 999 times the resistance of the coils and the slide-wire combined. This process, however, results in reducing the current flowing in the slide-wire. Hence automatically the shunt resistance is brought in the circuit, which keeps the current through the main battery circuit constant.

The resistance of the shunt required can be easily worked out as follows :—

Let the resistance of the potentiometer wire with the coils in series with it be x ohms, then the total resistance with the inclusion of R (= 999x ohms) is equal to 1000 x ohms. If the shunt resistance be S, then the equivaledt resistance of the combination is

$\frac{1000\,x.\,S}{1000\,x + S}$. In order to keep the battery current unaltered, this

equivalent resistance must be equal to x. Thus

$$\frac{1000\,x.\,S}{1000\,x + S} = x$$

Hence $$S = \frac{1000\,x}{999} \text{ ohms.}$$

In this way a resistance of requisite magnitude is inserted for the shunt.]

EXPERIMENT—31

Objects. To calibrate a voltmeter (of a given range) with a potentiometer.

Apparatus Required. A potentiometer, the given voltmeter, two storage batteries, two rheostats, a standard cell (cadmium cell, if available, otherwise a Daniell cell), a Weston galvanometer, two one-way keys, one two-way key, and connection wires.

Formula Employed. The error in the voltmeter reading is given by—

$$V' - V = V' - \frac{E.\,l_2}{l_1}$$

where
V' = P. D. between two points read by the voltmeter.
V = P. D. between the same two points as read by the potentiometer.
E = E.M F of the standard cell *
l_1 = Length of the potentiometer wire corresponding to the E. M. F. of the standard cell.
l_2 = Length of the potentiometer wire corresponding to the P. D. (V) measured by the potentiometer.

[**Note.** E/l_1 gives the potential gradient along the wire]

PRINCIPLE AND THEORY OF THE EXPERIMENT

The calibration of a voltmeter with a potentiometer means the measurement of potential difference between any two points by means of the voltmeter and the measurement of the same potential difference between the same two points by a potentiometer, and then to examine how far the two values agree. Potentiometer being by far the more accurate instrument, the error in the voltmeter reading can be easily determined.

For this purpose, the potentiometer wire is calibrated, in the usual way, with the help of a standard cell (not shown in fig.–64). Let the length of the potentiometer wire for no deflection in the galvanometer be l_1. If E be the E. M. F. of the standard cell

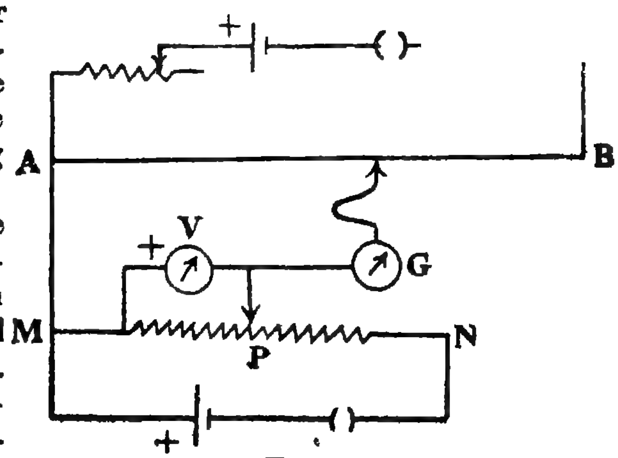

Fig. 64
Principle of calibration of a voltmeter

$$E = k\,l_1$$

where k is the potential gradient along the potentiometer wire.

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* The E. M. F. of the Daniell cell (which is often used as a standard cell for ordinary laboratory practice) is 1 08 volts.

Now let an auxiliary circuit be set up as shown in fig.-64, in which a constant current is maintained through a rheostat MN. The potential difference between M and the variable point P is measured with the help of the potentiometer. Let the null-point in the galvanometer be obtained on the potentiometer wire at a length l_2. Then the potential difference V between the points M and P is given by

$$V = k\, l_2 = El_2/l_1.$$

If the potential difference between the same two points M and P as measured by the voltmeter to be calibrated be V′ the error in the voltmeter reading is (V′—V).

In this way by shifting the point P and measuring the potential differences between M and the new positions of P with the help of the voltmeter as well as the potentiometer, the voltmeter can be calibrated in the required range and a calibration curve of the voltmeter can be drawn between the observed voltmeter readings (V′) and the errors (V′—V).

Method

(i) Set up the apparatus as shown in fig.-65 (a). Connect the shortage battery E_1, fully charged and of fairly large capacity

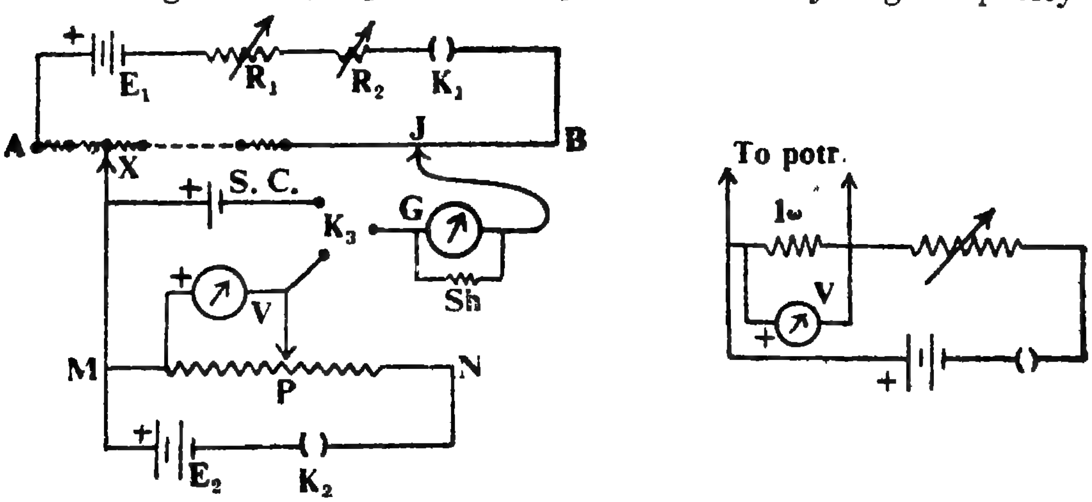

Fig. 65
Connections for calibration of a voltmeter

(so that it gives practically a constant current through the potentiometer wire) to the ends A and B of the potentiometer wire through two rheostats* R_1 and R_2 and a plug-key K_1.

* Instead of two rheostats, only one may be employed, but with the former there is greater facility in adjusting current through the potentiometer wire, R_1 may be of the order, say 200 ohms and R_2 of 10 ohms, the former is used for rough adjustment while the latter for finer one. The finer adjustment is not easily attained with a single rheostat.

Prepare an auxiliary circuit* as shown below the potentiometer wire (see fig.–65a). E_2 is a second storage battery, also giving a constant current, connected through K_2, to the fixed terminals M and N of a rheostat. The variable contact P is connected as shown.

S. C. is a standard cell (say, a Daniell cell) whose positive end, as well as the higher potential terminal M of the rheostat, are connected *towards* the higher potential end A of the potentiometer. The negative pole of the standard cell, as also the variable point P, are connected through the two-way key K_3 and the shunted galvanometer to the jockey J sliding along the potentiometer wire.

(ii) Close the key K_1 of the main circuit and connect the negative terminal of the standard cell by means of the two-way key (K_3) to the galvanometer. Place the contact-maker X at one end A of the potentiometer wire and place the jockey at 80 on the slide-wire†. Adjust the rheostats (R_1 and R_2) in the main circuit till there is no deflection in the galvanometer. By this procedure, we obtain a potential gradient of 1 millivolt per division along the potentiometer wire. Thus the instrument is made direct-reading‡ and the calculations are very much simplified.

(iii) Next connect the variable point P to the jockey and determine the total length (l_2) of the potentiometer wire when the balance-point is obtained on the wire. Note down the reading (V)′ of the voltmeter. Calculate the error with the help of the formula given above.

* The auxiliary circuit may be slightly modified, if desired, to one as shown in fig.–65 (b). It includes in addition one fixed resistance of, say, one ohm. Thus, in this case, the potential difference is measured between the ends of this resistance. This P. D. can be varied by operating the rheostat included in the circuit for this purpose. The fundamental principle in the two arrangements is essentially the same, the only advantage of the latter arrangement is that the connections are less confusing. The superiority of the former method lies in the fact that very little current is drawn from the battery E_2 throughout the entire experiment.

† This length of 1080 divs. of the potentiometer wire corresponds to l_1 of the formula given above.

‡ If a ten-wire potentiometer is used, the standardisation can be done at 540 divs. of the wire. In this case the potentiometer shall read 2 millivolts per division.

(iv) By altering the position of the variable point P, continue the above process till the entire range of the voltmeter is covered in suitable steps.*

(v) Now plot a graph† between the observed values (V′) of the voltmeter, represented on the x-axis, and the errors (V′ − V), represented on the y-axis.

Observations

[A] *Readings for the calibration of the potentiometer wire.*

Length of the potentiometer wire corresponding to the E. M. F. of the standard cell			Remarks
No. of coils	Length of the slide wire	Equivalent length (l_1)	(1) E. M. F. of the standard cell (E) = ...‡ volt
			(2) Potential gradient (E/l_1) = ... volt/cm.

* The calibration of the potentiometer wire should be checked now and then to see that the potential gradient established in the beginning remains unaltered. For this purpose, bring the standard cell in the circuit, keep the sliding contact-makers X and J *exactly at the same positions as in the first calibration process*, and test whether there is no deflection in the galvanometer. If the balance point has been disturbed, adjust the rheostat R_2 so that the null-point is again obtained at the same position.

† Join the consecutive points on the graph by straight lines. Since the voltmeter range has been divided in fairly small intervals, the relation between errors and the corresponding voltmeter readings will be more or less linear.

‡ Take the E. M. F. of the standard cell 1·0184 volt for the cadmium cell, or 1·08 volt for the Daniell cell.

[B] *Readings for the calibration of the voltmeter.*

S. No.	Length of the potr. wire corresponding to the P. D. between M and P			P. D. as read by the potentiometer (V)	P. D. as read by the voltmeter (V′)	Error* in the reading of the voltmeter (V′ − V)
	No. of coils	Length of the slide wire	Equivalent length			

Calculations Potential gradient, k = ... volt/cm.

Set I.

$$V = kl_2 = \ldots \text{ volt.}$$

[**Note**—Make similar calculations for the remaining readings.]

Result—The calibration curve (obtained by plotting the errors against the voltmeter readings) for the given voltmeter is attached herewith.

Precautions and Sources of Error

(1) The sucess of the experiment depends upon the constancy of the E. M. F.'s of the two storage batteries. They should have large capacity and should be fully charged. *Their voltages should be ascertained before inserting them in the circuit.*

(2) The ends of the connection wires should be cleaned and they should be firmly secured between the binding terminals. The wires connected to the higher potential points should all be led *towards* the same end of the potentiometer wire.

(3) The potential difference at the ends of the potentiometer wire should be greater than the maximum potential difference to be measured during the experiment. The rheostat in the main circuit should be so adjusted that this condition is fulfilled.

(4) In order to avoid unnecessary heating in different parts of the circuit two plug-keys should be used—one in the main circuit and the other in the auxiliary one.

* **Prefix +ve or −ve sign before each value of the error.**

(5) Change over from the standard cell to the auxiliary circuit should be done quickly with the help of the two-way key. Moreover, the calibration of the wire should be checked, now and then, during the course of the experiment by including the standard cell in the circuit. If the null-point with the standard cell is found to have shifted, it should be restored to the same position by adjusting the rheostat of low value in the main circuit.

(6) The contact of the jockey with the slide-wire should be mometary, and the jockey should not be moved along the wire while it is being pressed, otherwise the wire will be unevenly worn out and the uniformity of the wire will be impaired.

(7) During the early stages of locating the balance point the galvanometer should be kept shunted with a low resistance wire, so that excessive currents are avoided through the galvanometer. Exact position of the null-point should be determined with the shunt removed

(8) The potential gradient along the wire shall be uniform provided the wire is of constant thickness throughout its entire length. Hence the potentiometer should have its slide-wire of uniform thickness. If the potentiometer employed is a ten-wire potentiometer, the non-uniformity of the wire shall constitute a source of error.

EXPERIMENT—32

Object—To calibrate an ammeter (of a given range) with the help of a potentiometer.

Apparatus Required—A potentiometer, the given ammeter, two storage batteries, suitable rheostats, standard cell, Weston galvanometer, a standard one-ohm resistance, two-way key, single-way plug key, connection wires.

Formula Employed—Let the potential difference at the ends of the one-ohm coil be V, and let the null-point on the potentiometer correspond to a length l_1 of the wire then the current I flowing through the coil is given by.

$$I = \frac{V}{R} = V = k\, l_1$$

The potential gradient k is given by : $k = E/l$, where E is the E M. F. of the standard cell and l is the corresponding balancing length on the wire. Thus

$$I = \frac{E}{l} . \, l_1$$

The error of the ammeter $= I' - I$, where I' is the reading of the ammeter.

PRINCIPLE AND THEORY OF THE EXPERIMENT

The calibration of an ammeter with a potentiometer means literally the measurement of a current flowing in a circuit by an ammeter and its measurement with a potentiometer, and then to examine how far the two values agree. Potentiometer being a more accurate current-measurer, the error in the ammeter reading can be easily determined.

As a matter of fact, a potentiometer can accurately measure potential differences only ; it can be made to measure currents in an indirect manner. For this purpose let us examine fig.–66. AB is the potentiometer wire which has been previously calibrated with a standard cell. If E be the E. M. F. of the standard cell, which is balanced on a length l of the wire then the potential gradient, $k = E/l$, is known.

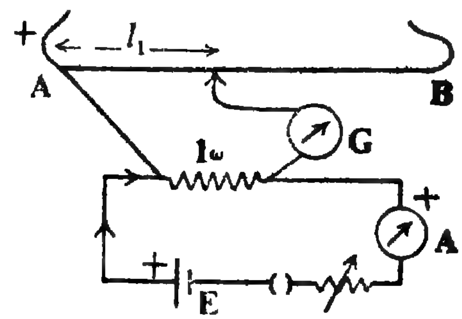

Fig. 66
Principle of calibration of an ammeter

Now an auxiliary circuit (as shown in the figure) is set up. In this circuit a standard one ohm* coil is also included. Let the current flowing through the coil be I, then, by Ohm's law a potential difference $V (= IR = I \times I = I)$ is created at its ends. This can be balanced on the potentiometer wire If l_1 is the balancing length of the wire, then

$$V (= I) = k l_1 = \frac{E}{l} \, l_1$$

Thus I is calculated. In this way the potentiometer becomes a current-measurer.

The same current is measured by the ammeter A included in the circuit. If it records a current I, the error in the instrument is equal to (I′—I).

In this way by operating a rheostat, also included in the auxiliary circuit the value of the current can be varied and the corresponding potential differences produced at the ends of the one-ohm coil can be measured. In this way the entire range of the ammeter can be calibrated, and a curve between the errors (represented along the y-axis) and the observed readings of the ammeter (represented along the x-axis) can be drawn.

* A resistance of 1 ohm is purposely employed. It eliminates calculation work.

Method

(i) Set up the apparatus as shown below (Fig -67)—connect the storage battery E_1 fully charged and of fairly large capacity (so that it gives a constant current through the potentiometer wire), to the ends A and B of the potentiometer wire through two rheostats* R_1 and R_2 and a plug-key K_1.

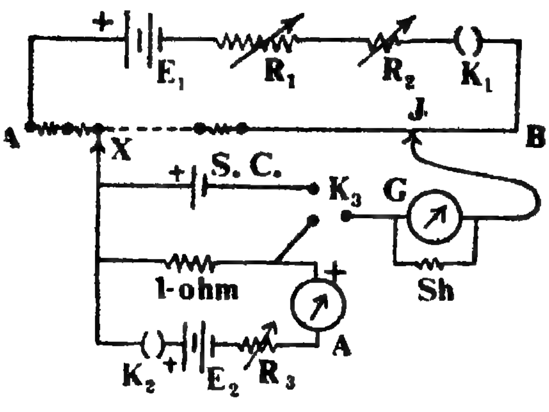

Fig. 67
Connections for the calibration of an ammeter

Prepare an auxiliary circuit as shown below AB. E_2 is a second storage battery, also giving a constant current connected through K_2, the rheostat R_3, the ammeter A to a standard 1-ohm resistance coil. The higher potential end of the coil is connected to the contact maker X, and the lower potential terminal to the two-way key as shown in the figure.

S. C. is a standard cell whose positive pole also is connected † to the contact maker X The negative pole is connected to the two-way key. This (or the lower potential terminal coming from the standard coil) can be connected through the shunted galvanometer to the jockey sliding over the potentiometer wire.

(ii) Close the key K_1 of the main circuit and connect the negative terminal of the standard cell by means of the two way key (K_3) to the galvanometer. Place the contact maker X at the end A of the potentiometer wire and place the jockey (J) at the division marked 80 on the slide-wire‡. Adjust the rheostats (R_1 and R_2) in the main circuit till there is no deflection in the galvanometer. By this procedure we obtain a potential gradient of 1 millivolt per division along the potentiometer wire. Thus the

* Instead of two rheostats, only one may be employed, but with the former arrangement the current flowing through the potentiometer can be adjusted with greater facility. R_1 may be of the order, say, 200 ohms and R_2 of 10 ohms; the former is manipulated for rough adjustment while the latter for finer one. The finer adjustment is not easily attained with a single rheostat.

† All the higher potentials terminals should be connected *towards* A, which is joined to the positive pole of the battery included in the main circuit.

‡ This length of 1080 divs. of the potentiometer wire corresponds to *l* of the formula given above.

potentiometer is made direct-reading* and the calculations are very much simplified.

(iii) Next connect the lower potential end of the one-ohm, coil to the jockey and determine the total length (l_1) of the potentiometer wire when the balance point is obtained on the wire Note down the reading of the ammeter. Calculate its error with the help of the formula given above.

(iv) By operating the rheostat included in the auxiliary circuit vary the current in suitable steps, and continue the above process till the ammeter is calibrated in its entire range†.

(v) Now plot a graph‡ between the observed values (I′) of the ammeter, represented on the x-axis, and the errors (I′—I), represented on the y-axis.

Observations

[A] *Readings for the calibration of the potentiometer wire.*

Length of the potentiometer wire corresponding to the E. M. F. of the standard cell			Remarks
No. of coils	Length of the slide wire	Equivalent length (l)	
			(1) E. M. F. of the standard cell (E) = ... **volt (2) Potential gradient (E/l) = ... volt/cm.

* If a ten-wire potentiometer is used, the standardisation can be done at 540 divs. of the wire. In this case the potentiometer shall read 2 millivolts per division.

† The calibration of the potentiometer wire should be checked now and then to see that the potential gradient established in the beginning remains unaltered. For this purpose, bring the standard cell in circuit, keep the sliding contact makers exactly at the same positions as in the first calibration process, and test whether there is no deflection in the galvanometer. If the balance point has been disturbed, adjust the rheostat R_2 so that the null-point is again had at the same position.

‡ Join the consecutive points on the graph by straight lines. Since the ammeter range has been broken up in steps of fairly small values, the relation between errors and the corresponding ammeter readings during these intervals will be more or less linear.

** Take the E M. F. of the standard cell 1·0184 volt for the cadmium cell, or 1·08 volt for the Daniell cell.

[B] *Readings for the calibration of the ammeter.*

S. No	Length of the potentiometer wire corresponding to the P. D. across the standard coil			P. D. across the 1-ohm coil	Accurate value of the current (I)	Ammeter reading (I′)	Error* (I′ − I)
	No. of coils	Length of the slide wire	Equivalent length				

Calculations Potential gradient, k = ... volt/cm.

Set I

$V_1 = k\ l_1 =$ ... volt.

Hence $I_1 = V_1 =$ ... amp.

[**Note**—Make similar calculations for the remaining readings.]

Result—The calibration curve (obtained by plotting the errors against the ammeter readings) for the given ammeter is attached herewith.

Precautions and Sources of Error.

(1) The success of the experiment depends on the constancy of the E. M. F.'s of the two storage batteries. They should have large capacity and should be fully charged. *Their voltage should be as certained before inserting them in the circuit.*

(2) The ends of the connection wires should be cleaned and they should be firmly secured between the binding terminals. The wires connected to the higher potential terminals should all be led towards the same end of the potentiometer wire.

(3) The potential difference at the ends of the potentiometer wire should be greater than the maximum potential difference to be

* Prefix +ve or −ve sign before the value of each error.

measured during the experiment. The rheostat in the main circuit should be so adjusted that this condition is fulfilled.

(4) In order to avoid unnecessary heating in different parts of the circuit two plug-keys should be used—one in the main circuit and the other in the auxiliary one.

(5) The ammeter should be connected in series in the circuit with the positively marked terminal to the higher potential point.

(6) Change over from the standard cell to the auxiliary circuit should be done quickly with the help of the two-way key. Moreover, the calibration of the wire should be checked, now and then, during the course of the experiment by including the standard cell in the circuit. If the null-point with the standard cell is found to have shifted, it should be restored to its initial position by adjusting the rheostat of low value in the main circuit.

(7) The contact of the jockey with the slide-wire should be momentary, and the jockey should not be moved along the wire while it is being pressed, otherwise the wire will be unevenly worn out and the uniformity of the wire will be impaired.

(8) During the early stages of locating the balance-point the galvanometer should be kept shunted with a low resistance wire, so that excessive currents are avoided through the galvanometer. Exact position of the null-point should be determined with the shunt removed.

(9) The potential gradient along the wire shall be uniform provided the wire is of constant thickness throughout its entire length. Hence the potentiometer should have its slide-wire of uniform diameter.

(10) The accurate measurement of current with the potentiometer depends on the accurate knowledge of the value of the standard resistance. If ordinary 1-ohm coil is employed, its value is not absolutely reliable, and the calibration of the ammeter shall be imperfect. For this purpose a standard resistance provided with separate current and potential terminals should be used.

EXPERIMENT—33

Object. To determine the internal resistance of a Leclanche cell with the help of a potentiometer.

Apparatus Required. Leclanche cell, a 10-wire potentiometer, storage battery, Weston galvanometer, rheostat, resistance box, a high resistance (of the order of 10,000 ohms), plug key, tapping key, and connection wires.

Formula Employed. The internal resistance (r) of the cell is calculated with the help of the following formula—

$$r = \left(\frac{l_1}{l_2} - 1 \right) R$$

where l_1 = Balancing length on the potentiometer wire when the Leclanche cell is on open circuit, i. e., the length corresponding to the E. M. F. of the cell.

l_2 = Balancing length on the potentiometer wire when the cell is in closed circuit, i. e., when a current is drawn from the cell.

R Resistance through which current from the Leclanche cell is drawn.

PRINCIPLE AND THEORY OF THE EXPERIMENT

When the poles of a cell are connected by an external resistance, a current begins to flow in the circuit. In the external circuit the current flows from the positive pole to the negative pole, while inside the cell the current is driven from the negative pole to the positive pole. During the passage of the current inside the cell the electrolyte offers some resistance to the flow of the current. This resistance offered by the cell is called its internal resistance and is denoted by the symbol r. The internal resistance of a cell depends on the area of the plates immersed in the electrolyte, the distance between them, the nature of the electrolyte, and also on the strength of the current which pases through the circuit. For very weak currents the internal resistance is practically independent of the strength of the current.

The internal resistance of a primary cell* can be determined by a potentiometer. Let AB represent a potentiometer wire in which a constant current is flowing A is at a higher potential than B. E is the Leclanche cell whose internal resistance is to be determined. The positive pole of this cell is connected to A and the negative pole through the galvanometer G to the jockey J which slides along the wire. With key K open let the cell be balanced and let l_1 be the corresponding length of the wire. Now, since the cell is on open circuit, the potential difference (V) between the points A and J balances the E. M. F. of the cell. Hence

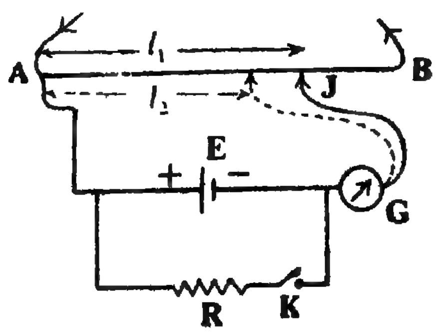

Fig. 68
Principle for the internal resistance of a cell;

$$E = V = kl_1 \quad \ldots \quad (1)$$

* The potentiometer method is unsuitable for determining the internal resistance of a secondary cell or any other cell whose resistance is very low. With such a cell a large current has to be drawn for producing a measurable fall of potential. Such excessive currents can damage the cell.

where k is the potential gradient along the wire and E is the E.M.F. of the Leclanche cell.

Let the cell be now short-circuited by a resistance R by depressing the key K. A current is drawn from the cell and consequently the P. D. (V_1) now existing between its poles is less than the E.M.F. The balancing point consequently shifts towards A. Let the new balancing length of the potentiometer wire be l_2, then

$$V_1 = kl_2 \qquad \ldots \qquad (2)$$

Hence

$$\frac{E}{V_1} = \frac{l_1}{l_2} \qquad \ldots \qquad (3)$$

Now applying Ohm's law to the circuit consisting of the cell and the external resistance R we have

$$\text{Current} = \frac{E}{R+r} = \frac{V_1}{R} \qquad \ldots \qquad (4)$$

where r is the internal resistance of the cell. From (4) we have

$$\frac{R+r}{r} = \frac{E}{V_1}$$

or

$$r = (E/V_1 - 1)\,R \qquad \ldots \qquad (5)$$

Substituting the value of E/V_1 from (3) in (5) we have

$$r = (l_1/l_2 - 1)\,R \qquad \ldots \qquad (6)$$

This is the required equation to determine the value of r.

Method

(i) Set up the apparatus as shown below—

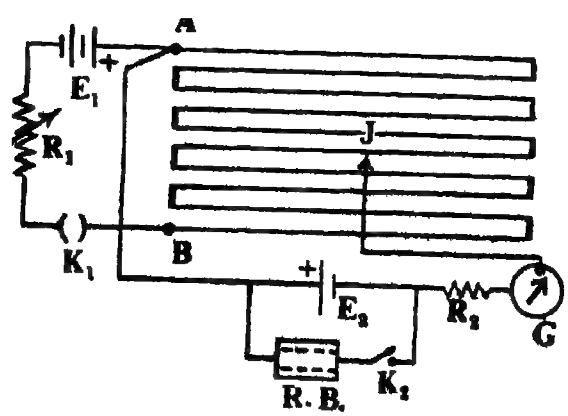

Fig. 69
Connections for the internal resistance of a cell

Connect the storage battery E_1, fully charged and of fairly large

capacity (so that it gives practically a constant current through the potentiometer wire) to the ends of the potentiometer wire through a rheostat R_1 and a plug-key K_1.

Connect the positive pole of the Leclanche cell E_2 *to the point A where the positive pole af the battery is also connected.* Connect the negative pole of E_2 through a high resistance* R_2, the galvanometer G to the jockey J sliding over the potentiometer wire. Connect a resistance box and a tapping key K_2 across the cell.

(ii) Close the key K_1 and bring the jockey near the end A. Press the jockey and note the deflection in the galvanometer. Now bring the jockey on the last wire near the end B and again note the deflection after pressing the jockey here. If the connections are correct the two deflections must be in opposite directions. If it is not so, the potential difference between the ends of the potentiometer wire is less than the E M.F. of the Leclanche cell In that case reduce the resistance in the main circuit by operating the rheostat R_1 and adjust its value till the balance point is obtained roughly on the last wire. Determine the exact position of the null-point by removing R_2 connected in series with the galvanometer. Open K_1 and measure the length of the wire from A to the point where the null-point has been obtained. This is l_1 of the formula given above.

(iii) Introduce a suitable resistance in the resistance box. Press the tapping key K_2 and obtain as before the new position of the exact balance point by shifting the jockey and determine the value of l_2.

(iv) Repeat the experiment with different values of R introduced in the resistance box, taking observations alternately with the Leclanche cell on open and closed circuit. Finally calculate the internal resistance of the cell for each set of observation separately.

* The high resistance R_2 may be of the order of 10,000 ohms. It has a special function to perform. It prevents the flow of excessive currents through the galvanometer, as well as it minimises the polarisation in the cell. This method suffers from all those defects which arise due to the polarisation taking place in the cell, hence polarisation has to be reduced as far as possible.

This resistance is used only upto the approximate balancing point. For locating the exact null-point this is removed from the circuit.

Observations

S. No.	Length of the potentiometer wire with the tapping key		Resistance introduced in the resistance box (R)	Internal resistance of the cell (r)
	open (l_1)	closed (l_2)		

Calculations

Set I

$$r_1 = (l_1/l_2 - 1) \text{ R}$$
$$= \dots\dots\dots \text{ ohm.}$$

[**Note.** Calculate similarly for other sets also.]

Result. From the values obtained for the internal resistance of the Leclanche cell it is found that it varies with the current drawn from the cell and its value lies between ...ohms and ...ohms.

Precautions and Sources of Error

(1) The ends of the connection wires should be carefully cleaned and they should be firmly secured between the binding terminals.

(2) The positive terminals of the battery as well as the Leclanche cell should be connected to the same end of the potentiometer wire.

(3) The storage battery should be fully charged and should have fairly large capacity so that it gives a practically constant current through the potentiometer wire and consequently the potential gradient also remains constant throughout the experiment.

(4) The rheostat in the main circuit should be so adjusted that the balance point with the Leclanche cell on the open circuit is obtained on the last wire. In this way maximum sensitivity of the instrument is utilised.

(5) A high resistance should be connected in series with the galvanometer, which should be disconnected when the approximate null-point is obtained.

(6) The jockey should be momentarily pressed on the potentiometer wire and it should not be moved along the wire when it is pressed, otherwise the uniformity of the wire shall be impaired.

(7) The unecessary heating of the potentiometer wire should be avoided by keeping the key in the moin circuit closed only when readings are taken.

(8) A tapping key should be inserted in the resistance box circuit. This should be pressed momentarily* when the null point is being, sought, and released again as soon as the jockey is raised from the wire for adjusting to a fresh position along the potentiometer wire.

(9) Apart from the polarisation affect on the internal resistance, the result shall also be adversely influenced by the non-uniformity of the potentiometer wire, which is highly probable in such a long wire. A non-uniform wire will not have a constant potential gradient along its entire length.

EXPERIMENT—34

Object. To compare two low resistances by means of a potentiometer.

Apparatus Required. Two low resistances, potentiometer, two storage batteries, two rheostats, Weston galvanometer, a six-terminal key (or a Pohl's commutator), two plug keys, and connection wire.

Formula Employed : $$\frac{R_1}{R_2} = \frac{l_1}{l_2}$$

where R_1, R_2, = The two resistances to be compared.

l_1, l_2 = Corresponding lengths of the potentiometer wire when balance points are obtained.

PRINCIPLE AND THEORY OF THE EXPERIMENT

If the two resistances to be compared are connected in series and a study current is allowed to flow through them, then by Ohm's law, the potential differences across them will be proportional to their resistances Now these potential differences can be accurately compared with a potentiometer, hence the resistances are thereby compared.

---

* If the tapping key is kept pressed for an appreciable time, the cell shall be polarised, and a gradual drift in the balance point shall be observed.

Let us refer to fig.-70, in which AB is the potentiometer wire carrying a steady current in the direction A to B so that A is at a higher potential than B. Let k be the potential gradient along the wire.

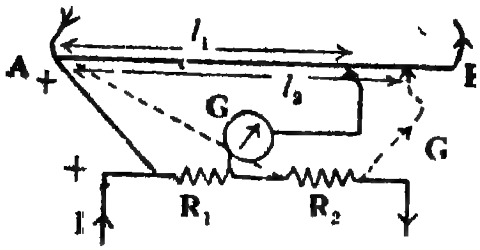

Fig 70
Principle for comparison of two resistances

Now R_1 and R_2 are the two resistances to be compared. Let them form part of an auxiliary circuit in which a steady current I is flowing. Let the higher potential terminal of R_1 be connected to A (*i e*. the higher potential end of the wire), and the lower potential terminal to the jockey. Let l_1 be the length of the potentiometer wire when a null-point is obtained. The potential difference V_1 at the ends of R_1 is given by,

$$V_1 = k\, l_1$$

But by Ohm's law $$V_1 = I\, R_1$$

Hence $$I\, R_1 = k\, l_1 \quad \ldots \quad (1)$$

Let R_1 be disconnected from the potentiometer wire, and let R_2 be now connected to A and the jockey, as shown by dotted lines. Let the potential difference V_2 across this resistance be balanced on a length l_2 of the potentiometer wire. Then

$$V_2 = k\, l_2$$

or $$I\, R_2 = k\, l_2 \quad \ldots \quad (2)$$

From (1) and (2) we have

$$\frac{R_1}{R_2} = \frac{l_1}{l_2} \quad \ldots \quad (3)$$

Method

(i) Set up the apparatus as shown in fig.-71. Connect the storage battery* E_1 to the ends A and B of the potentiometer wire through a rheostat Rh_1 and a plug-key K_1. The end A is connected to the positive pole of the battery so that the potential at A is higher than that at B.

* For the accuracy of the result it is essential that the potential gradient existent along the wire is constant throughout the experiment. It will be nearly so if the battery supplies a constant current. For this purpose a battery *fully charged* and of fairly *large capacity* should be employed.

Prepare an auxiliary circuit as shown below the potentiometer wire AB. E_2 is a second battery, also giving a constant current,

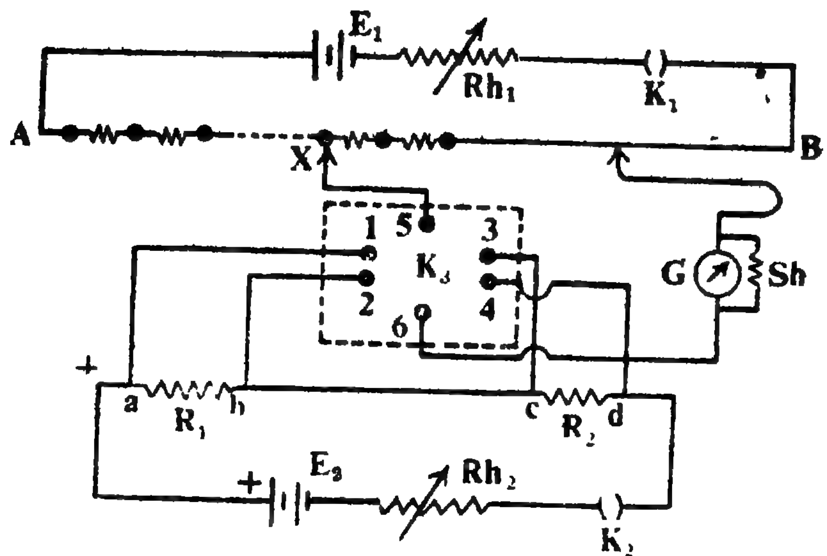

Fig. 71
Connections for the comparison of two resistances

connected to the two given resistances R_1 and R_2 through a rheostat Rh_2 and K_2.

K_3 is a six-way key*, whose terminals are numbered in the figure. Connect the higher potential terminal 'a' (marked +) of R_1 to terminal No 1, and lower potential terminal b to No. 2. Similarly connect c (higher potential terminal of R_2) to No. 3 and d to No 4. Connect terminal No. 5 to the contact maker X and No. 6 to the jockey J through the shunted galvanometer G.

(ii) Before doing the actual experiment secure first maximum sensitiveness for the potentiometer, *i. e., the potential gradient of the wire should be small, and at the same time the P. D. at the ends A and B should be greater than the P. D. to be balanced on the wire*

[**Note.** This can be easily accomplished as follows :

First of all estimate (roughly) which of the two resistances is greater, since if the P. D. across its ends can be balanced, that

* By this arrangement the two resistances R_1 and R_2 can be successively brought in circuit. When R_1 is used by connecting 5 to 1, and 6 to 2, the resistance R_2 is completely cut off from the circuit. Next time when R_2 is brought in circuit by connecting 5 to 3, and 6 to 4, the resistance R_1 is completely cut off from the main circuit. *A two-way key should not be employed* in place of this key, as in this case a resistance *is not completely cut off* from the potentiometer circuit, and consequently the positions of the null-points are erroneous.

across the smaller one will obviously be balanced, but the reverse may not always be true. For this purpose, adjust the rheostat Rh_2 to a low value. Determine approximately the balance point with R_1, and then with R_2. The equivalent lengths of the wire obtained in the two cases will indicate which resistance is greater. Let us suppose that R_1 is greater than R_2. Now switch this resistance R_1 to the potentiometer circuit. Place the contact maker X to the last stud of the coils and place the jockey on the wire. By adjusting the rheostat Rh_1 of the main circuit obtain a null-point on the slide wire. This procedure adjusts the P. D between A and B to be greater than that across the resistances R_1 and R_2, and at the same time secures maximum sensitivity of the potentiometer.]

(iii) Now obtain the exact balance points with R_1 and then with R_2, and note the corresponding lengths l_1 and l_2 of the wire.

(iv) Next **increase** the resistance in the rheostat Rh_2 and obtain the new values of l_1 and l_2. In this way, by slightly increasing the resistance in Rh_2 each time, take several readings, and calculate l_1/l_2 in each case. From this obtain the mean value of R_1/R_2.

Observations

S. No.	Length of the potentiometer wire corresponding to the Potential Difference						Ratio of the two resistances
	Across the first resistance (R_1)			Across the second resistance (R_2)			
	No. of coils	Length of wire	Equivalent length (l_1)	No. of coils	Length of wire	Equivalent length (l_2)	l_1/l_2
						Mean	

Calculations

Set I

$$\frac{R_1}{R_2} = \frac{l_1}{l_2} = \ldots \ldots$$

[**Note.** Calculate this ratio for each set separately and take the mean value.]

Result. The ratio of the two given resistances = ...

Precautions and Sources of Error

(1) The ends of the connection wires should be carefully cleaned and they should be firmly secured in the binding terminals. All the higher potential terminals should be led towards A.

(2) The P. D. between the ends A and B should be greater than the potential difference across the given resistances, which have to be balanced on the wire.

(3) The potential gradient of the potentiometer wire should remain constant throughout the experiment. To attain this, the battery in the main circuit should be fully charged and it should have a fairly large capacity. Such a battery will supply a fairly constant current.

(4) The potentiometer should be so adjusted that maximum sensitivity is attained, i. e., with the maximum value of the P. D. to be balanced, the null-point should be obtained (by adjusting the rheostat in the main circuit) with all the coils included. This will ensure maximum sensitivity, and at the same time it will ensure that the P. D. between A and B is greater than V_1 (across R_1) and V_2 (across R_2).

(5) The storage battery used in the auxiliary circuit should also be fully charged and should have a fairly large capacity, so that it sends steady current through R_1 and R_2.

(6) To avoid unnecessary heating in different parts of the circuit, each circuit should have a plug-key which should be closed only when readings are being taken.

(7) A six-way key should be employed to include either R_1 or R_2 in the potentiometer circuit. Its connections should be carefully done. The charge-over from R_1 to R_2 should be done quickly, so that effects due to heating and variations in the batteries are minimised.

(8) The jockey should be pressed momentarily, and it should not be moved along the wire in a pressed state, otherwise by uneven rubbing the wire shall lose its uniformity.

(9) The approximate null-point should be obtained with the galvanometer shunted with a low resistance. The shunt should be removed when the exact null-point is to be located.

ADDITIONAL EXPERIMENTS

Expt.—34 (1)

Object. To determine the value of a low resistance with a potentiometer.

[**Note.** In the experiment just described the two low resistances to be compared are of the same order of magnitude. However, if the resistances to be compared are of the order, say, 1 ohm and 0.01 ohm, the following method is adopted.]

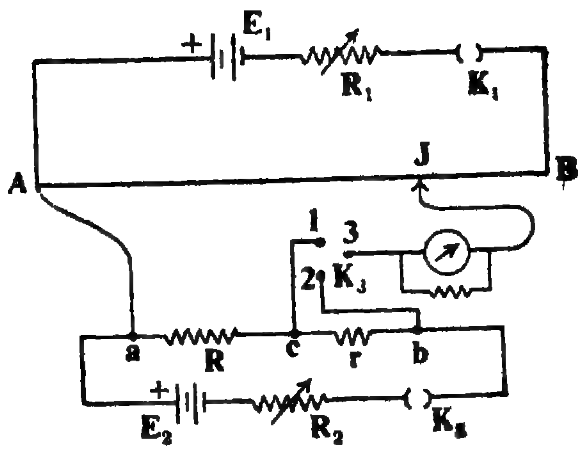

Fig. 72
Connections for the determination of a low resistance

The diagram is self-explanatory. r is the small resistance to be measured. In series with this is a resistance R of, say, 2 ohms. K_3 is a two-way key.

First balance the P. D. (v_1) between a and b i. e. at the ends of (R + r) by connecting 2 to 3. Let the balancing length of the potentiometer wire be l_1. Then balance the P. D. (v_2) between a and c (i. e , at the ends of R alone). Let the new balancing length of the wire be l_2. If I be the steady current flowing through R and r, we have.

$$\frac{V_1}{V_2} = \frac{I(R+r)}{IR} = \frac{l_1}{l_2}$$

where $$r = (l_1/l_2 - 1)R$$

In this way r can be determined.

[**Note.** (1) The above procedure can be adopted for determining the *specific resistance of copper.* For this purpose R replaces a standard resistance of, say, 0·1 or 0·01 ohm, and in place of r a copper wire is connected. In this case, the two resistances should have, for the sake of greater accuracy, four terminals each, two for leading the current in the conductor and two for measuring the potential difference. Then, as described just now, the resistance r of the copper wire is calculated with the help of the formula

$$r = (l_1/l_2 - 1)R$$

Knowing the length of the copper wire (only that length should be measured across which the P. D. has been measured), and the diameter, its specific resistance can be calculated.

(2) Using a decimal-ohm box in place of R and an ammeter in place of r, the *resistance of the ammeter* can be measured.

It should be added here that great accuracy can be attained in such determinations only when a very sensitive potentiometer, e. g. a Crompton's potentiometer is employed.]

EXPERIMENT—35

Object. To measure the thermo-electric e. m. f. generated in a copper-iron thermocouple for a known difference of temperature between its junctions.

Apparatus Required. A potentiometer, a standard cadmium cell, an accumulator, a copper-iron thermocouple, resistance box, rheostat, a sensitive galvanometer, a two-way key, plug-key, and connecting wires.

Description of the Apparatus

(a) **Potentiometer.** Since the magnitude of the thermo-electric e. m. f. is of the order of a few millivolts*, the ordinary potentiometer method cannot be employed here. Such e. m f's can therefore be measured by a suitable modification of the ordinary method so as to produce a potential difference of the order of a microvolt across each cm. of the potentiometer wire which is small enough to admit of sufficiently accurate measurement.

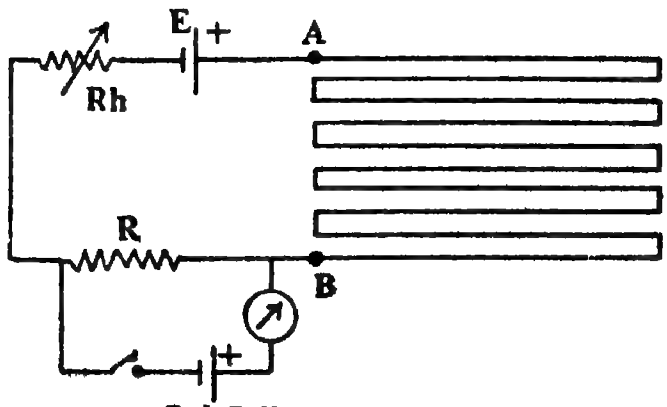

Fig. 73
Calibration of a potentiometer for direct reading

Let us take a ten-wire potentiometer such that its wire AB (fig.–73) is 1000 cm long and it has a resistance of 0·01 ohm per cm. If the resistance R has a value 1018 ohms, the potential difference across it will be exactly 1·018 volts when a current of 1 milli amp. flows through the circuit. Let the standard Weston cadmium cell, whose e. m. f. is 1 018 volts, be connected across R through a galvanometer and a key. Let the key be closed and the rheostat Rh be adjusted till the galvanometer shows no deflection. After this adjustment the wire AB carries a current of 1 milliamp. and hence it has a potential fall of $0{\cdot}01 \times 10^{-3} = 10 \times 10^{-6}$ (or 10 microvolts) per cm. of wire. The potentiometer can thus measure a smallest potential difference of 1 microvolt per mm and a maximum potential difference of 10 millivolt.

* For instance, in a copper-iron thermo-couple the e. m. f. generated when the junctions are maintained at 0°C and 100°C is only 1·3 millivolt.

[Note—(i) If a standard cadmium cell is not available in the laboratory, a Daniell cell (e. m. f. = 1 08 volt) can be used. In that case the resistance R should have a value equal to 1080 ohms to give the above constants to the potentiometer.

(ii) If the millivolt potentiometer (or students' potentiometer is available in the laboratory, it can be employed for the measurement of the thermal e. m. f.'s, but the procedure adopted above is more instructive.]

(b) **Standard Cadmium Cell.** The Weston cadmium cell is shown in the accompanying figure. Two tubes are arranged as shown, each being provided with an external lead of platinum which is in contact with the bottom layers. These layers consist of pure mercury in one limb, and an amalgam of pure mercury and cadmium in the other. Above the pure mercury is a layer of a paste of mercurous sulphate. Above this and the cadmium amalgam is a layer in each tube of pure cadmium sulphate crystals. Finally, a layer of a saturated solution of pure cadmium sulphate occupies the upper parts of the two tubes.

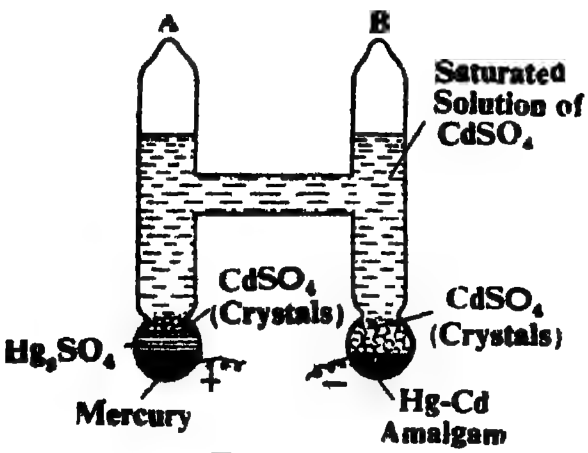

Fig. 74
Standard cadmium cell.

The following reactions take place in the cell :—

(i) $Cd = Cd^{++} + 2e$

(ii) $Cd^{++} + Cd\ SO_4 + Hg_2\ SO_4 = 2\ Cd\ SO_4 + Hg^+ + Hg^+$
(Depolariser)

(iii) $Hg^+ + Hg^+ - 2\ Hg + 2p$

where e and p represent respectively the elementary negative and positive charges.

The e. m. f. of the cell is constant at constant temperature. So that no current of any appreciable magnitude be drawn from the cell, the makers put a high resistance (of the order of 10,000 ohms) in series with it*. The International Conference on Electrical Units and Standards, 1908, adopted the following formula as giving most accurately the e. m. f. of the cell—

$E_t = 1{\cdot}0184 - 4\ 06 \times 10^{-5} (t-20) - 9\ 5 \times 10^{-7} (t-20)^2 + 10^{-8} (t-20)^3$ volt, where t is expressed in degrees centigrade.

* This precaution is necessary, for if a standard cell supplies more than a small current it is subject to polarisation and the value of the e. m. f. becomes uncertain.

The temperature coefficient is therefore small.

[Note—There is another standard cell, known as Clark cell which is identical with the Weston cell except that the cadmium is replaced in this case by zinc, cadmium sulphate by zinc sulphate etc. The e. m. f. of this cell at 15°C is 1·4328 volt, but this cell has a large temperature coefficient—a fact which explains the more general use of the Weston cell.]

Formula Employed—The value of the thermo-electric e. m. f. (e) developed in a thermo-couple is obtained with the help of the following formula—

$$e = \frac{\rho E l}{R}$$

where ρ = Resistance per unit length of the potentiometer wire

E = E. M. F. of the standard cell

R = Resistance across which the standard cell is balanced

l = Length of the potentiometer wire when the thermo-electric e. m. f. is balanced on this.

PRINCIPLE AND THEORY OF THE EXPERIMENT

Seebeck discovered in 1821 that when two dissimilar metals (e. g., copper and iron) are joined and the two junctions are maintained at different temperatures, a current—known as thermo-electric current—flows round the circuit in a direction shown by arrows in fig.—75 (a). The value of the thermo-e.m. f. depends upon the metals constituting the thermo-couple and the difference of temperature between the two junctions. The thermo e. m. f. increases fig.—75 (b) as the temperature of the hot junction increases and reaches a maximum value at a characteristic temperature T_n known as the *neutral temperature*, beyond which the e. m. f. begins to decrease, till at a temperature T_i, called the *temperature of inversion*, the e. m. f. drops to zero and changes its sign*.

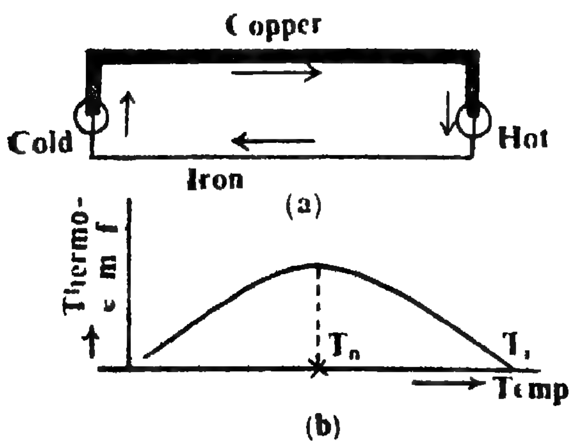

Fig. 75
Seebeck effect in copper iron thermo-couple.

* This curve is a parabola and can be represented by an equation of the type $e_t = at + bt^2$ where a, b are constants for a particular couple.

Let the electric connections be made as shown in fig.–76, which is self-explanatory. Now, if the standard cell circuit is closed by means of the two-way key, *and the jockey is put at A*, no deflection in the galvanometer can be obtained by adjusting the rheostat. Thus the e m.f. of the standard cell is balanced across the resistance R (= 1018 ohms). Hence if E be the e. m. f of the standard cell, and i be the current flowing through R (or the potentiometer wire), we have

$$E = i R \quad ...(1)$$

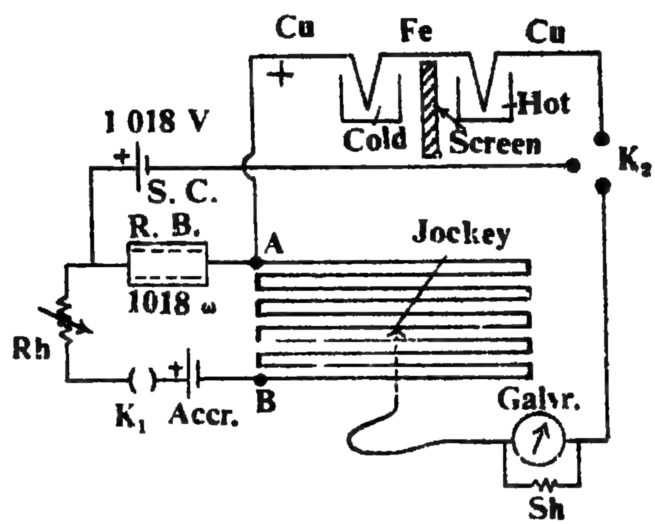

Fig. 76
Connection for a copper-iron thermocouple

Next let the standard cell circuit be broken, and the thermocouple circuit be connected to the galvanometer. Let the null-point be obtained at X, where AX = l. If e be the thermo-e m. f. and r be the resistance of the portion AX of the potentiometer wire, then

$$e = ir = i \rho l \quad ... \quad (2)$$

where ρ is the resistance per cm. of the wire*.

From (1) and (2) we have

$$e = \frac{\rho . E . l}{R} \quad ... \quad (3)$$

Equation (3) enables us to calculate the thermo-e. m. f. developed for this particular difference of temperature between the two junctions of the thermocouple.

Method

(i) Set up the electrical connections† as shown in fig.–76.

* A preliminary experiment gives the value of ρ by determining the resistance of the ten wires of the potentiometer.

† In this arrangement the most important connection is that of the positive end of the thermocouple to the potentiometer wire. The copper wire connected to the *cold end of the thermocouple* should be connected to A (which is connected to the positive pole of the accumulator) Note that *in a copper-iron thermocouple junction is positive.*

If a *copper-constantan thermocouple* is employed for this experiment, then its *hot end* (which is positive in this case) should be connected to A (the higher potential terminal of the potentiometer wire).

The rheostat in the accumulator circuit should be of a high value, and for R insert a resistance box from which a resistance of 1018 ohms can be unplugged.

(ii) Shunt the galvanometer and put the jockey at A. Adjust the rheostat till there is practically no deflection in the galvanometer. Remove the shunt and obtain the exact position of the null point by finally adjusting the rheostat. Now, the e. m. f. of the standard cell has been balanced by the potential difference across R.

(iii) Re-shunt the galvanometer and bring the thermocouple in circuit with the help of the two-way key. When the temperature of the hot junction has become steady, press the jockey on a wire and by adjusting the length of the potentiometer wire obtain the approximate position of the null point. Get the exact position by removing the shunt from the galvanometer. Note the length of the potentiometer wire from A to that point where the null point has been obtained.

(iv) Re-check the standardisation of the potentiometer by bringing in the standard cell again in circuit and repeating the process as above. Finally, repeat the experiment twice or thrice and obtain a mean value of l. Calculate the thermo-e. m. f. with the help of the above formula, taking the value of ρ as given.

Observations

S. No.	High resistance (R)	Length of the potentiometer wire corresponding to thermo-e m.f.			Thermo-e. m. f. (e)	Remarks
		No. of complete wires	Length of the remaining wire	Total length (l)		
						(1) E.M.F. of the standard cell = ...volt (2) Resistance per unit length of the potentiometer wire = ...ohm.

Calculations.

$$e = \frac{\sigma E l}{R}$$

$$= \text{...microvolts*}$$

Result. The value of the thermo-e. m. f. for copper-iron thermocouple when its junctions are at ......°C and ......°C = ... microvolts.

[Standard value = ... μV ; Error = ...%]

Precautions and Sources of Error

(1) Before making connections the ends of the connecting wires should be carefully cleaned with a sand paper and then firmly secured between the binding terminals.

(2) The accumulator should be fully charged and should have a large capacity so that its e. m. f. may remain constant for the duration of the experiment.

(3) A plug key should invariably be employed in the accumulator circuit so that the current flows only when it is desired. This eliminates the unnecessary heating of the potentiometer wires, and secondly there is no unnecessary drain on the accumulator, which consequently helps to maintain a constant potential gradient along the potentiometer wires.

(4) When the standard cell is being balanced across the resistance R, the jockey of the potentiometer should lie at the end A of the potentiometer wire i. e., the standard cell is to be balanced across the resistance R only.

(5) The leads coming from the thermocouple should be sufficiently long so that their free ends are at the same temperature.

(6) The jockey should be pressed on the potentiometer wire momentarily. In no case should it be dragged along in the pressed position otherwise the wire will be rubbed off non-uniformly and its diameter will not be the same throughout.

(7) The galvanometer employed in this experiment should be a sensitive one. It should always be shunted in the initial stages of locating the null point. The shunt should be removed when the exact null-point is sought. The first operation ensures safety of the instrument while the second one utilises its full sensitivity without any fear of damage to it.

(8) The potentiometer should be so standardised that the potential gradient along the wire is of the order of a microvolt per division, which admits of sufficiently accurate measurements.

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* Convert the result is micro-volts. 1 mv = 10^{-6} volt.

For this purpose the high resistance R and the rheostat should each be about a thousand ohms.

ADDITIONAL EXPERIMENT

Exp.—35 (a)

Object. To study the variation of the thermo-electric e. m. f. with temperature for a copper-iron thermocouple and to determine its neutral temperature.

The experiment has to be conducted as the main experiment described above. As before the cold junction is placed in cold water contained in a beaker, and the hot junction is put in mercury contained in a hard glass (or pyrex) test tube which is heated in a sand bath. A thermometer reading upto, say, 350°C is put in this tube. The mercury is heated upto 320 C and then the readings of the thermo-e m. f. and the temperatures of the hot and cold ends are recorded after every 10°C fall of temperature.

[**Note.** It is essential to check the standardisation of the potentiometer after every four or five observations.]

Finally a graph is drawn between the thermo-e. m. f. (along the y-axis) and the difference in temperature (along the x-axis) between the hot and cold junctions. The graph will be a parabola from which the neutral temperature, which corresponds to the maximum e. m f., is noted.

[**Note.** The neutral temperature of a copper-iron thermocouple is 270°C (it may be different for different specimens of iron and copper). It may be added here that the neutral temperature for a thermocouple is a constant (i.e. it is independent of the temperature of the cold junction). The temperature of hot junction at which the thermo-e m.f. is zero and reversal takes place (i.e., the temperature of inversion) is a variable one, being always as much above the neutral temperature as the cold junction is below it.]

Expt.—35 (b)

Object. To determine the melting point of wax by measuring the thermo-e m. f.'s of a copper iron thermocouple.

After performing the above experiment put the hot junction in melting wax, and when the wax solidifies, measure the thermo-e.m.f. and corresponding to this value read the temperature from the graph. To this add the temperature of the cold bath. This is the melting point of wax.

[**Note.** The experiment can be performed by placing the hot junction in boiling water only, and noting the readings after every 5°C fall in temperature. The melting point can be calculated with the help of the graph.]

AMMETERS AND VOLTMETERS

Ammeters and voltmeters* are classed together because there is no essential difference in the principle involved in their operation. Except in the case of electrostatic instruments, a voltmeter carries a current which is proportional to the potential difference which is to be measured, and this current produces the operating torque. In an ammeter this torque is produced by the current to be measured, or by a definite fraction of it. Thus, the only real difference between the two instruments is in the magnitude of the current producing the operating torque.

An ammeter is usually of *low resistance*, so that its connection in series with the circuit in which the current is to be measured does not appreciably alter the value of this current. A voltmeter, on the other hand, is connected in parallel with the potential difference to be measured, and must therefore have a *high resistance* so that the current drawn by it is small. As a matter of fact, a low range ammeter (i. e., one which gives full scale deflection for a very small current) may be used as a voltmeter if a high resistance is connected in series with it. The current which flows through it when it, together with its series resistance, is connected across the voltage to be measured, must be within its range when used as an ammeter.

[Example

A milliammeter, whose resistance is 1 ohm, gives a full-scale deflection for a current of 10 milliamperes. Calculate the resistance which must be connected in series with it in order that it may be used as a voltmeter for reading voltages upto 10 volts.

Let x be the required resistance. The current flowing through the instrument when 10 volts are applied to the instrument and with this resistance in series, must be 10 milliamperes (or 0 01 amp.)

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* For a detailed study of galvanometers, ammeters and voltmeters, read author's book "A Critical Study of Practical Physics and Viva-Voce".

Thus

$$0{\cdot}01 = \frac{10}{x + 1}$$

or $$x + 1 = \frac{10}{0{\cdot}01} = 1000 \text{ ohms.}$$

$$\therefore \quad x = 999 \text{ ohms.]}$$

The relative magnitudes of the resistance of the two types of instruments is also warranted by consideration of power loss occurring in them. For instance, if R_A is the resistance of an ammeter in which a current I is flowing, the power loss in the instrument is I^2R_A watts. Again, if R_V is the resistance of a voltmeter to which a voltage E is applied, the power loss in the instrument is E^2/R_V. Obviously, in order that the power loss in the instruments shall be small, R_A must be small and R_V should be large.

Voltmeter. As indicated above, a voltmeter is always connected in parallel with the two points whose potential difference is to be measured. The internal resistance of the instrument should therefore be large in order to avoid any appreciable rearrangement of current and potential drop in the circuit The current passing through the voltmeter is consequently very small for such a high internal resistance, and hence the heating in the coil is small.

The internal resistance of the voltmeter is made up of not only that of the copper coil, but greater part is due to a high resistance put in series with it. The chief reason for this is to avoid any error due to the heating in the moving coil. Such heating can take place either due to (1) variations in the room temperature, or (2) the Joule heating. Due to both these causes the resistance of the coil shall increase. unless the temperature coefficient of its material is small. Apparently manganin, due to its low temperature coefficient is preferable but it has a serious drawback. For the same coil resistance a manganin coil shall have less radiating surface. Again, the effect of Joule heating can be eliminated by making the resistance of the moving coil fairly low. Hence a compromise between the two requirements is necessary. This is effected by taking a coil wound with copper coil which has a comparatively low resistance. In series with this is put a high resistance wire of a material, usually manganin, whose temperature coefficient is small, so that although the resistance of the coil may change considerably, the change in total resistance is small. This series resistance can be constructed of a thicker wire than would be possible for the moving coil.

In an actual instrument often used in the laboratary, the copper coil is wound on a metallic frame, so that when the coil moves in the magnetic field of the horse-shoe magnet, it cuts lines of force, thereby generating eddy currents in the frame, which

oppose the motion of the coil bringing it quickly to rest and thus making the instrument a "dead-beat" one.

In such a moving coil instrument the direction of deflection depends on, the direction of the current. Hence, during insertion in the circuit, it should be carefully borne in mind that the positively marked terminal is connected to that point which has a higher potential, otherwise the needle may be damaged.

[**Note**. Sometimes the same voltmeter may be used to measure potential difference of different ranges. Thus if A and B terminals are connected to a difference of potential of 5 volts a full scale deflection is obtained in the instrument. If the resistance of the coil and the part L N of the resistance wire be 300 ohms, the current flowing in the coil will be 5/300 = 1/60 amp.

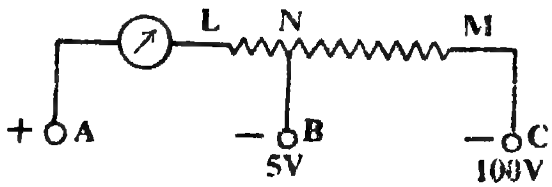

Fig. 77
A multiple-range voltmeter

If the same scale is to be employed to read 100 volts, the terminals A and C may be employed, so that an additional resistance NM is included in the circuit. Since the current is the same as before, we have

$$\frac{1}{60} = \frac{100}{R} \qquad \text{or} \qquad R = 6000 \text{ ohms}$$

where R is the total resistance (coil + external wire) of the circuit. Thus, to read 5 volts and 100 volts the scale will be divided in equal intervals, and each division will correspond to twenty times the value which corresponds to the lower range applied between AB.

If an instrument is intended to read millivolts, its internal resistance should be smaller, as can be easily worked out by pushing the above argument further.]

Ammeter. The resistance of an ammeter is small. This condition is achieved by connecting a low resistance in parallel with the moving coil. This resistance is referred to as a *shunt*, and in a fixed range ammeter it is contained inside the case. The value of the shunt resistance is small, hence the resistance of the whole instrument is also of the same order. The shunts are made of manganin since this material has a low temperature coefficient The dimensions of a particular manganin strip required to have a particular range ammeter are easily calculated out. If, in a particular case it is revealed that the shunt should have excessive width so that heating produced may be negligible, in that case not one, but several strips, are used in parallel.

It is easy to see that the shunt resistance is controlled by the ammeter range. The greater the range, the smaller is the resist-

ance of the shunt. In fact, superior types of ammeters are not provided with fixed shunts, but are provided with external ones, thus the range can be suitably varied by using a shunt of appropriate value. The instrument is made dead-beat, like the voltmeter by winding the coil on a metallic frame in which eddy currents are produced, and they bring about the required damping. The ammeter, like the voltmeter, is a uni-directional instrument, hence it has to be inserted in a circuit in such a way that its positively marked terminal is connected to the higher potential point of the circuit.

From a brief resume of these two important instruments it is clearly seen that they are essentially moving coil galvanometers with slight variations in their construction, which is necessitated by the particular role which they have to perform in electrical measurements. Below are described and discussed two experiments which bring out how a galvanometer can be adopted either for direct current measurements or for direct voltage measurements.

EXPERIMENT—36

Object. To convert a given Weston galvanometer into an ammeter of a given range.

Apparatus Required*. Weston galvanometer† a high resistance box (preferably of dial pattern), an accumulator, a high resistance voltmeter, plug-key, an ammeter of the same range as given for conversion.

Formula Employed. The shunt resistance, S, required for converting the galvanometer into an ammeter of a given range is calculated with the help of the formula—

$$S = \frac{I_g}{I - I_g} \cdot G$$

where G = Galvanometer resistance

I_g = Value of the current required to get a full-scale deflection in the galvanometer

I = Value of the current which has to be read by the galvanometer (i.e., its range)

* In this experiment the resistance of the galvanometer has to be known. If its value is not given, it has to be determined by Kelvin's method. In that case necessary apparatus for conducting this part of the experiment is also required [see expt.–13].

† For this experiment it is preferable to use a special type of galvanometer, the zero of whose scale lies on the extreme left as in the case of ammeter and voltmeter scales.

The length of the shunt wire can be calculated with the help of the formula—

$$S = l\rho$$

where l = Length of the shunt wire

ρ = Resistance *per unit length* of the shunt wire.

PRINCIPLE AND THEORY OF THE EXPERIMENT

The accompanying diagram represents a galvanometer of resistance G, in parallel with which a shunt of resistance S has been used. The main current I divides itself as shown in the figure. The currents obviously divided themselves in the inverse ratio of their resistances, that is,

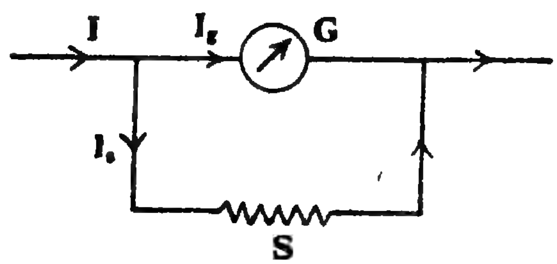

Fig. 78
Principle of an ammeter

$$\frac{I_s}{I_g} = \frac{G}{S}$$

or $$\frac{I_s + I_g}{I_g} = \frac{G + S}{S} \quad \text{[By adding 1 to both sides]}$$

or $$\frac{I}{I_g} = \frac{G + S}{S} \quad [\because I_s + I_g = I]$$

Thus the current I_g flowing through the galvanometer is a fraction of the main current and is equal to $\frac{S}{S + G}$. The value of S can be so adjusted that the fraction of the main current which the instrument is required to measure, is just sufficient to deflect the galvanometer needle through the whole range of the scale. The shunt resistance, from the above equation, is given by

$$S = \frac{I_g}{I - I_g} . G \quad \ldots \quad (1)$$

It is clear from this formula that if I_g is the current required by the galvanometer coil to produce a full range deflection of the needle, and if we wish to measure a higher current I with its help, we have to insert a shunt resistance S across the galvanometer coil, so that only I_g flows through the coil (thereby still producing the full range deflection), the remaining current being carried through the shunt.

Thus, to evaluate S to give a particular range (I) to the galvanometer, we have to determine the *figure of merit* of the

galvanometer, i. e., we have to know how much current should be sent through the galvanometer in order to produce a deflection of one division on the scale. Thus, if k be the figure of merit of the galvanometer, and n be the number of divisions on the scale, then

$$I_g = k.\, n. \qquad \ldots \qquad (2)$$

[Note. The following numerical example shall make the whole reasoning of the process very clear—

A galvanometer of resistance 30 ohms is provided with a pointer and a scale having 100 divisions. When a current of 2×10^{-4} amperes flows through the galvanometer, the needle is deflected through 1 division on the scale. What should be the resistance of the shunt so that the galvanometer may read 5 amps ?

From this problem it is clear that the figure of merit of the galvanometer is 2×10^{-4} amp. per division. Thus to produce a full scale deflection a current of $2 \times 10^{-4} \times 100 = 0{\cdot}02$ amp. is needed to pass through the galvanometer. If we wish to measure 5 amps. with it, we should use a shunt of resistance S such that 0·02 amp. current flows through the galvanometer, and the remaining current, $(5-0{\cdot}02) = 4{\cdot}98$ amp., passes through the shunt. Thus

$$\frac{S}{30} = \frac{0{\cdot}02}{4{\cdot}98} \qquad \therefore\ S = 0{\cdot}121 \text{ ohm}].$$

Method

[A] *Determination of the galvanometer resistance.*

[Note. If the galvanometer resistance is not given, determine it with the help of Kelvin's method, as described in expt.—13

[B] *Determination of the figure of merit of the galvanometer.*

For this set up the apparatus as shown in fig.–79. E is an accumulator connected in series with the galvanometer through a resistance box (preferably of the dial type) capable of giving high values for the resistance.

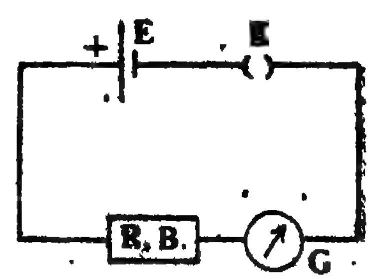

Fig. 79
Figure of merit of a galvanometer

Adjust a high resistance of about 5000 ohms in the box and close the key. A deflection will be produced in G. Now adjust the resistance in the box till a readable deflection is produced in the galvanometer. Note the resistance R and count the number of divisions of deflection. Measure the E. M. F., (E), of the cell *with a high resistance voltmeter*. Then

$$k = \frac{I}{n_1} = \frac{E}{n_1(R + G)}$$

where n_1 is the number of divisions through which the needle has been deflected when a current (say, I) flows through the circuit. If the total number of divisions on the galvanometer scale be n, then $I_g = kn$, which can be calculated out.

[C] *Determination of the shunt resistance and length of the shunt wire.*

Calculate the shunt resistance S from the equation (i) given above.

Now take a manganin wire and determine carefully the resistance for exactly one metre length of it with the post office box in the usual way. From this calculate ρ, the resistance per unit length of the wire. Then S/ρ will give the required length of the shunt wire. Cut a piece slightly longer than this calculated length and mark two points equidistant from the ends so that the length in between the marks is the calculated length. Connect the wire across the terminals of the galvanometer so that the marked points are just outside its binding terminals.

Now the galvanometer in conjunction with this length of shunt wire (of this thickness) has been converted into an ammeter which can read currents upto I amps.

[D] *Calibration of the converted galvanometer.*

Now set up an electrical circuit as shown in fig-80, in which A is an ammeter of nearly the same range as the converted galvanometer. Introduce a high resistance in the box and after pressing the key K take the reading of G and A. Convert the galvanometer reading to amperes and find the difference, if any, between the readings of the two instruments. This gives the error* of the galvanometer reading. In this way calibrate the whole dial of G and plot a graph taking the galvanometer readings as abscissae and corresponding ammeter readings as ordinates. This graph will be nearly a straight line and it will represent the calibration curve of the shunted galvanometer.

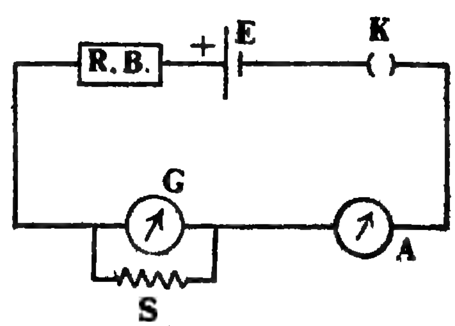

Fig. 80
Calibration of a galvanometer converted into an ammeter

* For more accurate work calibrate the shunted galvanometer with a potentiometer.

Observations

[A *Readings for the determination of the resistance of the galvanometer.*

[**Note.** See expt —13]

[**B**] *Readings for the figure of merit of the galvanometer.*

S. No.	Resistance introduced in the R. Box (R)	Deflection in the galvanometer (n_1)	Figure of Merit of the galvanometer (k)	Remarks
				1. No. of divisions on the galvanometer scale (n) = ... 2. E. M. F. of the cell (E) = ... volts.
		Mean		

[C] *Readings for resistance per unit length of the shunt wire.*

[**Note**—Make a table similar to one required for the resistance of the galvanometer.]

[D] *Calibration of the shunted galvanometer.*

S. No.	Reading of the shunted galvanometer		Ammeter reading (I′)	Error (I + I′)
	in divs.	in amps. (I)		

Calculations

(i) Current (I_g) required to produce a full scale deflection in the galvanometer $= k \times n = \ldots$ amp.

Now, (ii) Shunt resistance (S) $\dfrac{I_g \cdot G}{I - I_g}$ ... ohms.

Again, (iii) Resistance of the shunt wire per unit length, i. e., $\rho = \ldots$ ohm/cm.

(iv) Length of the shunt wire required $= S/\rho = \ldots$ cms.

Result. The length of the shunt wire of S. W. G. ...required to convert the given galvanometer into an ammeter of range ... amp. = ... cms.

Precautions and Sources of Error

[Note—For the precautions connected with the relevant experiments, see them at the places referred to above.]

(1) The accumulator used in this experiment should be fully charged and should be of a fairly large capacity, so that it gives a constant current.

(2) The resistance box should be a high resistance one and should preferably be of a dial pattern. *At no stage of the experiment should the resistance in the box be zero or small*, otherwise an excessive current shall flow through the galvanometer or ammeter which will consequently be damaged.

(3) The zero reading, if any, in the instruments should be carefully noted down and accounted for in the calculations. The ammeter used in the calibration part of the experiment should preferably be of the same range as the one which has been prepared with the shunted galvanometer.

(4) In this experiment the galvanometer is a uni-directional one, hence its positively marked terminal should be connected to the higher potential point of the circuit.

(5) While connecting the shunt wire across the galvanometer care should be taken to see that exactly the measured length is in parallel with the instrument.

EXPERIMENT—37

Object. To convert a Weston galvanometer into a voltmeter of a given range.

Apparatus Required*. Weston galvanometer†, a high resistance dial pattern resistance box, an accumulator, plug key, a high resistance voltmeter (to read the E M. F. of the cell), another voltmeter preferably of the same range as the one given for conversion.

* In this experiment the resistance of the galvanometer has to be known. If its value is not given (which should normally be given), in that case it has to be determined by Kelvin's method. In that case the necessary apparatus shall also be required. (See expt.—13).

† For this experiment it is preferable to use a special type of galvanometer, the zero mark of whose scale lies on the extreme left (and not in the centre as is usually the case) as in the case of ammeter and voltmeter scale.

Formula Employed—The series resistance R needed to convert the galvanometer into a voltmeter of a given range is calculated with the help of the formula—

$$R = \frac{V}{I_g} - G$$

where G = Galvanometer resistance.

V = P. D. that has to be read with the converted galvanometer (*e. g.*, the required range).

I_g = Value of the current required to get a full-scale deflection in the galvanometer.

PRINCIPLE AND THEORY OF THE EXPERIMENT

In the accompanying figure G is a galvanometer which requires a current, say, I_g to produce a full-scale deflection of its pointer. Now we have to read a potential difference of V volts with the help of this galvanometer. It is easy to understand that if we connect the galvanometer directly with V, excessive current shall flow and the coil of the galvanometer shall be burnt out. No current greater than I_g should be allowed to flow through G. Obviously the excessive current can be cut down by inserting a resistance of proper value so that the requisite current I_g flows through the galvanometer. If this resistance be R, we have from Ohm's law—

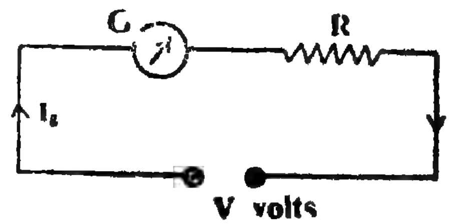

Fig. 81
Principle of a voltmeter

$$I_g = \frac{V}{G + R}$$

whence
$$R = \frac{V}{I_g} - G \quad \dots \quad (1)$$

Thus to evaluate the series resistance we have to know the value of I_g. For this purpose we should determine the *figure of merit* of the given galvanometer, *i. e.*, we should know how much current should be sent through the galvanometer in order that a deflection of one division is produced on the graduated scale. Thus, if k be the figure of merit of the galvanometer, and n be the number of divisions on the scale then

$$I_g = kn \quad \dots \quad (2)$$

[**Example**—In the numerical problem given in the body of the previous experiment let us calculate the series resistance which

shall convert the galvanometer into a voltmeter reading upto 5 volts.

Now, the current required to produce a full-scale deflection of the galvanometer = 0·02 ampere

If a P. D. of 5 volts is directly applied to the terminals of the galvanometer, the current flowing through it will be equal to 5/30 = .0 17 ampere nearly, which is more than eight times the normal current, hence the galvanometer coil shall be burnt out.

Hence to reduce this current to the normal value of 0·02 amp. and at the same time to convert the galvanometer into a voltmeter, a series resistance should be added. The resistance of this wire shall be given by

$$R = \frac{V}{I_g} - G = \frac{5}{0 \cdot 02} - 30 = 220 \text{ ohms. }]$$

Method

[A] *Determination of the galvanometer resistance.*

[**Note**—If the galvanometer resistance is not given, determine it by Kelvin's method. See expt.—13]

[B] *Determination of the figure of merit of the galvanometer.*

[**Note**—This has been fully discussed in the previous experiment]

[C] *Determination of the series resistance and the length of the wire*—From the above determinations calculate the value of R, the series resistance, with the help of equation (1) given above. Now take a manganin wire and determine the resistance* of *exactly* 1 metre length of the wire with a post-office box in the usual way.

From this calculate ρ the resistance per unit length (*i. e.* per cm.) of the wire. Hence the length of the wire required to be connected with the galvanometer $= R/\rho$. Connect this length in series with the galvanometer. Now the given galvanometer in conjunction with this length of resistance (of this thickness) has been converted into a voltmeter and can read voltages upto V volts.

* Alternatively, knowing the gauge number of the wire the resistance per metre can be obtained from the Tables of Constants.

[D] *Calibration of the converted galvanometer*—Set up the apparatus as shown in fig.-82, in which Rh is a rheostat, whose fixed terminals are connected to E. V is a voltmeter of nearly the same range as that of the converted galvanometer.

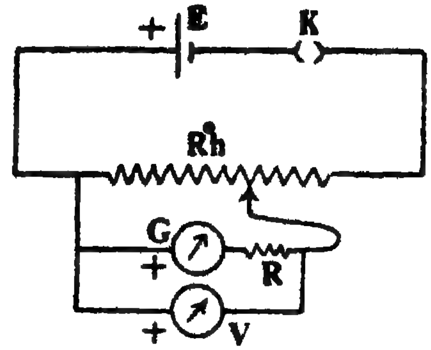

Fig. 82
Calibration of a galvanometer converted into a voltmeter

By shifting the position of the sliding contact of the rheostat, take a number of readings in G and their corresponding ones in V. Convert the galvanometer readings into volts and calculate the error†, if any, between the two values. In this way calibrate the whole dial of G and plot a graph between the galvanometer readings (represented on the x-axis) and the corresponding voltmeter readings (represented on the y-axis). This is the calibration curve of the galvanometer converted into a voltmeter.

Observations

[**Note.** Make appropriate tables with the help of those given in the previous experiment.]

Calculations

(i) Current (I_g) required to produce full scale deflection of the galvanometer = kn =..... amp.

(ii) Series resistance (R) $=V/I_g - G =$ ... ohm

(iii) Resistance of this wire per unit length, ρ =...ohm/cm

(iv) Length of the wire required to be put in series with the galvanometer $= R/\rho =$ ... cm

Result. The length of the manganin wire of S. W. G. ...... required to convert the given galvanometer into a voltmeter of range.........volts =.........cms.

Precautions and Sources of Error

[**Note.** For precautions connected with relevant experiments other than this, see them at their appropriate places referred to in the body of the text.]

(1) The accumulator used in this experiment should be fully charged and should be of a fairly large capacity, so that it gives a constant current throughout the experiment.

† For more accurate work the converted galvanometer should be calibrated with a potentiometer.

(2) The resistance box should be a high resistance one, and should preferably be of a dial pattern. *At no stage of the experiment should the resistance in the box be zero or small*, otherwise an excessive current shall flow through the galvanometer which will consequently be damaged.

(3) The zero reading if any in the galvanometer or the voltmeter should be carefully noted down and accounted for in the calculations. The voltmeter used in the calibration part of the experiment should preferably be of the same range as the one which has been prepared with the galvanometer.

(4) In this experiment the galvanometer used is a unidirectional one, hence its positively marked terminal should be connected to the higher potential point in the circuit. The same precaution should be observed with the voltmeter.

(5) While connecting the wire in series with the galvanometer it should be carefully noted that *only the marked length*, as required by calculation, is in series with the circuit, the extra portions of the wire on either end should be inside the appropriate binding terminals.

MISCELLANEOUS EXPERIMENTS

EXPERIMENT—38

Object. To determine the frequency of A. C. mains with the help of an electrical vibrator.

Apparatus Required. Electrical vibrator, a friction-less pulley, a uniform cord, a small pan, and a weight box.

Description of the Apparatus*. The *Electrical Vibrator* consists

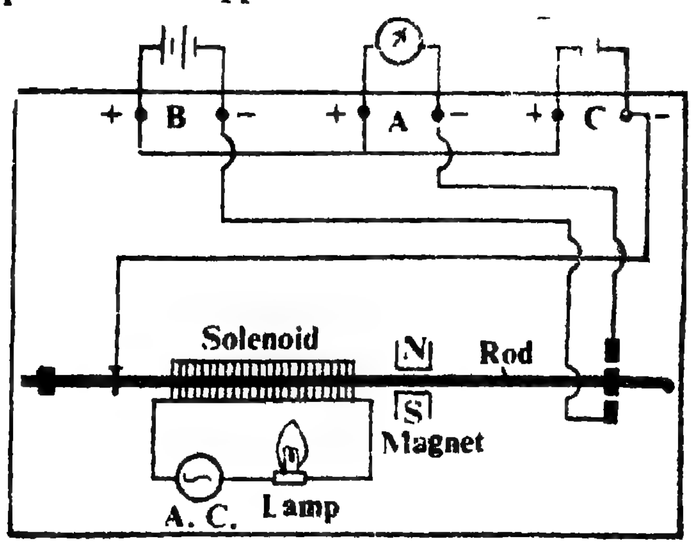

Fig. 83
An electric vibrator

* Since this vibrator can also be employed *for the determination of the capacity of a condenser*, terminals marked B (for connecting a Battery), A (for a micro-Ammeter), C (for a Condenser) are also provided on its baseboard. Internal connections are provided as shown in the figure. The steel rod carries near its hooked end a small iron piece with flat ends. When the rod is set vibrating, it makes, with the help of this iron piece, alternate contact with the terminals provided nearby. This operation during one half cycle of the alternating current charges the condenser, while during the other half cycle the condenser is discharged through the microammeter. For performing this experiment there is no need of using the thread.

of a solenoid through which passes a steel rod one end of which can be clamped, the other end ends in a hook to which a string under tension can be attached. The solenoid is energised by current drawn from the A. C. mains through a suitable bulb resistance. The steel rod passes through the pole-pieces of a permanent horse-shoe magnet mounted on the baseboard.

Formula Employed. The frequency (n) of the A. C. mains is given by the formula—

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}} = \frac{1}{2l}\sqrt{\frac{Mg}{m}}$$

where T = Tension applied to the string
= Mg (M = mass hung at the end of the string)
m = Mass per unit length of the string
l = Length of one loop of the vibrating string

PRINCIPLE AND THEORY OF THE EXPERIMENT

When the solenoid is energised by passing an alternating current through it, the steel rod placed inside it gets magnetised longitudinally with its polarity reversing during each half cycle of the current. The magnetic field supplied by the permanent horse-shoe magnet produces oscillations by interacting with the magnetised rod, the necessary energy being derived from the electric supply. The length of the steel rod can be adjusted so as to get resonant vibration indicated by a large amplitude of vibration of its free end. The vibrations are communicated to the stretched string which begins to vibrate in a number of segments*, the frequency of the string being the same as of the rod, which is vibrating with the frequency of the A. C. mains If l be the length of one loop of the string, the frequency of the string is given by—

$$n = \frac{\nu}{\lambda} = \frac{\nu}{2l} = \frac{1}{2l}\sqrt{\frac{T}{m}} \qquad \left(\because \nu = \sqrt{T/m}\right)$$

where T is the tension and m is the mass per unit length of the string. This is also the frequency of the A. C. mains.

Method

(i) After inserting a 25-watt lamp in the socket provided for it on the baseboard, switch on the current and adjust the length of the steel rod so that it is thrown in resonant vibration as evidenced by the amplitude attained by the free end.

* Stationary waves are produced in the string forming nodes and antinodes. Thus the string is divided in several segments. If the length of one segment be l, we have $l = \lambda/2$, where λ is the wavelength of the waves travelling along the string.

(ii) Now switch off the current and tie a uniform cord to the road. Pass the cord over a frictionless pulley attached to the table, and to its free end tie a light pan. Put some weight on the pan.

(iii) Switch on the current when the string will be found to vibrate in a number of loops, which can be sharply defined by displacing the vibrator thereby altering the length of the cord. Mark the position of the nodes and measure the distance between the consecutive nodes, and thus determine mean length* of a loop.

(iv) Repeat this process by keeping the tension constant and altering the length of the cord vibrating in resonance with the rod. Calculate the mean value of the length (l) of one loop.

(v) Weigh the pan and compute the total tension applied to the cord Also weigh an *exactly* measured length (say, 2 metres) of the cord in a chemical balance, and thereby calculate m, the mass per unit length of the cord

Calculate the frequency† from the formula given above.

Observations

S. No.	No of loops	Length of the loops	Length of one loop (l)	Mass of the pan (m_1)	Mass placed on the pan (m_2)	Total mass hung M (m_1+m_2)	Remarks
1.							Mass of 200 cms of cord =...gm ∴ m =...gm/cm
2.							
3.							
		Mean					

[Note. Make similar tables for other values of the tension.]

Calculations

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}} = \frac{1}{2l}\sqrt{\frac{Mg}{m}}$$

$$= \ldots\ldots\ldots\ldots \text{cycles/sec.}$$

* As the exact position of the first and the last node cannot be ascertained, they can be omitted in this measurement.

† The experiment may be repeated by altering tension, and thereby calculating the mean value of the frequency.

[**Note.** If a number of readings for T and l have been taken then the frequency can be calculated by finding T and l^2 *for each set separately* and then calculating the mean value of T/l^2 from these values and putting this value of T/l^2 in the formula

$$n^2 = \frac{1}{4m}\left(\frac{T}{l^2}\right).]$$

Result. The frequency of A.C. mains = ...cycles/sec.

Precautions and Sources of Error

(1) For this experiment a cord possessing a fairly constant mass per unit length should be employed. Hence a fishing cord, which fulfils this condition satisfactorily, should be preferred.

(2) Initially when the steel rod vibrates, its length should be adjusted so that it vibrates in resonance with the frequency of the A. C. mains This is accomplished when the free end vibrates with maximum amplitude.

(3) The length of the cord should be so adjusted that the nodes formed on it are well-defined. Due to uncertainty in the exact location of the first and the last nodes, they should not be taken into account while measuring the length of a loop.

(4) The pulley employed in this experiment should be a frictionless one, otherwise the tension acting on the string shall be different from the one actually applied. This will then constitute a source of error.

ADDITIONAL EXPERIMENT

Expt.—38 (a)

As indicated above, the electrical vibrator can also be employed to determine the capacity of a condenser. For this purpose connect a battery at B, a micrometer* at A, and the given condenser at C. A study of the internal connections of the vibrator shall reveal that during one half of the cycle the vibrating rod makes contact with the battery and the condenser, thus charging it to a voltage E. During the next half cycle the condenser plates are short-circuited through the microammeter and thus the condenser gets discharged through it. If n be the frequency of the A. C. mains, this process of charge and discharge of the condenser is repeated n times per second. Thus, the charge passing through the microammeter per second (i. e., the value of the current flowing

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* If a micrometer is not available in the laboratory, a sensitive galvanometer can be employed, but in that case the figure of merit of the galvanometer should be known. If it is not known, it should be determined as described in expt.-36.

through it) is equal to nCE coulombs. Thus if I be the current recorded by the microammeter, we have $I = nCE$, or

$$C = \frac{I}{nE} = \frac{\text{Current}}{\text{Frequency} \times \text{Voltage}}$$

[**Note.** For instance, in a particular experiment the microammeter registered a constant current of 200 μA, when the e. m. f. of the cell was 2 volts. Then

$$C = \frac{I}{nE} = \frac{200 \times 10^{-6}}{50^{*} \times 2} = 2 \times 10^{-6} \text{ farads, or } = 2\mu f.]$$

For the success of the experiment it may be necessary to adjust the contact of the vibrating rod with the flat discs provided, so that the microammeter registers a steady deflection when the vibrator is working. Moreover, the e. m. f. of the cell should be recorded with the help of a high resistance voltmeter.

EXPERIMENT—39

Object. To determine the frequency of A. C. mains by means of a sonometer.

Apparatus Required. A vertical pattern sonometer, a solenoid with a soft iron core, a pan (or a hanger), half kgm-weights, chemical balance and weight box.

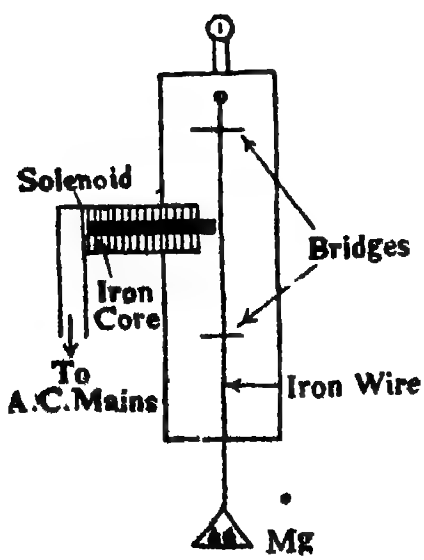

Fig. 84
Vertical sonometer for frequency of A. C. mains

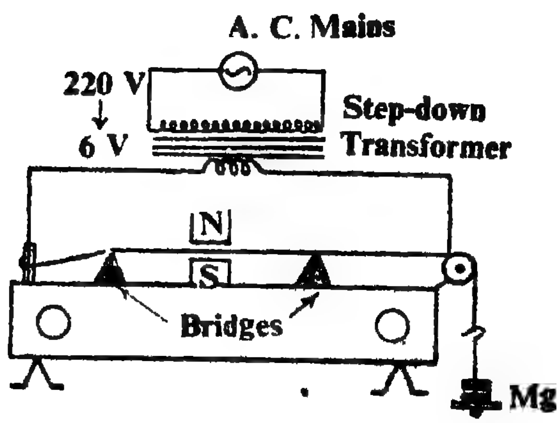

Fig. 85
Horizontal sonometer for frequency of A. C. mains

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* The frequency available for the city supply is generally 50 cycles/sec.

Description of the Apparatus. The apparatus consists of a vertical pattern sonometer on which is stretched an *iron* wire. A solenoid having a large number of turns of insulated copper wire and carrying a soft iron core along its axis is clamped near the middle of the segment of the wire between the two bridges. The lower end of the wire carries a pan on which suitable weights can be placed.

[**Note.** Another variation of the apparatus* is the usual horizontal pattern sonometer on which is stretched a *brass* wire. The alternating voltage is stepped down to, say, 6 volts by means of a step-down transformer and then is connected to the wire as shown. The wire passes between the pole-pieces N and S of a permanent horse shoe magnet. The wire therefore experiences an alternating force due to the field of the magnet on the current in the wire, and for a particular length of the wire between the bridges it is thrown into resonance as is evidenced by a large amplitude. This condition is achieved when the frequency of the alternating current passing through the wire is equal to its mechanical frequency of vibration, which is given by the formula $n = \frac{1}{2l}\sqrt{\frac{T}{m}}$]

Formula Employed†. The frequency (n) of the mains is given by the formula—

$$n = \frac{1}{4l}\sqrt{\frac{T}{m}}$$

where l = Length of the sonometer wire between the two bridges when it is thrown in resonant vibration.

T = Tension applied to the wire.

m = Mass per unit length of the wire.

PRINCIPLE AND THEORY OF THE EXPERIMENT

If an alternating current is passed through a solenoid having a soft iron core, the core is temporarily magnetised twice during each cycle of alternation—first with one polarity when the oscillation of the current is in one direction, and then with the opposite polarity when the current flows in the opposite direction. When the sonometer wire is held close to the core, it will be pulled twice during each cycle, and consequently if the frequency of the alternating current be n, the wire shall be pulled 2n times per second. If the length and tension of the wire be so adjusted that its natural frequency is also 2n, the wire will be thrown in resonant

* The vertical pattern is preferable to the horizontal one, since friction at the pulley is completely eliminated.

† Carefully note the difference in the two formulae.

vibration and the amplitude of vibration of the wire will be maximum. If the tension applied to the wire be T and m be the mass per unit length of the wire, the frequency of vibration N of the wire is given by—

$$N = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

where l is the resonant length of the wire.

The frequency (n) of the A. C. mains will be equal to N/2. Hence

$$n = \frac{1}{4l}\sqrt{\frac{T}{m}}$$

Method

(i) Before starting the actual experiment, have an idea of the breaking stress for the material of the wire from the Table of Physical Constants. From this value calculate the breaking tension (= breaking stress × area of cross-section of the wire) for your wire. During subsequent experiment the *weight in the pan should not exceed half the breaking tension.*

(ii) Suspend the sonometer from the nail on the wall and see that the pan provided below stays clear of the wall. Put a suitable load on the pan. Switch on the current and adjust the core of the solenoid near the middle of the wire between the bridges.

(iii) With the help of the bridges adjust the length of the wire till it begins to vibrate under the influence of the magnetic field provided by the core. During this adjustment the core should always be placed near about the middle of the vibrating wire.

Now by a slight delicate adjustment attain a position when the wire is thrown in violent resonant vibration and the amplitude is maximum.

(iv) Switch off the current and measure the length of the vibrating wire by holding a scale on the bridges and avoiding the error due to parallax. Record the tension, *which should include the mass of the pan or the hanger.*

(v) Change the tension in suitable steps and obtain the corresponding lengths of the vibrating wire. Now weigh in a chemical balance a known length (say, 100 cms) of the sonometer wire and thus calculate m, mass of the wire per unit length.

(vi) Calculate the frequency of the A. C. mains as indicated below.

Observations

S. No.	Tension* applied to the wire (T)	Length of the resonating wire (l)	l^2	Remarks
				Mass of 100 cms. of wire = ...gm.
				∴ m = ...gm/cm.
Mean		Mean		

Calculations

Substituting the mean values of T and l^2 in the formula we have—

$$n^2 = \frac{1}{4m} \cdot \left(\frac{T}{l^2} \right)$$

$$= \dots \dots \dots$$

Hence $= \dots \dots$ cycles/sec.

Result. The frequency of the A. C. mains = ... cycles/sec.

Precautions and Sources of Error

(1) The sonometer wire should be uniform and free from kinks.

(2) For bringing the wire in resonant vibration, start with a small length of the wire and increase the length in small steps. The solenoid should be so placed that its iron core is situated close to the middle of the vibrating portion of the wire.

(3) While finding out the tension of the wire, do not forget to add the mass of the pan or of the hanger. If a sonometer employs a spring balance note down its zero error, if any.

(4) While increasing the tension of the wire, be careful that the wire is not stretched beyond the elastic limit. For this purpose, before starting the experiment have an idea of the magnitude of the breaking load of the given wire from the Table of Physical Constants.

* This includes the mass of the pan (or the hanger).

(5) In the derivation of the formula $\nu = \sqrt{T/m}$ it has been assumed that the wire is perfectly flexible. Hence due to the rigidity of the experimental wire an error shall creep in the result.

(6) If the wire is not uniform or if its composition is variable then also the result will be erroneous.

(7) The tension on the two sides of the bridges may not be the same.

[**Note** If the horizontal pattern of the sonometer is employed, there will be an additional source of error. There may be friction at the pulley, hence the value of the tension is less than that actually applied. This consequently affects the value of the frequency.]

EXPERIMENT—40

Object. To determine the impedance of a given A. C. circuit.

Apparatus Required. An inductance, a condenser, a resistance, A. C. ammeter and voltmeter and flexible cord for making electrical connection.

Formula Employed. The impedance of the circuit is given by the following formula :—

$$Z = \frac{E_*}{I_*}$$

where Z = The required impedance

E_* = Virtual E. M. F. (as measured by A. C. voltmeter)

I_* = Virtual Current (as measured by A. C. ammeter)

PRINCIPLE AND THEORY OF THE EXPERIMENT

Let a harmonically varying voltage, $E_0 \sin \omega t$, be applied to a circuit containing an inductance L, a resistance R, and a capacitance C in series, as shown in fig.–86. The value of the current flowing in this circuit is obtained by solving the potential equation of the circuit. The potential equation of the circuit is

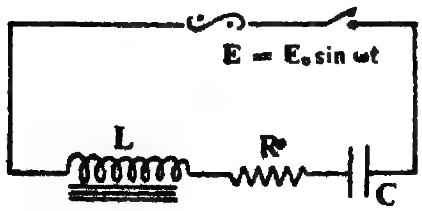


Fig. 86
Circuit containing L, C, and R

$$L\frac{dI}{dt} + IR + V = E_0 \sin \omega t \quad \dots (1)$$

where I is the instantaneous value of the current and V is the potential difference between the coatings of the condenser at that instant. If Q be the charge on the condenser at that moment, the above equation reduces to

$$L\frac{dI}{dt} + IR + \frac{Q}{C} = E_0 \sin \omega t \qquad \ldots \qquad (2)$$

Let the solution* of this equation be

$$I = I_0 \sin(\omega t - \phi) \qquad \ldots \qquad (3)$$

where I_0 and ϕ are to be determined. From (3) we have

$$\frac{dI}{dt} = I_0 \omega \cos(\cos \omega t - \phi)$$

Now because $dQ = I \cdot dt = I_0 \sin(\omega t - \phi)\, dt$, we have on integrating this

$$Q = -\frac{I_0}{\omega} \cos(\omega t - \phi)$$

Substituting the values of these expressions in equation (2) we have

$$L I_0 \omega \cos(\omega t - \phi) + I_0 R \sin(\omega t - \phi) - \frac{I_0}{C\omega} \cos(\omega t - \phi) = E_0 \sin \omega t$$

or
$$I_0 R \sin(\omega t - \phi) + I_0 \left(L\omega - \frac{1}{C\omega} \right) \cos(\omega t - \phi) = E_0 \sin \omega t$$

Comparing the coefficients of $\sin \omega t$ and $\cos \omega t$ on the two sides of this equation we have

$$I_0 R \cos \phi + I_0 (L\omega - 1\ C\omega) \sin \phi = E_0 \qquad \ldots \qquad (4)$$

and
$$-I_0 R \sin \phi + I_0 (L\omega - 1/C\omega) \cos \phi = 0 \qquad \ldots \qquad (5)$$

Now, squaring and adding (4) and (5) we have—

$$I_0^2 [R^2 + (L\omega - 1/C\omega)^2] = E_0^2$$

whence
$$I_0 = \frac{E_0}{\sqrt{R^2 + (L\omega - 1/C\omega)^2}} \qquad \ldots \qquad (6)$$

Also from equation (5) we have

$$\tan \phi = \frac{L\omega - 1/C\omega}{R} \qquad \ldots \qquad (7)$$

---

* The only part of the solution of equation (2) which is of importance to us is that in which the current has the same periodicity as the electromotive force, any other being quickly damped out.

Thus from equation (6) it is clear that effective resistance, or the impedance of the circuit is given by

$$Z = \sqrt{R^2 + (L\omega - 1/c\omega)^2} \quad \ldots \quad (8)$$

where $L\omega$ is the inductive reactance and $1/c\omega$ is the capacitive reactance. From equation (6) we have

$$Z = \frac{E_0}{I_0} = \frac{E_0/\sqrt{2}}{I_0/\sqrt{2}} = \frac{E_*}{I_*} \quad \ldots \quad (9)$$

where E_* and I_* are the virtual voltage and virtual current respectively. Normal A. C. measuring instruments† measure virtual values of voltage and current.

Thus by measuring E_* with an ordinary A. C. voltmeter and I_* with an A. C. ammeter, the impedance (Z) of the circuit can be evaluated.

Method

(i) Set up the apparatus as shown in fig.–87. Connect the primary of the step-down transformer to the A. C. mains. The secondary is connected through a rheostat to a choke coil (L), a condenser (C), and a resistance (R). Connect the A. C. ammeter (A) in series and the A. C. voltmeter (V) in parallel with this circuit.

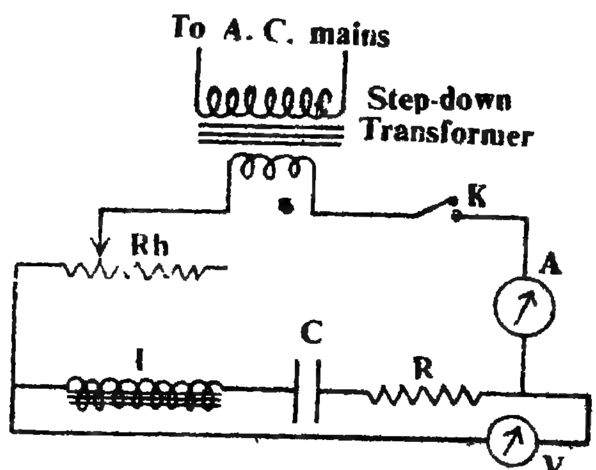


Fig. 87
Impedance of an A. C. circuit

(ii) When the connections have been properly made, switch on the current and for a particular setting of the rheostat record the readings of the ammeter and the voltmeter. In this way by adjusting the rheostat take several readings for the values of the current and the voltage.

(iii) Plot a graph between virtual volts V_* (represented along the y—axis) and virtual amperes I_* (represented along the

† For a detailed study of these instruments read author's book "A Critical Study of Practical Physics and Viva-Voce."

x-axis). The graph shall be a straight line. The slope ($= \tan\theta$) of the straight line gives the impedance of the circuit.

(iv) Calculate also the value of the impedance by taking the known values of L, C, and R, and substituting these values in equation (8) given above.

Observations

S. N.	Voltmeter reading (V_*)	Ammeter reading (I_*)	Slope of the graph (Z)
1	...... volts	...... amps.	
⋮			...... ohms.
⋮			

Calculations

From the graph, $Z = \frac{V_*}{I_*} = \ldots$ ohms

Again $Z = \sqrt{R^2 + (L\omega - I/C\omega)^2}$

Here $R = \ldots$ ohms.

$L = \ldots$ henry

$C = \ldots$ farad

and $\omega = 2\pi n = 2\pi . 50$

Thus $Z = \ldots \ldots = \ldots$ ohms.

Result. The impedance of the given A. C. circuit ohms.

ADDITIONAL EXPERIMENTS

Expt.—40 (a)

Variation of impedance of the circuit with frequency and determination of L or C from the resonant frequency.

From the relation

$$I_* = \frac{E_*}{R^2 + L\omega - I/C\omega)^2}$$

it is clear that if the A. C. voltage be kept constant and its frequency be varied, the current amplitude changes. If we plot a graph between I on the y-axis and frequency (n) on the x-axis, we get a curve as shown in the accompanying figure. Obviously the maximum current amplitude is obtained when the impedance of the circuit is minimum. The impedance is minimum when

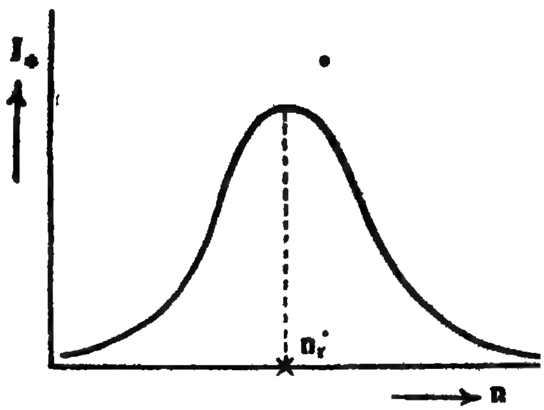


Fig. 88
Variation of current with frequency

$$L\omega - \frac{1}{C\omega} = 0 \text{ or } \omega^2 = \frac{1}{LC}$$

$$\text{Since } \omega = 2\pi n\ ;\ n^2 = \frac{1}{4\pi LC}$$

$$\text{or} \qquad n = \frac{1}{2\pi\sqrt{LC}}$$

When the frequency of the applied source is equal to this frequency, the circuit is said to be in resonance, and under this condition the current amplitude is maximum and the current and voltage are in phase with each other since $\tan\phi = 0$. Since the inductive reactance cancels the effect of the capacitive reactance, the current in this case is determined purely by the ohmic resistance.

Now, to conduct this experiment an alternating voltage source of variable frequency is needed. For this purpose, a valve oscillator can be employed as a variable frequency source. The electrical connections are made as above, and after adjusting the source for the lowest frequency which it can produce, the readings of the voltmeter and the ammeter are taken. The frequency (n) of the source is varied in steps and the corresponding values of the voltage (E) and the current (I) are recorded. The value of the impedance (Z = E/I) is calculated for each value of the frequency (n). Finally a graph is drawn between these two quantities (n and Z) and the frequency corresponding to minimum impedance is noted. Now, resonant frequency

$$n_r = \frac{1}{2\pi\sqrt{LC}}$$

Hence knowing n_r from the graph L or C can be calculated if the other quantity is given.

Expt.—40 (b)

Determination of the frequency of the alternating voltage.

For this purpose a variable condenser of known capacity replaces the one used in the main circuit above. If the variable capacity is changed, the current in the circuit as read by the ammeter also changes. Now the capacity is varied in steps and its values as well as the corresponding values of the current are noted down. These are depicted on a graph from which the value of capacity C_r giving the maximum current in the circuit is noted. The frequency of the voltage is calculated from the formula

$$n_r = \frac{1}{2\pi\sqrt{L C_r}}$$

[**Note.** This method is more convenient if the frequency of the source is, say, of the order of 1000 cycles per sec. If the frequency to be measured is of the supply mains, which is generally 50 cycles per second, the method adopted in Expt.—38 or 39 is always preferred.]

EXPERIMENT—41

Object. To draw the characteristic curves between grid voltage and plate current of a triode valve and with the help of these curves to determine the values of the amplification factor, the plate resistance, and the mutual conductance of the valve.

Apparatus Required. A triode valve, characteristic-curve-apparatus fitted with meters, etc., a 2-volt accumulator, high tension battery, low tension battery, rheostats and plug-keys.

Description of the Triode Valve. When a metal is heated to a high temperature, it begins to emit electrons. This phenomenon,

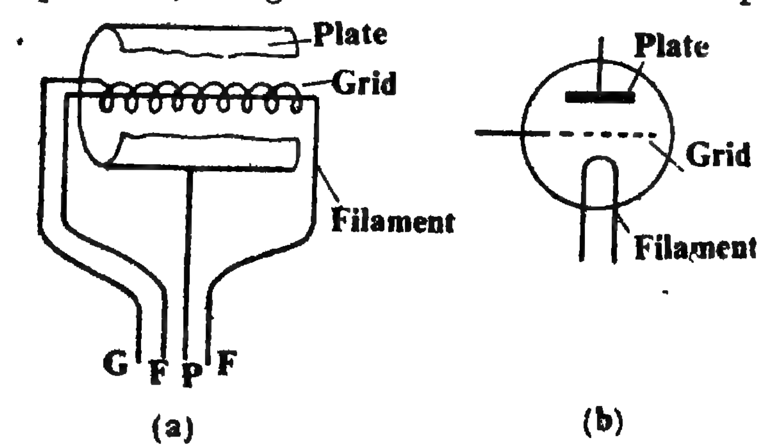


Fig. 89
A triode valve (sectional diagram)

called the *therminoic emission*, is utilised in the construction of

valves. The triode or the three-electrode valve, invented by Lee de Forest in 1907, consists of three components—the filament, the grid, and the anode or the plate, all mounted in a glass or metallic tube which is either highly evacuated or contains a trace of an inert gas.

In one pattern of the triode valve (fig -89 a) the grid is an open spiral wire surrounding the filament, and the plate is a cylinder of thin metal enveloping the grid and the filament. Fig -89 (b) depicts the conventional mode of representing the valve in diagrams.

The filaments are generally of two types :—

(i) The *directly heated type*, in which the filament is either a pure tungsten wire, or a thoriated one, or coated with special active material, such as alkaline earth metals and their oxides.

(ii) The *indirectly heated type*, in which the filament consists of a metal tube with insulated heater wire of pure tungsten at the centre. The metallic tube is externally coated with electron-emitting oxides.

The grid is usually made of spiral or mesh of molybdenum wire wound in grooves in the supporting wire. The plate is usually a circular or flattened cylinder of nickel or iron.

When the filament is heated by passing an electric current through it the electrons emitted by it are attracted by the plate which is always maintained at a high positive potential with respect to the filament. The grid may be raised to a positive or negative potential with respect to the filament. Consequently the electrons coming from the filament will either be attracted or repelled by the grid. Thus the electronic current flowing from the filament is determined jointly by the potentials of the plate and the grid, but as the grid is situated closer to the filament than the plate, it is much more effective in controlling the plate current. Thus the grid acts as a control electrode in a triode valve.

Formula Employed*

(i) The *Amplification Factor* (μ) is determined by the formula—

$$\mu = \left| \frac{\Delta E_p}{\Delta E_g} \right|_{I_p} \quad \ldots \quad . \quad (1)$$

which means that if the plate voltage is increased by an amount ΔE_p, the grid voltage has to be decreased by ΔE_g in order to

---

*The valve parameters are approximately constant over the straight part of the characteristic curves. (See fig.-92) hence their valves are determined only in this region.

keep the plate current I_p constant. As a matter of fact, amplification factor of a triode valve is a measure of the effectiveness of the grid with respect to the plate (or the anode) in controlling the plate (or anode) current, and may be defined as *the ratio of the change in the anode voltage, required to produce a certain change in the anode current, to the change in the grid voltage which would cause the same change in the anode current.*

(ii) The *Plate Resistance* (r_p) is calculated by the formula—

$$r_p = \left| \frac{\Delta E_p}{\Delta I_p} \right|_{E_g} \qquad \ldots \qquad (2)$$

which means that for a constant grid voltage E_g, a change in plate voltage by ΔE_p results in a corresponding change in plate current by ΔI_p. Plate resistance may be defined as the *reciprocal of the rate of variation of the anode current with anode voltage, when the grid voltage is kept constant.*

(iii) The *Mutual Conductance* (g_m) is evaluated from the formula—

$$g_m = \left| \frac{\Delta I_p}{\Delta E_p} \right| E_p \qquad \ldots \qquad (3)$$

which means that for a fixed plate voltage E_p, if the grid voltage changes by ΔE_g, the corresponding change in the plate current is ΔI_p. *Mutual conductance may be defined as the rate of change of anode current with grid voltage, when the plate (or anode) voltage is kept constant.*

PRINCIPLE AND THEORY OF THE EXPERIMENT

When the filament (of a directly heated valve) or the separate heater-cathode (of an indirectly heated one) is electrically heated, it becomes a source of electrons which accumulate in the region surrounding the filament. If an external field is applied which removes the electrons as fast as they are produced, the electrons begin to drift in a continuous stream which constitutes an electric current. However, if the external field is not sufficiently great to remove the electrons as fast as they are produced, a cloud of electrons will perpetually be formed near the filament surface, and will consequently exert a repulsive force on those electrons which are just to leave the surface. This electron cloud is known as *Space Charge*. If the external field is withdrawn, the space charge may attain a value which repels all the electrons as soon as they are emitted. Under this circumstance the flow of the current will cease.

In a triode valve the external field round the filament is jointly applied by the grid and the plate. Voltages applied to the grid

have a larger effect on the electrons emitted by the filament than the voltages applied to the plate since the grid is situated nearer the filament. The number of electrons drawn away from the filament is dependent on the plate voltage as well as the grid voltage, and since the grid does not obstruct the passage of electrons flowing through it, the electrons reach the plate constituting the plate current.

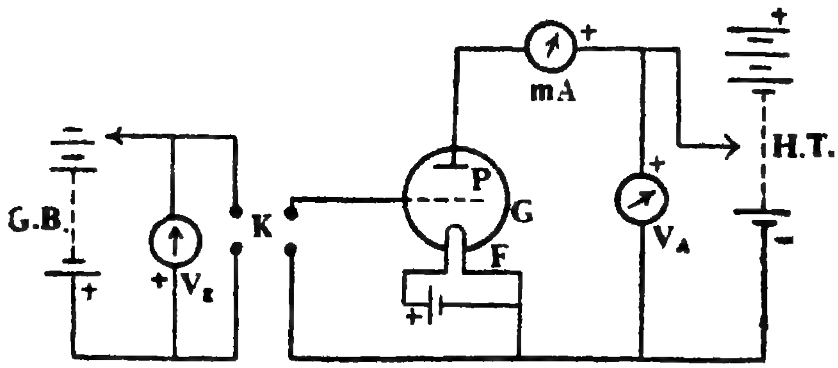


Fig. 90
Connections for a triode valve

Thus if the filament temperature is kept constant, the plate current I_p is a function of the plate voltage E_p as well as the grid voltage E_g. The most important characteristic curve* of a triode is the curve showing the variation of plate current with the variation of grid voltage for any fixed value of plate voltage. These curves can be studied with the help of the arrangement shown in fig.–90.

The filament F is heated by a 2-volt accumulator (or in accordance with the specifications prescribed by the maker for that particular valve), and the anode (plate) is connected to a high tension battery (H. T.) in series with a milliammeter (mA). The grid is connected to a variable grid-bias battery (G.B.) through a reversing key K. The characteristic curves obtained are of the type shown in the accompanying figure.

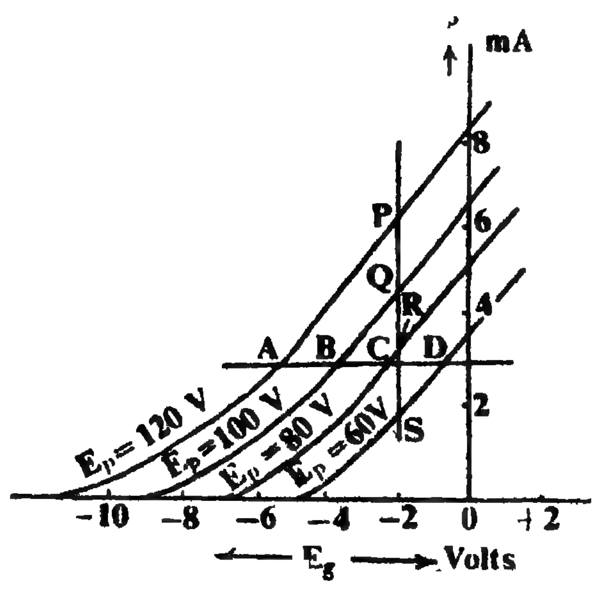


Fig. 91
Characteristic curves of triode

* This is also known as a *mutual characteristic* to distinguish it from *anode characteristic* which depicts the variation of anode current with anode potential for any fixed grid voltage.

The tube parameters* are calculated as follows :—

(i) *Amplification Factor*. Draw a line parallel to the x-axis cutting the straight portions of the curves at A, B, C, and D respectively. As the plate current I_p has the same value at A, B, C, and D, the amplification factor is given by the following expression :—

$$\mu = \left|\frac{\Delta E_p}{\Delta E_g}\right|_{I_p} = \frac{V_A - V_B}{AB} = \frac{V_B - V_C}{BC} = \ldots \text{ etc.}$$

The mean of these values may be taken as the amplification factor of the valve. Amplification factor has no unit and its value is always greater than unity.

(ii) *Plate Resistance*—Now draw a vertical line cutting the curves at P, Q, R, S respectively. Since E_g, the grid potential, is constant for all these points, the value of the plate resistance is given by—

$$r_p = \left|\frac{\Delta E_p}{\Delta I_p}\right|_{E_g} = \frac{V_P - V_Q}{PQ} = \frac{V_Q - V_R}{QR} = \ldots \text{ ohms.}$$

The unit of plate resistance is "ohms".

(iii) *Mutual Conductance*—For this purpose consider any one curve, and select out two points such as B and Q. The plate voltage E_p remains constant, hence the mutual conductance.

$$gm = \left|\frac{\Delta I_p}{\Delta E_g}\right|_{E_p} = \frac{i_p}{e_g} \text{ mho}$$

where i_p is the difference in the plate current, and e_g is the difference in the grid voltage for these points. Values of mutual conductance are calculated on the four curves separately. The unit of mutual conductance is "mho".

Note:

(i) Before starting the actual experiment ascertain the specifications prescribed by the manufacturer for the particular valve under experimental study. These specifications should be

* The experimentally determined values of these parameters can be employed to verify the relation—

$$\mu = r_p \times g_m$$

strictly followed. Now insert the valve in its socket and make the connections* as shown in fig.-90.

[Note—The high tension voltage may be had from a battery of dry cells, or it may be obtained from the D.C. mains, if available in the laboratory, with the help of resistances as shown in fig.-92. R is a fixed resistance which helps in creating a potential drop across the rheostat Rh, which is being used as a potential divider, from which suitable voltages can be tapped.

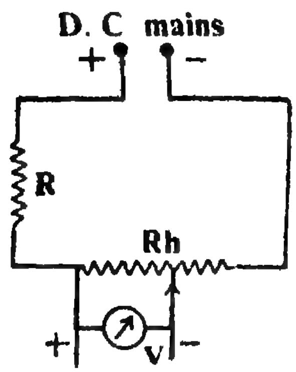


Fig. 92
High tension
for use with
a triode

If an A.C. mains is available in the laboratory, the anode potentials can be taken by using a rectifier set, which may incorporate, for instance, a metal rectifier which changes the alternating current into a unidirectional pulsating current, which is further smoothened by a capacity and choke combination.]

(ii) Adjust the plate voltage to any convenient value (say, 60 volts), and connect the grid bias to the reversing key in such a way that the grid becomes negative with respect to the filament. Adjust the variable tap of the grid bias in such a way that the plate current is zero. Now change the grid bias in equal steps till the voltage applied to the grid is zero. Reverse the key now so that the grid becomes positive with respect to the filament. Increase the grid voltage in steps to the maximum *permissible value*. Record each value of the grid voltage and the corresponding anode current. Also note down the anode voltage†.

(iii) Plot a curve between the grid voltage and the plate current taking the various grid voltage as abscissae and the corresponding values of the plate current as ordinates.

(iv) Now change the plate potential to, say, 80, 100, volts etc. and take a few more sets of observations for the variation of the plate current with the grid potential, and draw similar curves‡ on the same graph paper.

(v) Calculate the tube parameters as explained above.

---

* If the apparatus supplied in the laboratory for this experiment is a ready-made one, then study the internal connections carefully, this is very essential.

† It is essential that throughout this measurement, the plate voltage, fixed earlier, is maintained constant.

‡ A typical set of such curves obtained with a particular type of triode is illustrated on the graph at the end of this experiment.

Observations

S. No	Grid potential	Plate current when the plate potential is kept at a constant value of		
		60 volts	80 volts	100 volts
1.	... volt	... mA.	... mA.	... mA.

Calculations—From the graph

(i) *Amplification Factor,* $\mu = \dfrac{V_A - V_B}{AB} = \dots = \dots$

Similarly $\mu = \dfrac{V_B - V_C}{BC} = \dots = \dots$

... etc. ... etc. ...

$\therefore$ **Mean** $\mu = \dots$.

(ii) *Plate Resistance,* $r_g = \dfrac{V_P - V_Q}{PQ} = \dots = \dots$ ohms

... etc. ... etc. ...

$\therefore$ **Mean** $r_p = \dots$ ohms

(iii) *Mutual Conductance,* $g_m = \dfrac{i_p}{e_g}$

(a) For curve no. 1, $g_m \dots = \dots$ ohms

... etc. ... etc. ...

Result—The characteristic curves for the grid voltage and the plate current of the given triode valve are shown in fig.–93, and the values of the tube parameters are as follows :—

(i) Amplification factor = ...
(ii) Plate resistance = ... ohms
(iii) Mutual conductance = ... mhos

Precautions and Sources of Error

(1) The specifications prescribed by the manufacturer for the given triode should be strictly followed. If a specific value of the heating current for the filament has been prescribed, this should be adjusted to this value by including in this circuit a rheostat and an ammeter.

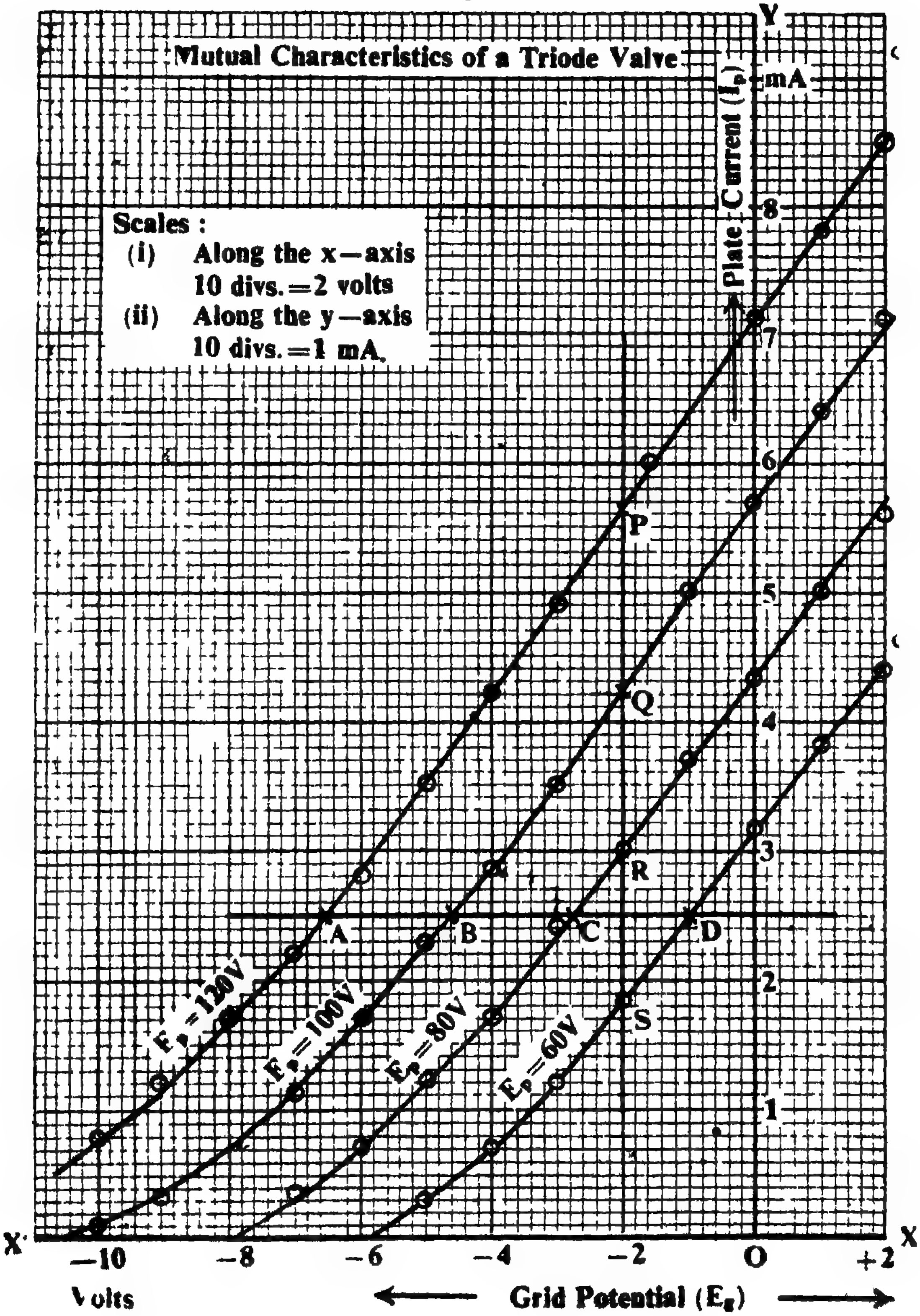
(2) The negative marked terminal of the milliammeter should be connected to the plate of the triode.

(3) While taking observations for the anode current with different grid voltages, the anode potential should be adjusted, if necessary, to its initial value. Moreover, it is well to arrange that the grid circuit is never broken while there is a high potential on the anode.

(4) The maximum voltage applied to the grid should not be more than 20 volts, otherwise the filament may be broken due to excessive mechanical strain.

(5) The characteristic curves should be drawn smooth on the graph paper and for the evaluation of the tube parameters the straight portions of the curves should be employed.

Note. [See the graph drawn in fig.-93.)



Mutual Characteristics of a Triode Valve
Scales :
(i) Along the x—axis
10 divs. =2 volts
(ii) Along the y—axis
10 divs. =1 mA
Y
Plate Current (I_p)
mA
8
7
6
5
4
3
2
1
P
Q
R
S
A
B
C
D
E_p =120V
E_p =100V
E_p =80V
E_p =60V
X'
X
—10
—8
—6
—4
—2
O
+2
Volts
Grid Potential (E_g)

Fig. 93

TABLES OF

PHYSICAL CONSTANTS

AND SOME

MATHEMATICAL FUNCTIONS

HEAT

Table—1. *Thermal Constants (Solids)*

Substance	Melting point *	Boiling point *	Coefficient of linear expansion **	Specific heat †	Latent heat ‡	Thermal conductivity ¶
Aluminium	658	1800	$25 \cdot 50 \times 10^{-6}$	·22	92·4	·504
Bismuth	269	1560	13·3	·03	13·0	·0194
Brass	900	...	18·9	·09	...	·260
Carbon	3500	3927	·63	·16	...	·3
Copper	1084	2360	16·7	·093	43	·918
Glass (soft)	1100	...	8·5	·16	...	·0025
Gold	1063	2360	13·9	·032	16	·7
Ice	0	100	50·7	·5	79·7	·005
India rubber	...	...	70·4	·38	...	·00045
Invar	1500	...	0·9	·12	...	...
Iron (cast)	1100	...	10·2	·12	28	·114
Iron (wrought)	1530	2450	11·9	·12	49	·144
Lead	327	1755	29·1	·031	·5	·083
Platinum	1774	4300	8·9	·032	27	·166
Silica (fused)	1700	...	0·42	·17	200	·0024
Silver	961	2152	18·8	·056	22	·974
Steel	1400	...	11·0	·11	...	·115
Tungsten	3387	4830	4·4	·03	...	·35
Zinc	419	913	26·3	·092	27	·265

* °C under normal pressure. ** per °C. † cal. per gm. per °C. ‡ cal. per gm. at normal melting point. ¶ cal. per sec. per sq. cm. per unit temperature gradient.

Table—2. *Thermal Constants (Liquids)*

Substance	Freezing point *	Boiling point *	Specific heat **	Latent heat ‡	Coefficient of cubical expansion †	Thermal conductivity ¶
					$\times 10^{-5}$	$\times 10^{-4}$
Alcohol (Ethyl)	—115	78.3	·55	205	110	4·2
,, (Methyl)	—95	64·7	·60	267	122	5·0
Benzene	5·5	80·2	·34	95	124	3·3
Castor oil	...	265	...	...	4·32	4·3
Ether	—123	34·6	·56	88	163	3·1
Glycerine	17	290	·58	...	53	6·8
Mercury	—38·9	357	·033	68	18·2	...
Olive oil	...	...	·47	...	70	4·0
Paraffin oil	...	...	·52	...	90	3·0
Turpentine	...	159	·42	70	94	3·2
Water	0	100	1·00	539	15·0	14·7

* °C. normal pressure. ** cal. per gm. per °C. ‡ cal./gm. at normal B. P. † per °C. ¶ cal. per sec. per sq. cm. per temp. gradient.

Table—3. *Thermal Constants (Gases)*

Substance	Coef. of cubical expansion*	Sp. heat† C_p	Ratio of sp. heats (r)	Liquefaction temp. ‡	Solidification temp. ‡	Thermal conductivity ¶
	$\times 10^{-4}$					$\times 10^{-4}$
Air	36·7	·242	1·40	—193	...	·58
CO_2	37·4	·20	1·30	—31	—57	·35
He	36·6	1·25	1·63	—269	—272	3·4
H_2	36·6	3·42	1·41	—253	—259	3·2
N_2	36·7	·235	1·41	—196	—210	·58
O_2	36·7	·22	1·40	—183	—219	·58
Steam	...	·49	1·31	...	...	·52

* per °C (at constant pressure). † cals. per °C ‡ °C ¶ cals. per sec. per sq. cm. per unit temp. gradient.

Table—4. *Cubical Expansion of Water*

Temperature range	Cubical expansion	Temperature range	Cubical expansion
5°—10°C	$5\ 3 \times 10^{-5}$	40°—60°C	$45{\cdot}8 \times 10^{-5}$
10°—20°C	15·0 ,,	60°—80°C	58·7 ,,
20°—40°C	30·2 ,,	80°—100°C	70 0 ,,

Table—5. *Boiling Point (°C) of Water under various Barometric Pressures*

Barometric height (mm.)	0	1	2	3	4	5	6	7	8	9
680	96 91	·95	·99	·03*	·07	·11	·15	·19	·23	·27
690	97·31	·35	·39	·43	·47	·51	·55	·59	·63	·67
700	97 71	75	·79	·83	·87	91	·95	·98	02*	·06
710	98·10	·14	·18	·22	·26	30	·34	·37	·41	·45
720	98·49	·53	·57	·61	·64	·68	·72	·76	·80	·84
730	98·87	·91	·95	·99	03*	·07	·10	·14	·18	·22
740	99 25	·29	·33	·37	·41	·44	·48	·52	·55	·59
750	99·63	·67	·70	·74	·78	·82	·85	·89	.93	·96
760	100 00	·04	·07	·11	·15	·18	·22	·26	·29	·33

* For entries marked with an asterisk, the integral number advances by 1 degree centrigrade.

Table—6. *Magnetic Elements at selected Indian Stations.*

Station	Declination	Dip	H
Agra ...	0° 10′ E	40° 40′	·348
Aligarh ...	0° 20′ E	41° 50′	·346
Allahabad ...	0° 20′ W	37° 10′	·353
Bareilly ...	0° 20′ E	42° 20′	·344
Bombay ...	0° 20′ W	25° 30′	·376
Calcutta ...	0° 00′	31° 30′	·382
Chandausi ...	0° 30′ E	42° 40′	·343
Dehradun ...	0° 50′ E	45° 50′	·332
Delhi ...	0° 40′ E	42° 52′	·345
Gorakhpur ...	0° 20′ W	39° 40′	·358
Gwalior ...	0° 20′ E	39° 00′	·353
Jaipur ...	0° 30′ E	40° 30′	·347
Kanpur ...	0° 00′	38° 39′	·363
Khurja ...	0° 30′ E	42° 10′	·343
Lucknow ...	0° 10′ W	40° 00′	·354
Meerut ...	0° 40′ E	43° 30′	·339
Varanashi ...	0° 30′ W	37° 10′	·364

ELECTRICITY

Table—7. *Wire Resistances*

S.W G No.	Diameter (mm.)	Resistance (ohms/metre) Copper	Constantan*	Manganin†
10	3·25	0021	·057	·051
12	2 64	0032	·086	·077
14	2 03	·0054	·146	·131
16	1·63	·0083	·228	·204
18	1 22	·0148	·405	·361
20	·914	·0260	·722	·645
22	·711	·0435	1·20	1 07
24	·559	·070	1·93	1·73
26	·457	·105	0·89	2 58
28	374	·155	4·27	3·82
30	·315	·222	6 08	5·45
32	·274	·293	8·02	7·18
34	·234	·404	11 1	9 9
36	·193	·590	16 2	14 5
38	·152	·950	26 0	23 2
40	·122	1 48	40 6	36 3
42	·102	2 10	58 5	53·4
44	·081	3 30	91·4	81·7
46	·061	5 90	162 5	145·5

* 60 Cu, 40 Ni. † 84 Cu, 4 Ni, 12 Mn.

Table—8. *Electrical Resistances*

Substance	Specific resistance (ohm-cm)	Temp. coef. of resistance (per °C)	Substance	Specific resistance (ohm cm)	Temp coef. of resistance (per °C)
Brass*	6 6x10^{-6}	10 $\times 10^{-4}$	Mercury	95 8 $\times 10^{-6}$	8·9 $\times 10^{-4}$
Constantan†	49 1 ,,	-4to+·1,,	Nichrome	110 ,,	3 5 ,,
Copper	1·78 ,,	42 8 ,,	Phos. bronze††	5–10 ,,	35 ,,
German silver‡	26·6 ,,	2·3–6 ,,	Platinoid‡‡	34·4 ,,	2 5 ,,
Iron	12·0 ,,	62 ,,	Platinum	11·0 ,,	37 ,,
Manganin**	44 5 ,,	·02–·5 ,,	Silver	1·65 ,,	40 ,,

* 70 Cu, 30 Zn † 60 Cu, 40 Ni. ‡ 62 Cu, 1·5 Ni, 22 Zn. ** 84 Cu, 4Ni, 12 Mn. †† 92·5 Cu, 7 Sn, ·5 P. ‡‡ German silver with about 1% tungsten.

Table—9. *Specific Conductivity* of Standard Solutions*

Solution	0°C	8°C	12°C	16°C	20°C	24°C
NaCl, Sat.	·1345	·1688	·1872	·2063	·2260	·2462
KCl, 1/n.	·06541	·07954	·08869	·09441	·10207	·10984
KCl, 1/10n.	·00715	·00888	00979	·01072	·01167	·01264
KCl, 1/100n	·00078	·00097	·00107	·001173	001278	001386

* *Unit* : $ohm^{-1}\ cm^{-1}$.

KCl 1/n = normal KCl = 74·59 gm/litre

NaCl Sat = saturated NaCl at temp. t of the experiment.

Table—10. *Electro-chemical Equivalent of Elements*

Element	Atomic weight.	Valency	E.C.E. (gm./coulomb)
Copper	63·57	2	·0003295
Gold	197·2	3	·006812
Hydrogen	1 0080	1	·00001045
Lead	207·21	2	·0010736
Nickel	58·69	3	·0002027
Oxygen	16·00	2	·0000829
Silver.	107·88	1	·0011180

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170						5	9	13	17	21	26	30	34	38
						0212	0253	0294	0334	0374	4	8	12	16	20	24	28	32	36
11	0414	0453	0492	0531	0569						4	8	12	16	20	23	27	31	35
						0607	0645	0682	0719	0755	4	7	11	15	18	22	26	29	33
12	0792	0828	0864	0899	0934						3	7	11	14	18	21	25	28	32
						0969	1004	1038	1072	1106	3	7	10	14	17	20	24	27	31
13	1139	1173	1206	1239	1271						3	6	10	13	16	19	23	26	29
						1303	1335	1367	1399	1430	3	7	10	13	16	19	22	25	29
14	1461	1492	1523	1553	1584						3	6	9	12	15	19	22	25	28
						1614	1644	1673	1703	1732	3	6	9	12	14	17	20	23	26
15	1761	1790	1818	1847	1875						3	6	9	11	14	17	20	23	26
						1903	1931	1959	1987	2014	3	6	8	11	14	17	19	22	25
16	2041	2068	2095	2122	2148						3	6	8	11	14	16	19	22	24
						2175	2201	2227	2253	2279	3	5	8	10	13	16	18	21	23
17	2304	2330	2355	2380	2405						3	5	8	10	13	15	18	20	23
						2430	2455	2480	2504	2529	3	5	8	10	12	15	17	20	22
18	2553	2577	2601	2625	2648						2	5	7	9	12	14	17	19	21
						2672	2695	2718	2742	2765	2	4	7	9	11	14	16	18	21
19	2788	2810	2833	2856	2878						2	4	7	9	11	13	16	18	20
						2900	2923	2945	2967	2989	2	4	6	8	11	13	15	17	19
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1		4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

ANTILOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
·05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
·08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
·10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
·15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
·20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
·25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
·30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
·35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
·40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
·45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
·46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
·47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6

ANTILOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
50	[illegible]	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
51	[illegible]	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
53	[illegible]	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
54	[illegible]	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
55	3548	[illegible]	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
71	[illegible]	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
79	[illegible]	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	[illegible]	4	6	8	10	12	15	17	19
96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

NATURAL SINES

Degrees	0′ 0°.0	6′ 0°.1	12′ 0°.2	18′ 0°.3	24′ 0°.4	30′ 0°.5	36′ 0°.6	42′ 0°.7	48′ 0°.8	54′ 0°.9	Mean Differences 1	2	3	4	5
0	0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	15
1	0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15
2	0349	0366	0384	0401	0419	0436	0454	0471	0488	0506	3	6	9	12	15
3	0523	0541	0558	0576	0593	0610	0628	0645	0663	0680	3	6	9	12	15
4	0698	0715	0732	0750	0767	0785	0802	0819	0837	0854	3	6	9	12	15
5	0872	0889	0906	0924	0941	0958	0976	0993	1011	1028	3	6	9	12	15
6	1045	1063	1080	1097	1115	1132	1149	1167	1184	1201	3	6	9	12	14
7	1219	1236	1253	1271	1288	1305	1323	1340	1357	1374	3	6	9	[illegible]	14
8	1392	1409	1426	1444	1461	1478	1495	1513	1530	1547	3	6	9	12	14
9	1564	1582	1599	1616	1633	1650	1668	1685	1702	1719	3	6	9	12	14
10	1736	1754	1771	1788	1805	1822	1840	1857	1874	1891	3	6	9	12	[illegible]
11	1908	1925	1942	1959	1977	1994	2011	2028	2045	2062	3	6	9	11	14
12	2079	2096	2113	2130	2147	2164	2181	2198	2215	2232	3	6	9	11	14
13	2250	2267	2284	2300	2317	2334	2351	2368	2385	24[illegible]	3	6	8	11	14
14	2419	2436	2453	2470	2487	2504	2521	2538	2554	25[illegible]1	3	6	8	11	14
15	2588	2605	2622	2639	2656	2672	2689	2706	2723	2740	3	6	8	11	14
16	2756	2773	2790	2807	2823	2840	2857	2874	2890	2907	3	6	8	[illegible]	14
17	2924	2940	2957	2974	2990	3007	3024	3040	3057	3074	3	6	8	[illegible]	[illegible]
18	3090	3107	3123	3140	3156	3173	3190	3206	3223	3239	3	5	[illegible]	11	14
19	3256	3272	3289	3305	3322	3338	3355	3371	3387	3404	3	5	[illegible]	11	14
20	3420	3437	3453	3469	3486	3502	3518	3535	3551	3567	3	5	[illegible]	11	14
21	3584	3600	3616	3633	3649	3665	3681	3697	3714	3730	3	5	8	11	14
22	3746	3762	3778	3795	3811	3827	3843	3859	3875	3891	3	5	8	11	14
23	3907	3923	3939	3955	3971	3987	4003	4019	4035	4051	3	5	8	11	14
24	4067	4083	4099	4115	4131	4147	4163	4179	4195	4210	3	5	8	11	13
25	4226	4242	4258	4274	4289	4305	4321	4337	4352	4368	3	5	8	11	13
26	4384	4399	4415	4431	4446	4462	4478	4493	4509	4524	3	5	8	10	13
27	4540	4555	4571	4586	4602	4617	4633	4648	4664	4679	3	5	8	10	13
28	4695	4710	4726	4741	4756	4772	4787	4802	4818	4833	3	5	8	10	13
29	4848	4863	4879	4894	4909	4924	4939	4955	4970	4985	3	5	8	10	13
30	5000	5015	5030	5045	5060	5075	5090	5105	5120	51[illegible]	3	5	8	10	13
31	5150	5165	5180	5195	5210	5225	5240	5255	5270	5284	2	5	7	10	12
32	5299	5314	5329	5344	5358	5373	5388	5402	5417	5432	2	5	7	10	12
33	5446	5461	5476	5490	5505	5519	5534	5548	5563	5577	2	5	7	10	12
34	5592	5606	5621	5635	5650	5664	5678	5693	5707	5721	2	5	7	10	12
35	5736	5750	5764	5779	5793	5807	5821	5835	5850	5864	2	5	7	10	12
36	5878	5892	5906	5920	5934	5948	5962	5976	5990	6004	2	5	7	9	12
37	6018	6032	6046	6060	6074	6088	6101	6115	6129	6143	2	5	7	9	12
38	6157	6170	6184	6198	6211	6225	6239	6252	6266	6280	2	5	7	9	11
39	6293	6307	6320	6334	6347	6361	6374	6388	6401	6414	2	4	7	9	11
40	6428	6441	6455	6468	6481	6494	6508	6521	6534	6547	2	4	7	9	11
41	6561	6574	6587	6600	6613	6626	6639	6652	6665	6678	2	4	7	9	11
42	6691	6704	6717	6730	6743	6756	6769	6782	6794	6807	2	4	6	9	11
43	6820	6833	6845	6858	6871	6884	6896	6909	6921	6934	2	4	6	8	11
44	6947	6959	6972	6984	6997	7009	7022	7034	7046	7059	2	4	6	8	10

Degrees	0′ 0°·0	6′ 0°·1	12′ 0°·2	18′ 0°·3	24′ 0°·4	30′ 0°·5	36′ 0°·6	42′ 0°·7	48′ 0°·8	54′ 0°·9	Mean Differences. 1	2	3	4	5
45	·7071	7059	7046	7034	7022	7009	6997	6984	6972	6959	2	4	6	8	10
46	·6947	6934	6921	6909	6896	6884	6871	6858	6845	6833	2	4	6	8	11
47	6820	6807	6794	6782	6769	6756	6743	6730	6717	6704	2	4	6	9	11
48	6691	6678	6665	6652	6639	6626	6613	6600	6587	6574	2	4	7	9	11
49	·6561	6547	6534	6521	6508	6494	6481	6468	6455	6441	2	4	7	9	11
50	6428	6414	6401	6388	6374	6361	6347	6334	6320	6307	2	4	7	9	11
51	·6293	6280	6266	6252	6239	6225	6211	6198	6184	6170	2	5	7	9	11
52	·6157	6143	6129	6115	6101	6088	6074	6060	6046	6032	2	5	7	9	12
53	6018	6004	5990	5976	5962	5948	5934	5920	5906	5892	2	5	7	9	12
54	·5878	5864	5850	5835	5821	5807	5793	5779	5764	5750	2	5	7	9	12
55	·5736	5721	5707	5693	5678	5664	5650	5635	5621	5606	2	5	7	10	12
56	·5592	5577	5563	5548	5534	5519	5505	5490	5476	5461	2	5	7	10	12
57	·5446	5432	5417	5402	5388	5373	5358	5344	5329	5314	2	5	7	10	12
58	·5299	5284	5270	5255	5240	5225	5210	5195	5180	5165	2	5	7	10	12
59	·5150	5135	5120	5105	5090	5075	5060	5045	5030	5015	3	5	8	10	13
60	·5000	4985	4970	4955	4939	4924	4909	4894	4879	4863	3	5	8	10	13
61	·4848	4833	4818	4802	4787	4772	4756	4741	4726	4710	3	5	8	10	13
62	·4695	4679	4664	4648	4633	4617	4602	4586	4571	4555	3	5	8	10	13
63	·4540	4524	4509	4493	4478	4462	4446	4431	4415	4399	3	5	8	10	13
64	·4384	4368	4352	4337	4321	4305	4289	4274	4258	4242	3	5	8	11	13
65	·4226	4210	4195	4179	4163	4147	4131	4115	4099	4083	3	5	8	11	13
66	·4067	4051	4035	4019	4003	3987	3971	3955	3939	3923	3	5	8	11	14
67	·3907	3891	3875	3859	3843	3827	3811	3795	3778	3762	3	5	8	11	14
68	·3746	3730	3714	3697	3681	3665	3649	3633	3616	3600	3	5	8	11	14
69	·3584	3567	3551	3535	3518	3502	3486	3469	3453	3437	3	5	8	11	14
70	·3420	3404	3387	3371	3355	3338	3322	3305	3289	3272	3	5	8	11	14
71	·3256	3239	3223	3206	3190	3173	3156	3140	3123	3107	3	6	8	11	14
72	·3090	3074	3057	3040	3024	3007	2990	2974	2957	2940	3	6	8	11	14
73	·2924	2907	2890	2874	2857	2840	2823	2807	2790	2773	3	6	8	11	14
74	·2756	2740	2723	2706	2689	2672	2656	2639	2622	2605	3	6	8	11	14
75	·2588	2571	2554	2538	2521	2504	2487	2470	2453	2436	3	6	8	11	14
76	·2419	2402	2385	2368	2351	2334	2317	2300	2284	2267	3	6	8	11	14
77	·2250	2233	2215	2198	2181	2164	2147	2130	2113	2096	3	6	9	11	14
78	·2079	2062	2045	2028	2011	1994	1977	1959	1942	1925	3	6	9	11	14
79	·1908	1891	1874	1857	1840	1822	1805	1788	1771	1754	3	6	9	11	14
80	·1736	1719	1702	1685	1668	1650	1633	1616	1599	1582	3	6	9	12	14
81	·1564	1547	1530	1513	1495	1478	1461	1444	1426	1409	3	6	9	12	14
82	·1392	1374	1357	1340	1323	1305	1288	1271	1253	1236	3	6	9	12	14
83	·1219	1201	1184	1167	1149	1132	1115	1097	1080	1063	3	6	9	12	14
84	·1045	1028	1011	0993	0976	0958	0941	0924	0906	0889	3	6	9	12	14
85	·0872	0854	0837	0819	0802	0785	0767	0750	0732	0715	3	6	9	12	15
86	·0698	0680	0663	0645	0628	0610	0593	0576	0558	0541	3	6	9	12	15
87	·0523	0506	0488	0471	0454	0436	0419	0401	0384	0366	3	6	9	12	15
88	·0349	0332	0314	0297	0279	0262	0244	0227	0209	0192	3	6	9	12	15
89	·0175	0157	0140	0122	0105	0087	0070	0052	0035	0017	3	6	9	12	15
90	·0000														

NATURAL COSINES

Degrees.	0′ 0°0	6′ 0°1	12′ 0°2	18′ 0°3	24′ 0°4	30′ 0°5	36 0°6	42 0°7	48′ 0°8	54′ 0°9	Mean Differences 1	2	3	4	5
0	1 000	1 000	1 000	1 000	1 000	1 000	9999	9999	9999	9999	0	0	0	0	0
1	9998	9998	9998	9997	9997	9997	9996	9996	9995	9995	0	0	0	0	0
2	9994	9993	9993	9992	9991	9990	9990	9989	9988	9987	0	0	0	1	1
3	9986	9985	9984	9983	9982	9981	9980	9979	9978	9977	0	0	1	1	1
4	9976	9974	9973	9972	9971	9969	9968	9966	9965	9963	0	0	1	1	1
5	9962	9960	9959	9957	9956	9954	9952	9951	9949	9947	0	1	1	1	2
6	9945	9943	9942	9940	9938	9936	9934	9932	9930	9928	0	1	1	1	2
7	9925	9923	9921	9919	9917	9914	9912	9910	9907	9905	0	1	1	2	2
8	9903	9900	9898	9895	9893	9890	9888	9885	9882	9880	0	1	1	2	2
9	9877	9874	9871	9869	9866	9863	9860	9857	9854	9851	0	1	1	2	2
10	9848	9845	9842	9839	9836	9833	9829	9826	9823	9820	1	1	2	2	3
11	9816	9813	9810	9806	9803	9799	9796	9792	9789	9785	1	1	2	2	3
12	9781	9778	9774	9770	9767	9763	9759	9755	9751	9748	1	1	2	3	3
13	9744	9740	9736	9732	9728	9724	9720	9715	9711	9707	1	1	2	3	3
14	9703	9699	9694	9690	9686	9681	9677	9673	9668	9664	1	1	2	3	4
15	9659	9655	9650	9646	9641	9636	9632	9627	9622	9617	1	2	2	3	4
16	9613	9608	9603	9598	9593	9588	9583	9578	9573	9568	1	2	2	3	4
17	9563	9558	9553	9548	9542	9537	9532	9527	9521	9516	1	2	3	3	4
18	9511	9505	9500	9494	9489	9483	9478	9472	9466	9461	1	2	3	4	5
19	9455	9449	9444	9438	9432	9426	9421	9415	9409	9403	1	2	3	4	5
20	9397	9391	9385	9379	9373	9367	9361	9354	9348	9342	1	2	3	4	5
21	9336	9330	9323	9317	9311	9304	9298	9291	9285	9278	1	2	3	4	5
22	9272	9265	9259	9252	9245	9239	9232	9225	9219	9212	1	2	3	4	6
23	9205	9198	9191	9184	9178	9171	9164	9157	9150	9143	1	2	3	5	6
24	9135	9128	9121	9114	9107	9100	9092	9085	9078	9070	1	2	4	5	6
25	9063	9056	9048	9041	9033	9026	9018	9011	9003	8996	1	3	4	5	6
26	8988	8980	8973	8965	8957	8949	8942	8934	8926	8918	1	3	4	5	6
27	8910	8902	8894	8886	8878	8870	8862	8854	8846	8838	1	3	4	5	7
28	8829	8821	8813	8805	8796	8788	8780	8771	8763	8755	1	3	4	6	7
29	8746	8738	8729	8721	8712	8704	8695	8686	8678	8669	1	3	4	6	7
30	8660	8652	8643	8634	8625	8616	8607	8599	8590	8581	1	3	4	6	7
31	8572	8563	8554	8545	8536	8526	8517	8508	8499	8490	2	3	5	6	8
32	8480	8471	8462	8453	8443	8434	8425	8415	8406	8396	2	3	5	6	8
33	8387	8377	8368	8358	8348	8339	8329	8320	8310	8300	2	3	5	6	8
34	8290	8281	8271	8261	8251	8241	8231	8221	8211	8202	2	3	5	7	8
35	8192	8181	8171	8161	8151	8141	8131	8121	8111	8100	2	3	5	7	8
36	8090	8080	8070	8059	8049	8039	8028	8018	8007	7997	2	3	5	7	9
37	7986	7976	7965	7955	7944	7934	7923	7912	7902	7891	2	4	5	7	9
38	7880	7869	7859	7848	7837	7826	7815	7804	7793	7782	2	4	5	7	9
39	7771	7760	7749	7738	7727	7716	7705	7694	7683	7672	2	4	6	7	9
40	7660	7649	7638	7627	7615	7604	7593	7581	7570	7559	2	4	6	8	9
41	7547	7536	7524	7513	7501	7490	7478	7466	7455	7443	2	4	6	8	10
42	7431	7420	7408	7396	7385	7373	7361	7349	7337	7325	2	4	6	8	10
43	7314	7302	7290	7278	7266	7254	7242	7230	7218	7206	2	4	6	8	10
44	7193	7181	7169	7157	7145	7133	7120	7108	7096	7083	2	4	6	8	10

NATURAL COSINES

Degrees	0′ 0°.0	6′ 0°.1	12′ 0°.2	18′ 0°.3	24′ 0°.4	30′ 0°.5	36′ 0°.6	42′ 0°.7	48′ 0°.8	54′ 0°.9	Mean Differences 1	2	3	4	5
45	7071	7083	7096	7108	7120	7133	7145	7157	7169	7181	2	4	6	8	10
46	7193	7206	7218	7230	7242	7254	7266	7278	7290	7302	2	4	6	8	10
47	7314	7325	7337	7349	7361	7373	7385	7396	7408	7420	2	4	6	8	10
48	7431	7443	7455	7466	7478	7490	7501	7513	7524	7536	2	4	6	8	10
49	7547	7559	7570	7581	7593	7604	7615	7627	7638	7649	2	4	6	8	9
50	7660	7672	7683	7694	7705	7716	7727	7738	7749	7760	2	4	6	7	9
51	7771	7782	7793	7804	7815	7826	7837	7848	7859	7869	2	4	5	7	9
52	7880	7891	7902	7912	7923	7934	7944	7955	7965	7976	2	4	5	7	9
53	7986	7997	8007	8018	8028	8039	8049	8059	8070	8080	2	3	5	7	9
54	8090	8100	8111	8121	8131	8141	8151	8161	8171	8181	2	3	5	7	8
55	8192	8202	8211	8221	8231	8241	8251	8261	8271	8281	2	3	5	7	8
56	8290	8300	8310	8320	8329	8339	8348	8358	8368	8377	2	3	5	6	8
57	8387	8396	8406	8415	8425	8434	8443	8453	8462	8471	2	3	5	6	8
58	8480	8490	8499	8508	8517	8526	8536	8545	8554	8563	2	3	5	6	8
59	8572	8581	8590	8599	8607	8616	8625	8634	8643	8652	1	3	4	6	7
60	8660	8669	8678	8686	8695	8704	8712	8721	8729	8738	1	3	4	6	7
61	8746	8755	8763	8771	8780	8788	8796	8805	8813	8821	1	3	4	6	7
62	8829	8838	8846	8854	8862	8870	8878	8886	8894	8902	1	3	4	5	7
63	8910	8918	8926	8934	8942	8949	8957	8965	8973	8980	1	3	4	5	6
64	8988	8996	9003	9011	9018	9026	9033	9041	9048	9056	1	3	4	5	6
65	9063	9070	9078	9085	9092	9100	9107	9114	9121	9128	1	2	4	5	6
66	9135	9143	9150	9157	9164	9171	9178	9184	9191	9198	1	2	3	5	6
67	9205	9212	9219	9225	9232	9239	9245	9252	9259	9265	1	2	3	4	6
68	9272	9278	9285	9291	9298	9304	9311	9317	9323	9330	1	2	3	4	5
69	9336	9342	9348	9354	9361	9367	9373	9379	9385	9391	1	2	3	4	5
70	9397	9403	9409	9415	9421	9426	9432	9438	9444	9449	1	2	3	4	5
71	9455	9461	9466	9472	9478	9483	9489	9494	9500	9505	1	2	3	4	5
72	9511	9516	9521	9527	9532	9537	9542	9548	9553	9558	1	2	3	3	4
73	9563	9568	9573	9578	9583	9588	9593	9598	9603	9608	1	2	2	3	4
74	9613	9617	9622	9627	9632	9636	9641	9646	9650	9655	1	2	2	3	4
75	9659	9664	9668	9673	9677	9681	9686	9690	9694	9699	1	1	2	3	4
76	9703	9707	9711	9715	9720	9724	9728	9732	9736	9740	1	1	2	3	3
77	9744	9748	9751	9755	9759	9763	9767	9770	9774	9778	1	1	2	3	3
78	9781	9785	9789	9792	9796	9799	9803	9806	9810	9813	1	1	2	2	3
79	9816	9820	9823	9826	9829	9833	9836	9839	9842	9845	1	1	2	2	3
80	9848	9851	9854	9857	9860	9863	9866	9869	9871	9874	0	1	1	2	2
81	9877	9880	9882	9885	9888	9890	9893	9895	9898	9900	0	1	1	2	2
82	9903	9905	9907	9910	9912	9914	9917	9919	9921	9923	0	1	1	2	2
83	9925	9928	9930	9932	9934	9936	9938	9940	9942	9943	0	1	1	1	2
84	9945	9947	9949	9951	9952	9954	9956	9957	9959	9960	0	1	1	1	2
85	9962	9963	9965	9966	9968	9969	9971	9972	9973	9974	0	0	1	1	1
86	9976	9977	9978	9979	9980	9981	9982	9983	9984	9985	0	0	1	1	1
87	9986	9987	9988	9989	9990	9990	9991	9992	9993	9993	0	0	0	1	1
88	9994	9995	9995	9996	9996	9997	9997	9997	9998	9998	0	0	0	0	0
89	9998	9999	9999	9999	9999	1.000	1.000	1.000	1.000	1.000	0	0	0	0	0
90	1.000														

NATURAL TANGENTS

Degrees.	0′ 0°·0	6′ 0°·1	12′ 0°·2	18′ 0°·3	24′ 0°·4	30′ 0°·5	36′ 0°·6	42′ 0°·7	48′ 0°·8	54′ 0°·9	Mean Differences. 1	2	3	4	5
0	0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	15
1	0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15
2	0349	0367	0384	0402	0419	0437	0454	0472	0489	0507	3	6	9	12	15
3	0524	0542	0559	0577	0594	0612	0629	0647	0664	0682	3	6	9	12	15
4	0699	0717	0734	0752	0769	0787	0805	0822	0840	0857	3	6	9	12	15
5	0875	0892	0910	0928	0945	0963	0981	0998	1016	1033	3	6	9	12	15
6	·1051	1069	1086	1104	1122	1139	1157	1175	1192	1210	3	6	9	12	15
7	·1228	1246	1263	1281	1299	1317	1334	1352	1370	1388	3	6	9	12	15
8	1405	1423	1441	1459	1477	1495	1512	1530	1548	1566	3	6	9	12	15
9	1584	1602	1620	1638	1655	1673	1691	1709	1727	1745	3	6	9	12	15
10	1763	1781	1799	1817	1835	1853	1871	1890	1908	1926	3	6	9	12	15
11	1944	1962	1980	1998	2016	2035	2053	2071	2089	2107	3	6	9	12	15
12	2126	2144	2162	2180	2199	2217	2235	2254	2272	2290	3	6	9	12	15
13	2309	2327	2345	2364	2382	2401	2419	2438	2456	2475	3	6	9	12	15
14	2493	2512	2530	2549	2568	2586	2605	2623	2642	2661	3	6	9	12	16
15	2679	2698	2717	2736	2754	2773	2792	2811	2830	2849	3	6	9	13	16
16	2867	2886	2905	2924	2943	2962	2981	3000	3019	3038	3	6	9	13	16
17	·3057	3076	3096	3115	3134	3153	3172	3191	3211	3230	3	6	10	13	16
18	·3249	3269	3288	3307	3327	3346	3365	3385	3404	3424	3	6	10	13	16
19	3443	3463	3482	3502	3522	3541	3561	3581	3600	3620	3	7	10	13	16
20	·3640	3659	3679	3699	3719	3739	3759	3779	3799	3819	3	7	10	13	17
21	3839	3859	3879	3899	3919	3939	3959	3979	4000	4020	3	7	10	13	17
22	4040	4061	4081	4101	4122	4142	4163	4183	4204	4224	3	7	10	14	17
23	4245	4265	4286	4307	4327	4348	4369	4390	4411	4431	3	7	10	14	17
24	4452	4473	4494	4515	4536	4557	4578	4599	4621	4642	4	7	11	14	18
25	4663	4684	4706	4727	4748	4770	4791	4813	4834	4856	4	7	11	14	18
26	·4877	4899	4921	4942	4964	4986	5008	5029	5051	5073	4	7	11	15	18
27	5095	5117	5139	5161	5184	5206	5228	5250	5272	5295	4	7	11	15	18
28	5317	5340	5362	5384	5407	5430	5452	5475	5498	5520	4	8	11	15	19
29	5543	5566	5589	5612	5635	5658	5681	5704	5727	5750	4	8	12	15	19
30	5774	5797	5820	5844	5867	5890	5914	5938	5961	5985	4	8	12	16	20
31	6009	6032	6056	6080	6104	6128	6152	6176	6200	6224	4	8	12	16	20
32	6249	6273	6297	6322	6346	6371	6395	6420	6445	6469	4	8	12	16	20
33	6494	6519	6544	6569	6594	6619	6644	6669	6694	6720	4	8	13	17	21
34	6745	6771	6796	6822	6847	6873	6899	6924	6950	6976	4	9	13	17	21
35	7002	7028	7054	7080	7107	7133	7159	7186	7212	7239	4	9	13	18	22
36	7265	7292	7319	7346	7373	7400	7427	7454	7481	7508	5	9	14	18	23
37	7536	7563	7590	7618	7646	7673	7701	7729	7757	7785	5	9	14	18	23
38	7813	7841	7869	7898	7926	7954	7983	8012	8040	8069	5	9	14	19	24
39	8098	8127	8156	8185	8214	8243	8273	8302	8332	8361	5	10	15	20	24
40	8391	8421	8451	8481	8511	8541	8571	8601	8632	8662	5	10	15	20	25
41	8693	8724	8754	8785	8816	8847	8878	8910	8941	8972	5	10	16	21	26
42	9004	9036	9067	9099	9131	9163	9195	9228	9260	9293	5	11	16	21	27
43	9325	9358	9391	9424	9457	9490	9523	9556	9590	9623	6	11	17	22	28
44	9657	9691	9725	9759	9793	9827	9861	9896	9930	9965	6	11	17	23	29

NATURAL TANGENTS

Degrees	0 0°·0	6′ 0°·1	12′ 0°·2	18′ 0°·3	24′ 0°·4	30′ 0°·5	36′ 0°·6	42′ 0°·7	48′ 0°·8	54′ 0°·9	Mean Differences 1	2	3	4	5
45	1·0000	0035	0070	0105	0141	0176	0212	0247	0283	0319	6	12	18	24	30
46	1·0355	0392	0428	0464	0501	0538	0575	0612	0649	0686	6	12	18	25	31
47	1·0724	0761	0799	0837	0875	0913	0951	0990	1028	1067	6	13	19	25	32
48	1·1106	1145	1184	1224	1263	1303	1343	1383	1423	1463	7	13	20	27	33
49	1·1504	1544	1585	1626	1667	1708	1750	1792	1833	1875	7	14	21	28	34
50	1·1918	1960	2002	2045	2088	2131	2174	2218	2261	2305	7	14	22	29	36
51	1·2349	2393	2437	2482	2527	2572	2617	2662	2708	2753	8	15	23	30	38
52	1·2799	2846	2892	2938	2985	3032	3079	3127	3175	3222	8	16	24	31	39
53	1·3270	3319	3367	3416	3465	3514	3564	3613	3663	3713	8	16	25	33	41
54	1·3764	3814	3865	3916	3968	4019	4071	4124	4176	4229	9	17	26	34	43
55	1·4281	4335	4388	4442	4496	4550	4605	4659	4715	4770	9	18	27	36	45
56	1·4826	4882	4938	4994	5051	5108	5166	5224	5282	5340	10	19	29	38	48
57	1·5399	5458	5517	5577	5637	5697	5757	5818	5880	5941	10	20	30	40	50
58	1·6003	6066	6128	6191	6255	6319	6383	6447	6512	6577	11	21	32	43	53
59	1·6643	6709	6775	6842	6909	6977	7045	7113	7182	7251	11	23	34	45	56
60	1·7321	7391	7461	7532	7603	7675	7747	7820	7893	7966	12	24	36	48	60
61	1·8040	8115	8190	8265	8341	8418	8495	8572	8650	8728	13	26	38	51	64
62	1·8807	8887	8967	9047	9128	9210	9292	9375	9458	9542	14	27	41	55	68
63	1·9626	9711	9797	9883	9970	2·0057	2·0145	2·0233	2·0323	2·0413	15	29	44	58	73
64	2·0503	0594	0686	0778	0872	0965	1060	1155	1251	1348	16	31	47	63	78
65	2·1445	1543	1642	1742	1842	1943	2045	2148	2251	2355	17	34	51	68	85
66	2·2460	2566	2673	2781	2889	2998	3109	3220	3332	3445	18	37	55	73	92
67	2·3559	3673	3789	3906	4023	4142	4262	4383	4504	4627	20	40	60	79	99
68	2·4751	4876	5002	5129	5257	5386	5517	5649	5782	5916	22	43	65	87	108
69	2·6051	6187	6325	6464	6605	6746	6889	7034	7179	7326	24	47	71	95	119
70	2·7475	7625	7776	7929	8083	8239	8397	8556	8716	8878	26	52	78	104	131
71	2·9042	9208	9375	9544	9714	9887	3·0061	3·0237	3·0415	3·0595	29	58	87	116	145
72	3·0777	0961	1146	1334	1524	1716	1910	2106	2305	2506	32	64	96	129	161
73	3·2709	2914	3122	3332	3544	3759	3977	4197	4420	4646	36	72	108	144	180
74	3·4874	5105	5339	5576	5816	6059	6305	6554	6806	7062	41	81	122	163	204
75	3·7321	7583	7848	8118	8391	8667	8947	9232	9520	9812	46	93	139	186	232
76	4·0108	0408	0713	1022	1335	1653	1976	2303	2635	2972	53	107	160	213	267
77	4·3315	3662	4015	4374	4737	5107	5483	5864	6252	6646					
78	4·7046	7453	7867	8288	8716	9152	9594	5·0045	5·0504	5·0970					
79	5·1446	1929	2422	2924	3435	3955	4486	5026	5578	6140					
80	5·6713	7297	7894	8502	9124	9758	6·0405	6·1066	6·1742	6·2432					
81	6·3138	3859	4596	5350	6122	6912	7720	8548	9395	7·0264					
82	7·1154	2066	3002	3962	4947	5958	6996	8062	9158	8·0285					
83	8·1443	2636	3863	5126	6427	7769	9152	9·0579	9·2052	9·3572					
84	9·5144	9·677	9·845	10·02	10·20	10·39	10·58	10·78	10·99	11·20					
85	11·43	11·66	11·91	12·16	12·43	12·71	13·00	13·30	13·62	13·95					
86	14·30	14·67	15·06	15·46	15·89	16·35	16·83	17·34	17·89	18·46					
87	19·08	19·74	20·45	21·20	22·02	22·90	23·86	24·90	26·03	27·27					
88	28·64	30·14	31·82	33·69	35·80	38·19	40·92	44·07	47·74	52·08					
89	57·29	63·66	71·62	81·85	95·49	114·6	143·2	191·0	286·5	573·0					
90	∞														

Mean differences cease to be sufficiently accurate

POWERS, ROOTS & RECIPROCALS

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$
1	1	1	1	1	1
2	4	8	1·414	1·260	·5000
3	9	27	1·732	1·442	·3333
4	16	64	2	1·587	·2500
5	25	125	2·236	1·710	·2000
6	36	216	2·449	1·817	·1667
7	49	343	2·646	1·913	·1429
8	64	512	2·828	2·000	·1250
9	81	729	3·000	2·080	·1111
10	100	1000	3·162	2·154	·1000
11	121	1331	3·317	2·224	·09091
12	144	1728	3·464	2·289	·08333
13	169	2197	3·606	2·351	·07692
14	196	2744	3·742	2·410	·07143
15	225	3375	3·873	2·466	·06667
16	256	4096	4·000	2·520	·06250
17	289	4913	4·123	2·571	·05882
18	324	5832	4·243	2·621	·05556
19	361	6859	4·359	2·668	·05263
20	400	8000	4·472	2·714	·0500
21	441	9261	4·583	2·759	·04762
22	484	10648	4·690	2·802	·04545
23	529	12167	4·796	2·844	·04348
24	576	13824	4·899	2·884	·04167
25	625	15625	5·000	2·924	·0400
26	676	17576	5·099	2·962	·03846
27	729	19683	5·196	3·000	·03704
28	784	21952	5·292	3·037	·03571
29	841	24389	5·385	3·072	·03448
30	900	27000	5·477	3·107	·03333
31	961	29791	5·568	3·141	·03226
32	1024	32768	5·657	3·175	·03125
33	1089	35937	5·745	3·208	·03030
34	1156	39304	5·831	3·240	·02941
35	1225	42875	5·916	3·271	·02857
36	1296	46656	6·000	3·302	·02778
37	1369	50653	6·083	3·332	·02703
38	1444	54872	6·164	3·362	·02632
39	1521	59319	6·245	3·391	·02564
40	1600	64000	6·325	3·420	·0250
41	1681	68921	6·403	3·448	·02439
42	1764	74088	6·481	3·476	·02381
43	1849	79507	6·557	3·503	·02326
44	1936	85184	6·633	3·530	·02273
45	2025	91125	6·708	3·557	·02222
46	2116	97336	6·782	3·583	·02174
47	2209	103823	6·856	3·609	·02128
48	2304	110592	6·928	3·634	·02083
49	2401	117649	7·000	3·659	·02041
50	2500	125000	7·071	3·684	·020

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$
51	2601	132651	7·141	3·708	·01961
52	2704	140608	7·211	3·733	·01923
53	2809	148877	7·280	3·756	·01887
54	2916	157464	7·348	3·780	·01852
55	3025	166375	7·416	3·803	·01818
56	3136	175616	7·483	3·826	·01786
57	3249	185193	7·550	3·849	·01754
58	3364	195112	7·616	3·871	·01724
59	3481	205379	7·681	3·893	·01695
60	3600	216000	7·746	3·915	·01667
61	3721	226981	7·810	3·936	·01639
62	3844	238328	7·874	3·958	·01613
63	3969	250047	7·937	3·979	·01587
64	4096	262144	8·000	4·000	·01562
65	4225	274625	8·062	4·021	·01538
66	4356	287496	8·124	4·041	·01515
67	4489	300763	8·185	4·062	·01493
68	4624	314432	8·246	4·082	·01471
69	4761	328509	8·307	4·102	·01449
70	4900	343000	8·367	4·121	·01429
71	5041	357911	8·426	4·141	·01408
72	5184	373248	8·485	4·160	·01389
73	5329	389017	8·544	4·179	·01370
74	5476	405224	8·602	4·198	·01351
75	5625	421875	8·660	4·217	·01333
76	5776	438976	8·718	4·236	·01316
77	5929	456533	8·775	4·254	·01299
78	6084	474552	8·832	4·273	·01282
79	6241	493039	8·888	4·291	·01266
80	6400	512000	8·944	4·309	·01250
81	6561	531441	9·000	4·327	·01235
82	6724	551368	9·055	4·344	·01220
83	6889	571787	9·110	4·362	·01205
84	7056	592704	9·165	4·380	·01190
85	7225	614125	9·220	4·397	·01176
86	7396	636056	9·274	4·414	·01163
87	7569	658503	9·327	4·431	·01149
88	7744	681472	9·381	4·448	·01136
89	7921	704969	9·434	4·465	·01124
90	8100	729000	9·487	4·481	·01111
91	8281	753571	9·539	4·498	·01099
92	8464	778688	9·592	4·514	·01087
93	8649	804357	9·644	4·531	·01075
94	8836	830584	9·695	4·547	·01064
95	9025	857375	9·747	4·563	·01053
96	9216	884736	9·798	4·579	·01042
97	9409	912673	9·849	4·595	·01031
98	9604	941192	9·899	4·610	·01020
99	9801	970299	9·950	4·626	·01010
100	10000	1000000	10·000	4·642	·0100

SQUARE ROOTS FROM 1 TO 10

	0	1	2	3	4	5	6	7	8	9	Mean Differences.								
											1	2	3	4	5	6	7	8	9
1·0	1·000	1·005	1·010	1·015	1·020	1·025	1·030	1·034	1·039	1·044	0	1	1	2	2	3	3	4	4
1·1	1·049	1·054	1·058	1·063	1·068	1·072	1·077	1·082	1·086	1·091	0	1	1	2	2	3	3	4	4
1·2	1·095	1·100	1·105	1·109	1·114	1·118	1·122	1·127	1·131	1·136	0	1	1	2	2	3	3	4	4
1·3	1·140	1·145	1·149	1·153	1·158	1·162	1·166	1·170	1·175	1·179	0	1	1	2	2	3	3	3	4
1·4	1·183	1·187	1·192	1·196	1·200	1·204	1·208	1·212	1·217	1·221	0	1	1	2	2	2	3	3	4
1·5	1·225	1·229	1·233	1·237	1·241	1·245	1·249	1·253	1·257	1·261	0	1	1	2	2	2	3	3	4
1·6	1·265	1·269	1·273	1·277	1·281	1·285	1·288	1·292	1·296	1·300	0	1	1	2	2	2	3	3	3
1·7	1·304	1·308	1·311	1·315	1·319	1·323	1·327	1·330	1·334	1·338	0	1	1	2	2	2	3	3	3
1·8	1·342	1·345	1·349	1·353	1·356	1·360	1·364	1·367	1·371	1·375	0	1	1	1	2	2	3	3	3
1·9	1·378	1·382	1·386	1·389	1·393	1·396	1·400	1·404	1·407	1·411	0	1	1	1	2	2	3	3	3
2·0	1·414	1·418	1·421	1·425	1·428	1·432	1·435	1·439	1·442	1·446	0	1	1	1	2	2	2	3	3
2·1	1·449	1·453	1·456	1·459	1·463	1·466	1·470	1·473	1·476	1·480	0	1	1	1	2	2	2	3	3
2·2	1·483	1·487	1·490	1·493	1·497	1·500	1·503	1·507	1·510	1·513	0	1	1	1	2	2	2	3	3
2·3	1·517	1·520	1·523	1·526	1·530	1·533	1·536	1·539	1·543	1·546	0	1	1	1	2	2	2	3	3
2·4	1·549	1·552	1·556	1·559	1·562	1·565	1·568	1·572	1·575	1·578	0	1	1	1	2	2	2	3	3
2·5	1·581	1·584	1·587	1·591	1·594	1·597	1·600	1·603	1·606	1·609	0	1	1	1	2	2	2	3	3
2·6	1·612	1·616	1·619	1·622	1·625	1·628	1·631	1·634	1·637	1·640	0	1	1	1	2	2	2	2	3
2·7	1·643	1·646	1·649	1·652	1·655	1·658	1·661	1·664	1·667	1·670	0	1	1	1	2	2	2	2	3
2·8	1·673	1·676	1·679	1·682	1·685	1·688	1·691	1·694	1·697	1·700	0	1	1	1	1	2	2	2	3
2·9	1·703	1·706	1·709	1·712	1·715	1·718	1·720	1·723	1·726	1·729	0	1	1	1	1	2	2	2	3
3·0	1·732	1·735	1·738	1·741	1·744	1·746	1·749	1·752	1·755	1·758	0	1	1	1	1	2	2	2	3
3·1	1·761	1·764	1·766	1·769	1·772	1·775	1·778	1·780	1·783	1·786	0	1	1	1	1	2	2	2	3
3·2	1·789	1·792	1·794	1·797	1·800	1·803	1·806	1·808	1·811	1·814	0	1	1	1	1	2	2	2	2
3·3	1·817	1·819	1·822	1·825	1·828	1·830	1·833	1·836	1·838	1·841	0	1	1	1	1	2	2	2	2
3·4	1·844	1·847	1·849	1·852	1·855	1·857	1·860	1·863	1·865	1·868	0	1	1	1	1	2	2	2	2
3·5	1·871	1·873	1·876	1·879	1·881	1·884	1·887	1·889	1·892	1·895	0	1	1	1	1	2	2	2	2
3·6	1·897	1·900	1·903	1·905	1·908	1·910	1·913	1·916	1·918	1·921	0	1	1	1	1	2	2	2	2
3·7	1·924	1·926	1·929	1·931	1·934	1·936	1·939	1·942	1·944	1·947	0	1	1	1	1	[illegible]	2	2	2
3·8	1·949	1·952	1·954	1·957	1·960	1·962	1·965	1·967	1·970	1·972	0	1	1	1	1	2	2	2	2
3·9	1·975	1·977	1·980	1·982	1·985	1·987	1·990	1·992	1·995	1·997	0	1	1	1	1	2	2	2	2
4·0	2·000	2·002	2·005	2·007	2·010	2·012	2·015	2·017	2·020	2·022	0	0	1	1	1	1	2	2	2
4·1	2·025	2·027	2·030	2·032	2·035	2·037	2·040	2·042	2·045	2·047	0	0	1	1	1	1	2	2	2
4·2	2·049	2·052	2·054	2·057	2·059	2·062	2·064	2·066	2·069	2·071	0	0	1	1	1	1	2	2	2
4·3	2·074	2·076	2·078	2·081	2·083	2·086	2·088	2·090	2·093	2·095	0	0	1	1	1	1	2	2	2
4·4	2·098	2·100	2·102	2·105	2·107	2·110	2·112	2·114	2·117	2·119	0	0	1	1	1	1	2	2	2
4·5	2·121	2·124	2·126	2·128	2·131	2·133	2·135	2·138	2·140	2·142	0	0	1	1	1	1	2	2	2
4·6	2·145	2·147	2·149	2·152	2·154	2·156	2·159	2·161	2·163	2·166	0	0	1	1	1	1	2	2	2
4·7	2·168	2·170	2·173	2·175	2·177	2·179	2·182	2·184	2·186	2·189	0	0	1	1	1	1	2	2	2
4·8	2·191	2·193	2·195	2·198	2·200	2·202	2·205	2·207	2·209	2·211	0	0	1	1	1	1	2	2	2
4·9	2·214	2·216	2·218	2·220	2·223	2·225	2·227	2·229	2·232	2·234	0	0	1	1	1	1	2	2	2
5·0	2·236	2·238	2·241	2·243	2·245	2·247	2·249	2·252	2·254	2·256	0	0	1	1	1	1	2	2	2
5·1	2·258	2·261	2·263	2·265	2·267	2·269	2·272	2·274	2·276	2·278	0	0	1	1	1	1	2	2	2
5·2	2·280	2·283	2·285	2·287	2·289	2·291	2·293	2·296	2·298	2·300	0	0	1	1	1	1	2	2	2
5·3	2·302	2·304	2·307	2·309	2·311	2·313	2·315	2·317	2·319	2·322	0	0	1	1	1	1	2	2	2
5·4	2·324	2·326	2·328	2·330	2·332	2·335	2·337	2·339	2·341	2·343	0	0	1	1	1	1	2	2	2

SQUARE ROOTS FROM 1 TO 10

	0	1	2	3	4	5	6	7	8	9	Mean Differences 1	2	3	4	5	6	7	8	9
55	2 345	2 347	2 349	2 352	2 354	2 356	2 358	2 360	2 362	2 364	0	0	1	1	1	1	1	2	2
56	2 366	2 369	2 371	2 373	2 375	2 377	2 379	2 381	2 383	2 385	0	0	1	1	1	1	1	2	2
57	2 387	2 390	2 392	2 394	2 396	2 398	2 400	2 402	2 404	2 406	0	0	1	1	1	1	1	2	2
58	2 408	2 410	2 412	2 415	2 417	2 419	2 421	2 423	2 425	2 427	0	0	1	1	1	1	1	2	2
59	2 429	2 431	2 433	2 435	2 437	2 439	2 441	2 443	2 445	2 447	0	0	1	1	1	1	1	2	2
60	2 449	2 452	2 454	2 456	2 458	2 460	2 462	2 464	2 466	2 468	0	0	1	1	1	1	1	2	2
61	2 470	2 472	2 474	2 476	2 478	2 480	2 482	2 484	2 486	2 488	0	0	1	1	1	1	1	2	2
62	2 4900	2 492	2 494	2 496	2 498	2 500	2 502	2 504	2 506	2 508	0	0	1	1	1	1	1	2	2
63	2 510	2 512	2 514	2 516	2 518	2 520	2 522	2 524	2 526	2 528	0	0	1	1	1	1	1	2	2
64	2 530	2 532	2 534	2 536	2 538	2 540	2 542	2 544	2 546	2 548	0	0	1	1	1	1	1	2	2
65	2 550	2 551	2 553	2 555	2 557	2 559	2 561	2 563	2 565	2 567	0	0	1	1	1	1	1	2	2
66	2 569	2 571	2 573	2 575	2 577	2 579	2 581	2 583	2 585	2 587	0	0	1	1	1	1	1	2	2
67	2 588	2 590	2 592	2 594	2 596	2 598	2 600	2 602	2 604	2 606	0	0	1	1	1	1	1	2	2
68	2 608	2 610	2 612	2 613	2 615	2 617	2 619	2 621	2 623	2 625	0	0	1	1	1	1	1	2	2
69	2 627	2 629	2 631	2 632	2 634	2 636	2 638	2 640	2 642	2 644	0	0	1	1	1	1	1	2	2
70	2 646	2 648	2 650	2 651	2 653	2 655	2 657	2 659	2 661	2 663	0	0	1	1	1	1	1	2	2
71	2 665	2 666	2 668	2 670	2 672	2 674	2 676	2 678	2 680	2 681	0	0	1	1	1	1	1	1	2
72	2 683	2 685	2 687	2 689	2 691	2 693	2 694	2 696	2 698	2 700	0	0	1	1	1	1	1	1	2
73	2 702	2 704	2 706	2 707	2 709	2 711	2 713	2 715	2 717	2 718	0	0	1	1	1	1	1	1	2
74	2 720	2 722	2 724	2 726	2 728	2 729	2 731	2 733	2 735	2 737	0	0	1	1	1	1	1	1	2
75	2 739	2 740	2 742	2 744	2 746	2 748	2 750	2 751	2 753	2 755	0	0	1	1	1	1	1	1	2
76	2 757	2 759	2 760	2 762	2 764	2 766	2 768	2 769	2 771	2 773	0	0	1	1	1	1	1	1	2
77	2 775	2 777	2 778	2 780	2 782	2 784	2 786	2 787	2 789	2 791	0	0	1	1	1	1	1	1	2
78	2 793	2 795	2 796	2 798	2 800	2 802	2 804	2 805	2 807	2 809	0	0	1	1	1	1	1	1	2
79	2 811	2 812	2 814	2 816	2 818	2 820	2 821	2 823	2 825	2 827	0	0	1	1	1	1	1	1	2
80	2 828	2 830	2 832	2 834	2 835	2 837	2 839	2 841	2 843	2 844	0	0	1	1	1	1	1	1	2
81	2 846	2 848	2 850	2 851	2 853	2 855	2 857	2 858	2 860	2 862	0	0	1	1	1	1	1	1	2
82	2 864	2 865	2 867	2 869	2 871	2 872	2 874	2 876	2 877	2 879	0	0	1	1	1	1	1	1	2
83	2 881	2 883	2 884	2 886	2 888	2 890	2 891	2 893	2 895	2 897	0	0	1	1	1	1	1	1	2
84	2 898	2 900	2 902	2 903	2 905	2 907	2 909	2 910	2 912	2 914	0	0	1	1	1	1	1	1	2
85	2 915	2 917	2 919	2 921	2 922	2 924	2 926	2 927	2 929	2 931	0	0	1	1	1	1	1	1	2
86	2 933	2 934	2 936	2 938	2 939	2 941	2 943	2 944	2 946	2 948	0	0	1	1	1	1	1	1	2
87	2 950	2 951	2 953	2 955	2 956	2 958	2 960	2 961	2 963	2 965	0	0	1	1	1	1	1	1	2
88	2 966	2 968	2 970	2 972	2 973	2 975	2 977	2 978	2 980	2 982	0	0	1	1	1	1	1	1	2
89	2 983	2 985	2 987	2 988	2 990	2 992	2 993	2 995	2 997	2 998	0	0	1	1	1	1	1	1	2
90	3 000	3 002	3 003	3 005	3 007	3 008	3 010	3 012	3 013	3 015	0	0	0	1	1	1	1	1	1
91	3 017	3 018	3 020	3 022	3 023	3 025	3 027	3 028	3 030	3 032	0	0	0	1	1	1	1	1	1
92	3 033	3 035	3 036	3 038	3 040	3 041	3 043	3 045	3 046	3 048	0	0	0	1	1	1	1	1	1
93	3 050	3 051	3 053	3 055	3 056	3 058	3 059	3 061	3 063	3 064	0	0	0	1	1	1	1	1	1
94	3 066	3 068	3 069	3 071	3 072	3 074	3 076	3 077	3 079	3 081	0	0	0	1	1	1	1	1	1
95	3 082	3 084	3 085	3 087	3 089	3 090	3 092	3 094	3 095	3 097	0	0	0	1	1	1	1	1	1
96	3 098	3 100	3 102	3 103	3 105	3 106	3 108	3 110	3 111	3 113	0	0	0	1	1	1	1	1	1
97	3 114	3 116	3 118	3 119	3 121	3 122	3 124	3 126	3 127	3 129	0	0	0	1	1	1	1	1	1
98	3 130	3 132	3 134	3 135	3 137	3 138	3 140	3 142	3 143	3 145	0	0	0	1	1	1	1	1	1
99	3 146	3 148	3 150	3 151	3 153	3 154	3 156	3 158	3 159	3 161	0	0	0	1	1	1	1	1	1

QUARE ROOTS FROM 10 TO 100

	0	1	2	3	4	5	6	7	8	9	Mean Differences 1	2	3	4	5	6	7	8	9
10	3 162	3 178	3 194	3 209	3 225	3 240	3 256	3 271	3 286	3 302	2	3	5	6	8	9	11	12	14
11	3 317	3 332	3 347	3 362	3 376	3 391	3 406	3 421	3 435	3 450	1	3	4	6	7	9	10	12	13
12	3 464	3 479	3 493	3 507	3 521	3 536	3 550	3 564	3 578	3 592	1	3	4	6	7	8	10	11	13
13	3 606	3 619	3 633	3 647	3 661	3 674	3 688	3 701	3 715	3 728	1	3	4	5	7	8	10	11	12
14	3 742	3 755	3 768	3 782	3 795	3 808	3 821	3 834	3 847	3 860	1	3	4	5	7	8	9	11	12
15	3 873	3 886	3 899	3 912	3 924	3 937	3 950	3 962	3 975	3 987	1	3	4	5	6	8	9	10	11
16	4 000	4 012	4 025	4 037	4 050	4 062	4 074	4 087	4 099	4 111	1	2	4	5	6	7	9	10	11
17	4 123	4 135	4 147	4 159	4 171	4 183	4 195	4 207	4 219	4 231	1	2	4	5	6	7	8	10	11
18	4 243	4 254	4 266	4 278	4 290	4 301	4 313	4 324	4 336	4 347	1	2	3	5	6	7	8	9	10
19	4 359	4 370	4 382	4 393	4 405	4 416	4 427	4 438	4 450	4 461	1	2	3	5	6	7	8	9	10
20	4 472	4 483	4 494	4 506	4 517	4 528	4 539	4 550	4 561	4 572	1	2	3	4	6	7	8	9	10
21	4 583	4 593	4 604	4 615	4 626	4 637	4 648	4 658	4 669	4 680	1	2	3	4	5	6	8	9	10
22	4 690	4 701	4 712	4 722	4 733	4 743	4 754	4 764	4 775	4 785	1	2	3	4	5	6	7	8	9
23	4 796	4 806	4 817	4 827	4 837	4 848	4 858	4 868	4 879	4 889	1	2	3	4	5	6	7	8	9
24	4 899	4 909	4 919	4 930	4 940	4 950	4 960	4 970	4 980	4 990	1	2	3	4	5	6	7	8	9
25	5 000	5 010	5 020	5 030	5 040	5 050	5 060	5 070	5 079	5 089	1	2	3	4	5	6	7	8	9
26	5 099	5 109	5 119	5 128	5 138	5 148	5 158	5 167	5 177	5 187	1	2	3	4	5	6	7	8	9
27	5 196	5 206	5 215	5 225	5 235	5 244	5 254	5 263	5 273	5 282	1	2	3	4	5	6	7	8	9
28	5 292	5 301	5 310	5 320	5 329	5 339	5 348	5 357	5 367	5 376	1	2	3	4	5	6	7	7	8
29	5 385	5 394	5 404	5 413	5 422	5 431	5 441	5 450	5 459	5 468	1	2	3	4	5	5	6	7	8
30	5 477	5 486	5 495	5 505	5 514	5 523	5 532	5 541	5 550	5 559	1	2	3	4	4	5	6	7	8
31	5 568	5 577	5 586	5 595	5 604	5 612	5 621	5 630	5 639	5 648	1	2	3	3	4	5	6	7	8
32	5 657	5 666	5 675	5 683	5 692	5 701	5 710	5 718	5 727	5 736	1	2	3	3	4	5	6	7	8
33	5 745	5 753	5 762	5 771	5 779	5 788	5 797	5 805	5 814	5 822	1	2	3	3	4	5	6	7	8
34	5 831	5 840	5 848	5 857	5 865	5 874	5 882	5 891	5 899	5 908	1	2	3	3	4	5	6	7	8
35	5 916	5 925	5 933	5 941	5 950	5 958	5 967	5 975	5 983	5 992	1	2	2	3	4	5	6	7	8
36	6 000	6 008	6 017	6 025	6 033	6 042	6 050	6 058	6 066	6 075	1	2	2	3	4	5	6	7	7
37	6 083	6 091	6 099	6 107	6 116	6 124	6 132	6 140	6 148	6 156	1	2	2	3	4	5	6	7	7
38	6 164	6 173	6 181	6 189	6 197	6 205	6 213	6 221	6 229	6 237	1	2	2	3	4	5	6	6	7
39	6 245	6 253	6 261	6 269	6 277	6 285	6 293	6 301	6 309	6 317	1	2	2	3	4	5	6	6	7
40	6 325	6 332	6 340	6 348	6 356	6 364	6 372	6 380	6 387	6 395	1	2	2	3	4	5	6	6	7
41	6 403	6 411	6 419	6 427	6 434	6 442	6 450	6 458	6 465	6 473	1	2	2	3	4	5	5	6	7
42	6 481	6 488	6 496	6 504	6 512	6 519	6 527	6 535	6 542	6 550	1	2	2	3	4	5	5	6	7
43	6 557	6 565	6 573	6 580	6 588	6 595	6 603	6 611	6 618	6 626	1	2	2	3	4	5	5	6	7
44	6 633	6 641	6 648	6 656	6 663	6 671	6 678	6 686	6 693	6 701	1	2	2	3	4	5	5	6	7
45	6 708	6 716	6 723	6 731	6 738	6 745	6 753	6 760	6 768	6 775	1	1	2	3	4	4	5	6	7
46	6 782	6 790	6 797	6 804	6 812	6 819	6 826	6 834	6 841	6 848	1	1	2	3	4	4	5	6	7
47	6 856	6 863	6 870	6 877	6 885	6 892	6 899	6 907	6 914	6 921	1	1	2	3	4	4	5	6	7
48	6 928	6 935	6 943	6 950	6 957	6 964	6 971	6 979	6 986	6 993	1	1	2	3	4	4	5	6	6
49	7 000	7 007	7 014	7 021	7 029	7 036	7 043	7 050	7 057	7 064	1	1	2	3	4	4	5	6	6
50	7 071	7 078	7 085	7 092	7 099	7 106	7 113	7 120	7 127	7 134	1	1	2	3	4	4	5	6	6
51	7 141	7 148	7 155	7 162	7 169	7 176	7 183	7 190	7 197	7 204	1	1	2	3	4	4	5	6	6
52	7 211	7 218	7 225	7 232	7 239	7 246	7 253	7 259	7 266	7 273	1	1	2	3	4	4	5	6	6
53	7 280	7 287	7 294	7 301	7 308	7 314	7 321	7 328	7 335	7 342	1	1	2	3	3	4	5	5	6
54	7 348	7 355	7 362	7 369	7 376	7 382	7 389	7 396	7 403	7 409	1	1	2	3	3	4	5	5	6

SQUARE ROOTS FROM 10 TO 100

	0	1	2	3	4	5	6	7	8	9	Mean Differences 1	2	3	4	5	6	7	8	9
55	7 416	7 423	7 430	7 436	7 443	7 450	7 457	7 463	7 470	7 477	1	1	2	3	3	4	5	5	6
56	7 483	7 490	7 497	7 503	7 510	7 517	7 523	7 530	7 537	7 543	1	1	2	3	3	4	5	5	6
57	7 550	7 556	7 563	7 570	7 576	7 583	7 589	7 596	7 603	7 609	1	1	2	3	3	4	5	5	6
58	7 616	7 622	7 629	7 635	7 642	7 649	7 655	7 662	7 668	7 675	1	1	2	3	3	4	5	5	6
59	7 681	7 688	7 694	7 701	7 707	7 714	7 720	7 727	7 733	7 740	1	1	2	3	3	4	4	5	6
60	7 746	7 752	7 759	7 765	7 772	7 778	7 785	7 791	7 797	7 804	1	1	2	3	3	4	4	5	6
61	7 810	7 817	7 823	7 829	7 836	7 842	7 849	7 855	7 861	7 868	1	1	2	3	3	4	4	5	6
62	7 874	7 880	7 887	7 893	7 899	7 906	7 912	7 918	7 925	7 931	1	1	2	3	3	4	4	5	6
63	7 937	7 944	7 950	7 956	7 962	7 969	7 975	7 981	7 987	7 994	1	1	2	3	3	4	4	5	6
64	8 000	8 006	8 012	8 019	8 025	8 031	8 037	8 044	8 050	8 056	1	1	2	2	3	4	4	5	6
65	8 062	8 068	8 075	8 081	8 087	8 093	8 099	8 106	8 112	8 118	1	1	2	2	3	4	4	5	6
66	8 124	8 130	8 136	8 142	8 149	8 155	8 161	8 167	8 173	8 179	1	1	2	2	3	4	4	5	5
67	8 185	8 191	8 198	8 204	8 210	8 216	8 222	8 228	8 234	8 240	1	1	2	2	3	4	4	5	5
68	8 246	8 252	8 258	8 264	8 270	8 276	8 283	8 289	8 295	8 301	1	1	2	2	3	4	4	5	5
69	8 307	8 313	8 319	8 325	8 331	8 337	8 343	8 349	8 355	8 361	1	1	2	2	3	4	4	5	5
70	8 367	8 373	8 379	8 385	8 390	8 396	8 402	8 408	8 414	8 420	1	1	2	2	3	4	4	5	5
71	8 426	8 432	8 438	8 444	8 450	8 456	8 462	8 468	8 473	8 479	1	1	2	2	3	4	4	5	5
72	8 485	8 491	8 497	8 503	8 509	8 515	8 521	8 526	8 532	8 538	1	1	2	2	3	3	4	5	5
73	8 544	8 550	8 556	8 562	8 567	8 573	8 579	8 585	8 591	8 597	1	1	2	2	3	3	4	5	5
74	8 602	8 608	8 614	8 620	8 626	8 631	8 637	8 643	8 649	8 654	1	1	2	2	3	3	4	5	5
75	8 660	8 666	8 672	8 678	8 683	8 689	8 695	8 701	8 706	8 712	1	1	2	2	3	3	4	5	5
76	8 718	8 724	8 729	8 735	8 742	8 746	8 752	8 758	8 764	8 769	1	1	2	2	3	3	4	5	5
77	8 775	8 781	8 786	8 792	8 798	8 803	8 809	8 815	8 820	8 826	1	1	2	2	3	3	4	4	5
78	8 832	8 837	8 843	8 849	8 854	8 860	8 866	8 871	8 877	8 883	1	1	2	2	3	3	4	4	5
79	8 888	8 894	8 899	8 905	8 911	8 916	8 922	8 927	8 933	8 939	1	1	2	2	3	3	4	4	5
80	8 944	8 950	8 955	8 961	8 967	8 972	8 978	8 983	8 989	8 994	1	1	2	2	3	3	4	4	5
81	9 000	9 006	9 011	9 017	9 022	9 028	9 033	9 039	9 044	9 050	1	1	2	2	3	3	4	4	5
82	9 055	9 061	9 066	9 072	9 077	9 083	9 088	9 094	9 099	9 105	1	1	2	2	3	3	4	4	5
83	9 110	9 116	9 121	9 127	9 132	9 138	9 143	9 149	9 154	9 160	1	1	2	2	3	3	4	4	5
84	9 165	9 171	9 176	9 182	9 187	9 192	9 198	9 203	9 209	9 214	1	1	2	2	3	3	4	4	5
85	9 220	9 225	9 230	9 236	9 241	9 247	9 252	9 257	9 263	9 268	1	1	2	2	3	3	4	4	5
86	9 274	9 279	9 284	9 290	9 295	9 301	9 306	9 311	9 317	9 322	1	1	2	2	3	3	4	4	5
87	9 327	9 333	9 338	9 343	9 349	9 354	9 359	9 365	9 370	9 375	1	1	2	2	3	3	4	4	5
88	9 381	9 386	9 391	9 397	9 402	9 407	9 413	9 418	9 423	9 429	1	1	2	2	3	3	4	4	5
89	9 434	9 439	9 445	9 450	9 455	9 460	9 466	9 471	9 476	9 482	1	1	2	2	3	3	4	4	5
90	9 487	9 492	9 497	9 503	9 508	9 513	9 518	9 524	9 529	9 534	1	1	2	2	3	3	4	4	5
91	9 539	9 545	9 550	9 555	9 560	9 566	9 571	9 576	9 581	9 586	1	1	2	2	3	3	4	4	5
92	9 592	9 597	9 602	9 607	9 612	9 618	9 623	9 628	9 633	9 638	1	1	2	2	3	3	4	4	5
93	9 644	9 649	9 654	9 659	9 664	9 670	9 675	9 680	9 685	9 690	1	1	2	2	3	3	4	4	5
94	9 695	9 701	9 706	9 711	9 716	9 721	9 726	9 731	9 737	9 742	1	1	2	2	3	3	4	4	5
95	9 747	9 752	9 757	9 762	9 767	9 772	9 778	9 783	9 788	9 793	1	1	2	2	3	3	4	4	5
96	9 798	9 803	9 808	9 813	9 818	9 823	9 829	9 834	9 839	9 844	1	1	2	2	3	3	4	4	5
97	9 849	9 854	9 859	9 864	9 869	9 874	9 879	9 884	9 889	9 894	1	1	1	2	3	3	4	4	5
98	9 899	9 905	9 910	9 915	9 920	9 925	9 930	9 935	9 940	9 945	0	1	1	2	2	3	3	4	4
99	9 950	9 955	9 960	9 965	9 970	9 975	9 980	9 985	9 990	9 995	0	1	1	2	2	3	3	4	4